When is the Long Run? - Historical Time and Adjustment Periods in Demand-led Growth Models

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Abstract

In recent years, Post-Keynesian models of growth and distribution have substantially shifted their focus from short to long-run analysis. While many authors have focused on the convergence of demand-led growth models to a fully-adjusted equilibrium, relatively little attention has been given to the time required to reach this long-run position. In order to fill the gap, this paper seeks to answer the question of *how long is the long run* in demand-led growth models. By making use of numerical integration, it analyses the time of adjustment from one steady-state to the other in two well-known heterodox models of growth and distribution: the Sraffian Supermultiplier and the fully-adjusted version of the Neo-Kaleckian model. The results show that the adjustment period is generally beyond an economically meaningful time span, suggesting that researchers and policy makers ought to pay more attention to the models' predictions during the traverse rather than focusing on steady-state positions.

Keywords: Neo-Kaleckian model; Sraffian Supermultiplier; time; adjustment period; traverse; effective demand; growth

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"If we throw away information about the time dimension, we are reducing still further our limited understanding of the relationship between these models and the real world."

Atkinson (1971, p.137)

1 Introduction

This paper takes Joan Robinson seriously. In her famous 1980 article, Robinson claimed that "to construct models that cannot be applied is merely an idle amusement" (p. 223-224). Yet, the construction of any supposedly realistic model cannot abstain from the consideration that historical time - rather than logical time - rules reality. Accordingly, it is "a common error to confuse a comparison of static positions with a movement between them" (*ibid.*, p. 228). This paper is chiefly interested in the duration of the movement between steady-state positions in demand-led growth models.

In recent years, Post-Keynesian models of growth and distribution have substantially shifted their focus from the short run - or from "chain*(s)* of short-period situations" (Kalecki, 1971, p. 165) - to long-run modeling. While Post-Keynesian growth theory benefited from this shift, gaining more rigor and coherency, more fundamental questions were often overlooked; in particular, few or no academic discussions can be found as regards the essential question of "what is the long run", how we define it and how long it actually is. In other terms, discussions about the nature and duration of the traverse were effectively marginalized, in the effort to provide more theoretically compelling modeling techniques of the growth process.

This contribution seeks to shed light on a dormant debate on traverse analysis and the persistence of out-of-equilibrium dynamics, thus recovering and deepening Joan Robinson's insights on the differences between logical and historical time in economic analysis. Accordingly, the main research goal is to analyze the time of adjustment in two leading demand-led models, the Sraffian Supermultiplier Model and the long-run version of the Neo-Kaleckian model presented by Lavoie (2016)[1]. More specifically, in accordance with the line of research pioneered by

[1] Some words on the rationale behind the choice of the two models are in order. First, the models are the two most widespread demand-led models, thus allowing to summarize the compatibilities and divergences of Kaleckian and Sraffian insights on growth in a relatively simple way. Second, both models reach a fully

Sato (1963, 1980), Sato (1966) and Atkinson (1971), the paper adopts the method of numerical integration to solve the systems of differential equations regulating the dynamics of the two models. In order to do that, we calibrate both models in line with the existing theoretical and empirical literature.

The paper is organized as follows. Section 2 presents the two models under scrutiny, i.e. the Sraffian Supermultiplier model and the long-run version of the Neo-Kaleckian model presented by Lavoie (2016). Section 3 discusses the adopted parameter calibration, then presenting the numerical solution of the two models and our main findings. Last, Section 4 concludes, discussing the implications of the results for model building and policy analysis.

2 Sraffian and Kaleckian Long-run Growth Models

This Section provides a synthetic review of the models under scrutiny. A more in-dept discussion of the Sraffian Supermultiplier model (Subsection 2.1), the economic intuitions upon which it relies and its implications can be found in Serrano (1995*b*); Serrano and Freitas (2017); Girardi and Pariboni (2016); Gallo (2019). As regards the long-run version of the Neo-Kaleckian model (Subsection 2.2) with autonomous demand and Harrodian dynamics, see Lavoie (2016); Allain (2015).

In order to make the Supermultiplier and Neo-Kaleckian frameworks fully comparable, the two models are presented for an open economy with government activity. Moreover, we include a coefficient of linear depreciation of physical capital. All relevant variables will be considered net of depreciation.[2]

2.1 The Sraffian Supermultiplier Model

Following Serrano and Freitas (2017), this Subsection presents the Sraffian Supermultiplier model assuming a closed economy without government activity. The model can be represented as a 3-equation in 3 variables - autonomous demand growth g_t^Z, the investment share h_t and

adjusted position - equaling the actual and normal rate of capacity utilization in the long run - thus preventing the emergence of the second Harrodian problem.

[2] For the derivation of variables from levels to growth rates, see Appendix A. The list of variables used in the paper is reported in Appendix B, while a list and description of parameters can be found in Table 1 below.

the rate of capacity utilization u_t:

$$g_t^Y = g_t^Z + \frac{h_t \gamma (u_t - u_n)}{s + m - h_t} \tag{1}$$

$$g_t^K = \frac{h_t u_t}{v} - \delta \tag{2}$$

$$g_t^Z = \overline{g^Z} \tag{3}$$

Equation (35) describes the evolution of economic activity as depending on autonomous demand growth g_t^Z plus an additional proportional rate of growth of output resulting from the supermultiplier when capacity utilization is not at its normal degree u_n, i.e. the second term of the equation. Moreover, s indicates the "tax-adjusted marginal propensity to save" (Girardi and Pariboni, 2015, p.526) and γ is "a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of the actual degree of capacity utilization" (Serrano and Freitas, 2017, p.74). Assuming a constant capital-capacity ratio v, the evolution of capital accumulation is given by the rate of growth of capacity output minus the depreciation rate (δ), as in Equation (2).[3] Lastly, Equation (3) constitutes the closure of the model for an exogenously given rate of growth of autonomous demand $\overline{g^Z}$.

The model settles in its long-run steady state when the fully-adjusted position (Vianello, 1985) is reached, i.e. $u_t = u_n$ and actual output and capital grow at the same pace - $g_t^Y = g_t^K$. Therefore, the long-run equilibrium position of the model is characterized by:

$$h^* = \frac{v}{u_n}(\overline{g^Z} + \delta) \tag{4}$$

$$u^* = u_n \tag{5}$$

$$g^{Z*} = \overline{g^Z} \tag{6}$$

Accordingly, in the long run all growth rates ought to equal the exogenous expansion of autonomous components of demand, i.e $g^* = g^{K*} = g^{Y*} = \overline{g^Z}$.

Let us now analyze more-in-depth the process of economic growth and out-of-equilibrium

[3] Under the assumption of fully-induced investment, it ought to be noted that this is a mere accounting identity, as showed in Appendix A.

dynamics. The adjustment to the long-run equilibrium is carried out by the two endogenous variables of the system, i.e. the rate of capacity utilization u_t and the investment share h_t. In line with Serrano and Freitas (2017), the two adjustment mechanisms[4] are modeled as follows:

$$\dot{u} = u_t(g_t^Y - g_t^K) \tag{7}$$

$$\dot{h} = h_t\gamma\left(u_t - u_n\right) \tag{8}$$

Substituting Equation (35 and 2) into Equation (7), we obtain the system of two first-order non-linear differential equations that will be solved numerically in Section (3):

$$\begin{cases} \dot{u} = & u_t\left[g_t^Z + \dfrac{h_t\gamma\left(u_t - u_n\right)}{s + m - h_t} - \dfrac{h_t}{v}u_t + \delta\right] \\ \dot{h} = & h_t\gamma\left(u_t - u_n\right) \end{cases} \tag{9}$$

Summarizing, discrepancies between actual and normal degrees of capacity utilization can only be of transient nature, producing growth effects in the short but not in the long run, in which the fully-adjusted position is reached.[5] More specifically, during the adjustment process when $u_t \gtreqless u_n$, it follows that $g_t^K \gtreqless g_t^Y \gtreqless \overline{g^Z}$. The rest of the paper will discuss whether the transiency of these effects is economically meaningful.

2.2 The Long-run Neo-Kaleckian Model with Autonomous Expenditures and a Harrodian Mechanism

The Lavoie (2016) long-run version of the Neo-Kaleckian model can be presented as the following 3-equations system in 3 variables - autonomous demand growth g_t^Z, animal spirits α_t and the

[4] Henceforth, changes of a variable over time will be denoted with the dot symbol, e.g. $\dot{u} = du/dt$.
[5] For a discussion of the stability of the system, see Appendix A.

autonomous demand-capital ratio z_t:

$$g_t^I = \alpha_t + \beta(u_t - u_n) \tag{10}$$

$$g_t^S = \frac{(s+m)u_t}{v} - z_t \tag{11}$$

$$g_t^Z = \overline{g^Z} \tag{12}$$

Equation (10) constitutes the conventional version of the Neo-Kaleckian investment function with a normal rate of capacity utilization. The term α_t captures animal spirits, which along with z, vary in the long run to prevent the emergence of the second Harrodian problem, as we will see later.[6] Equation (11) represents the saving function proposed by Lavoie (2016) in line with Serrano (1995*a*,*b*).[7] In other terms, it incorporates in the Neo-Kaleckian model a "non-proportional saving function with a constant term that in the long run grows at an exogenously given rate" (Lavoie, 2016, p.173).

In the short run, animal spirits α and the autonomous demand-capital ratio z are assumed to be constant. Accordingly the *ex-post* equality of the growth rates of investment and saving yields the following short-run equilibrium rate of growth of investment and saving:

$$g_{sr}^{I*} = g_{sr}^{S*} = \alpha + \beta(u_{sr}^* - u_n) \tag{13}$$

Solving for the short-run goods market equilibrium of $g_t^S = g_t^I$, it follows that the short-run rate of capacity utilization is equal to:

$$u_{sr}^* = \frac{(\alpha + z - \beta u_n)v}{s + m - \beta v} \tag{14}$$

If not by a fluke, the short-run rate of capacity utilization u_{SR}^* - that brings about the goods market equilibrium - will generally diverge from its long-run value u_n. More specifically, short-run discrepancies between the actual and normal rates of capacity utilization are given

[6] In order to make the two models fully compatibles, a small amendment to Lavoie (2016) is introduced, including a linear depreciation rate of the capital stock. The novelty does not alter significantly the long-run equilibrium results, as showed in Appendix A.

[7] See Appendix A for the derivation from levels to growth rates.

by:

$$u_{sr}^{*} - u_n = \frac{(\alpha + z)v - (s+m)u_n}{s+m-\beta v} \tag{15}$$

Consequently, the equilibrium accumulation rate in the short run is given by:

$$g_{sr}^{K*} = g_{sr}^{I*} - \delta = \alpha + \beta(u_{sr}^{*} - u_n) - \delta \tag{16}$$

However, during the traverse towards the long-run steady state, animal spirits α and the z ratio will vary, ensuring the long-run convergence of economic growth to autonomous demand growth ($g^{*} = g^{K*} = g^{Y*} = \overline{g^{Z}}$) and of the actual rate of capacity utilization towards its normal degree ($u = u_n$). Therefore, the long-run equilibrium position of the model is characterized by:

$$z^{*} = \frac{(s+m)u_n}{v} - \overline{g^{Z}} - \delta \tag{17}$$

$$\alpha^{*} = \overline{g^{Z}} + \delta \tag{18}$$

$$u^{*} = u_n \tag{19}$$

$$g^{Z*} = \overline{g^{Z}} \tag{20}$$

As mentioned above, the long-run adjustment process is carried out through changes in animal spirits and in the autonomous demand-capital ratio. More specifically, animal spirits react to discrepancies between the short-run equilibrium of the capacity utilization rate - i.e. the one that ensures the *ex-post* adjustment of saving to investment - and the normal degree. Furthermore, the z ratio adjusts to discrepancies between the exogenous growth rate of autonomous demand and the short-run equilibrium rate of economic growth[8]. Transforming Lavoie's (2016, p.178, 185) dynamic equations into differential ones, it follows that:

$$\dot{\alpha} = \alpha_t \mu (g_{sr}^{I*} - \alpha_t) = \alpha_t \beta \mu \left(u_{sr}^{*} - u_n\right) \tag{21}$$

$$\dot{z} = z_t \left(\overline{g^{Z}} - g_{sr}^{K*}\right) \tag{22}$$

Substituting Equations (15) and (16) into the above equations, we obtain the system of

[8] For the discussion of the derivation and the economic rationale of the two adjustments, see Lavoie (2016).

two first-order non-linear differential equations describing out-of-equilibrium dynamics in the Lavoie's proposal:

$$\begin{cases} \dot{\alpha} = & \alpha_t \beta \mu \left[\dfrac{(\alpha_t + z)v - (s+m)u_n}{s+m-\beta v} \right] \\ \dot{z} = & z_t \left[\overline{g^Z} - \alpha_t - \beta \left(\dfrac{(\alpha_t + z)v - (s+m)u_n}{s+m-\beta v} \right) + \delta \right] \end{cases} \tag{23}$$

As commented by Lavoie (2016, p.185-186), the system is dynamically stable "when there is short-run Keynesian stability as long as the effect of Harrodian instability is not overly strong", i.e. $s - \beta v > 0$ and $\mu < 1$ (as proven in Appendix A). Moreover, it is worth stressing that - similarly to the Supermultiplier model - "the growth rate of autonomous expenditures cannot be too large, for otherwise the share of autonomous consumption expenditures would need to be negative" (Lavoie, 2016, p.193). In our framework:

$$z^* > 0 \quad \Rightarrow \quad \overline{g^Z} < \frac{(s+m)u_n}{v} - \delta \tag{24}$$

3 Numerical Solution

Since an analytical solution to the two systems of differential equations cannot be found, the method of numerical integration is adopted. Accordingly, the first challenge is to provide a sound calibration of the models' structural parameters.

3.1 Parameter Calibration and Initial Values

Parameter values are set in accordance with the empirical evidences for the US economy in the post-war period, as well as in line with previous model calibrations.

As regards the annual growth rate of autonomous demand (g_z), the parameter value is obtained by averaging the empirical observation by Girardi and Pariboni (2016, p. 532) for the US economy. The capital-capacity ratio (v) and the normal rate of capacity utilization (u_n) are set in accordance to Setterfield and Budd (2011) and Skott and Ryoo (2008). As regards u_n, it is worth mentioning that the value matches the empirical evidences for other advanced capitalist economies, e.g. it is relatively close for the value (0.8104) estimated by Gallo (2019).[9]

The benchmark values of the propensities to save and to import are set in accordance to the empirical evidence in the US economy (Girardi and Pariboni, 2016). Moreover, it is worth stressing that these values are consistent with the empirical calibration of Fazzari et al. (2020), who set the imports and tax-adjusted propensity to consume equal to 0.5.

Choosing a value for the remaining sensitivities γ_u, β and μ is probably the most challenging task of the simulation exercise. are probably the challenging most difficult. In this version of paper, they are set in line with the existing literature Lavoie and Godley (2001); Skott and Ryoo (2008); however, as their values greatly influence the numerical solution of the two systems of differential equations under scrutiny, more attention should be given to them. Later versions of this paper will include sensitivity analysis and some robustness checks in order to assess how the numerical solution is affected by changes in the structural parameters.

The values assigned to the parameters in order to numerically solve our two systems of differential equations are summarized in Table (1).

Table 1: Parameter values

Par.	**Description**	**Value**	**Source**
g_z	Autonomous demand growth	0.0332	Author's calculation, based on Girardi and Pariboni (2016)
δ	Depreciation rate	0.084	Fazzari et al. (2020)
u_n	Normal rate of capacity utilization	0.8242	Setterfield and Budd (2011)
v	Capital-capacity ratio	0.94	Author's calculation, based on Fazzari et al. (2020)
s	Propensity to save	0.35	Author's calculation, based on Girardi and Pariboni (2016)
m	Propensity to import	0.15	Author's calculation, based on Girardi and Pariboni (2016)
γ_u	Sensitivity of the investment share to $u_t - u_n$	0.07	Author's calculation, based on Haluska et al. (2021)
β	Sensitivity of capital formation to $u_t - u_n$	0.05	Author's calculation, based on Lavoie and Godley (2001)
μ	Sensitivity of animal spirits to $u_t - u_n$	0.90	Author's calculation, based on Fazzari et al. (2020)

Source: author's calculation, various sources (see Appendix A)

Let us now discuss the rationale behind the choice of the initial conditions. Recalling that the main goal of the exercise it to show the persistence of a relatively small discrepancy

[9] Consistent with the models presented in Section 2, treating the normal degree of capacity as parametric implies that it is not affected by temporary changes in demand. For a more detailed critical discussion of the notion of normal capacity, the interested reader may refer to Ciccone (1986); Kurz (1986). For an empirical support of the idea that normal utilization is exogenous on the level of demand in the short run, see Haluska et al. (2021).

between the actual and normal rates of capacity utilization, we maintain animal spirits and the investment share at their long-run equilibrium level:

$$\alpha_0 = \alpha^* = \overline{g^Z} = 0.0332 \tag{25}$$

$$h_0 = h^* = \frac{(s+m)u_n}{v} - \overline{g^Z} = 0.1208 \tag{26}$$

Therefore, we start with a actual rate of capacity utilization 1% above its natural level, i.e. $u_0 = 1.01u_n = 0.8324$. As a consequence, the initial value of the z ratio will be given by $z_0 = (s+m)u_0/v - \overline{g^Z} = 0.1055$.

The initial conditions are summarized in Table (2).

Table 2: Initial conditions

Variable	**Description**	**Value**
u_0	Capacity utilization rate	0.8324
α_0	Animal spirits	0.1172
z_0	Autonomous demand-capital ratio	0.3254
h_0	Investment share	0.1336

Source: author's calculation

3.2 How Long is the Long Run?

This Section presents a graphical representation of the responses of the two models' endogenous variables to a relatively small divergence between the actual and the normal rate of capacity utilization. More specifically, the Section shows by means of numerical integration the behavior of the two systems to a 1% positive divergence between the actual rate of capacity utilization and the normal one (based on the values in Table 1 and 2).

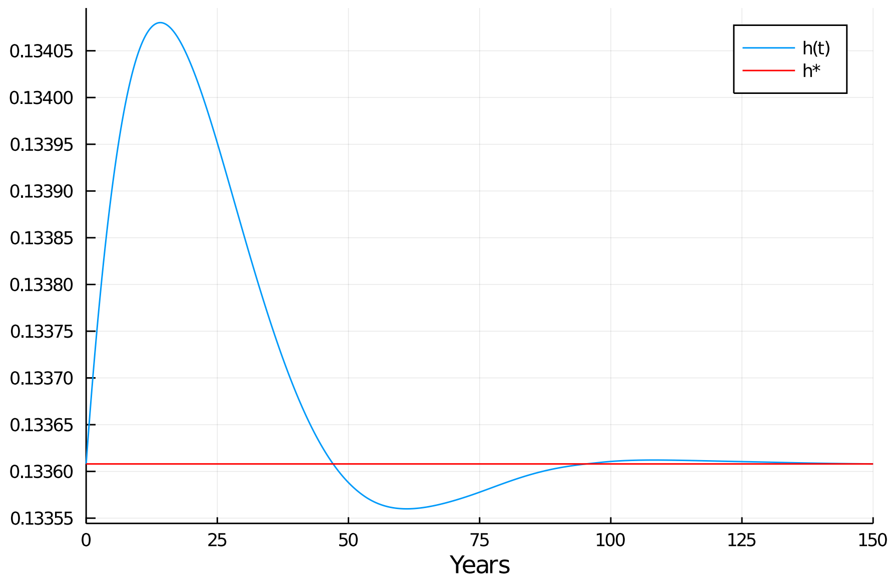
Before moving to the, an important consideration is in order. In the following Sections, I will focus on calendar time as in yearly frequency. The reader may well ask from where this comes from.

3.2.1 The Long Run in the Supermultiplier Model

As discussed in Subsection (2.1), the two adjusting variables of the Sraffian Supermultiplier model are the rate of capacity utilization and the investment share. In the long-run steady-state, they should come back, respectively, to the normal rate of capacity utilization and to an equilibrium h^* given by Equation (4). In our parametrization, $u_n = 0.8242$ and $h^* = 0.1572$ (see Section 3.1).

Figure (1) and (2) show the behavior of the two adjusting variables in the long run[10].

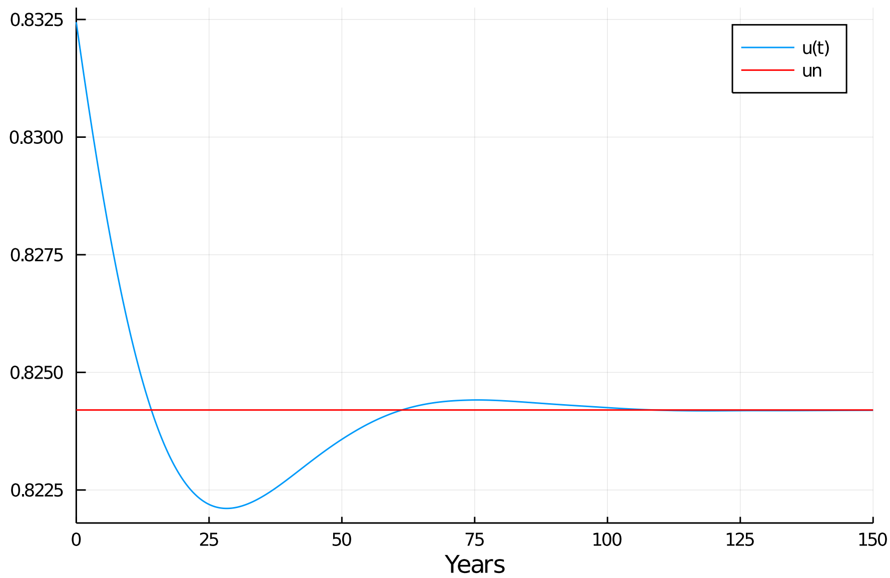
Figure 1: The long-run dynamic of the investment share



Source: authors' representation

[10] It is worth noting as Freitas and Serrano (2013, p. 41) report a graph that is very similar to the ones below. However, they express time as logical indexes $(t_0, t_1, ...)$ instead of historical time (months, quarters, years, etc.)

Figure 2: The long-run dynamic of the rate of capacity utilization



Source: authors' representation

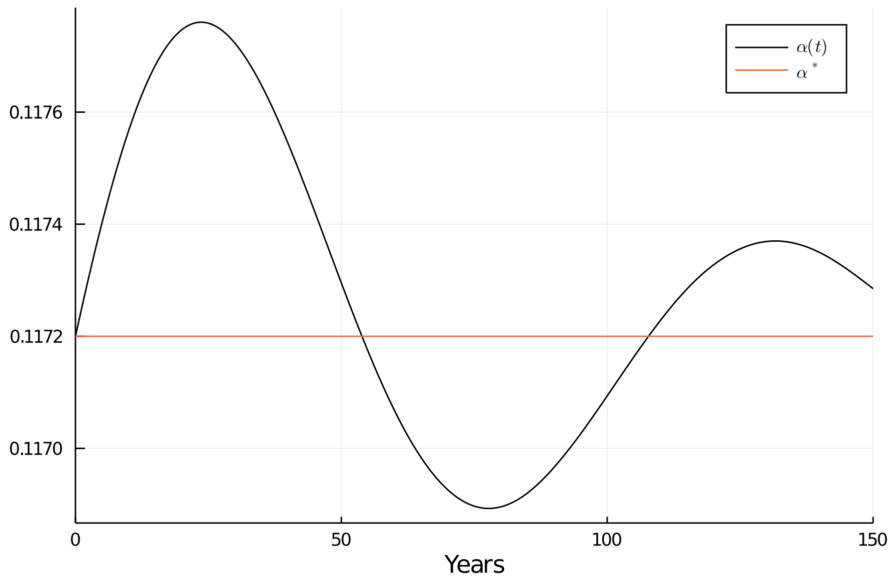
Given our parameter constellation, it takes 25 years for a 1% positive discrepancy between u and u_n to be reabsorbed (Figure 2). However, following this period, the model simulation predicts a long period of under-utilization of productive capacity (about 65 years). Only after about 150 years do the dynamics of the rate of capacity utilization and the investment share begin to stabilize around their steady-state values. In other terms, the simulation postulates that it takes a - generally speaking - very long period of time for the model to settle down in the fully-adjusted equilibrium.

3.2.2 The Long Run in the Amended Neo-Kaleckian Model

In the model presented in Subsection (2.2), the two adjusting variables are the autonomous demand-capital ratio z and the animal spirits proxy variable α. In the steady state, their values are given by Equations (17 and 18); in our numerical example, their value is 0.0308 and 0.0432, respectively.

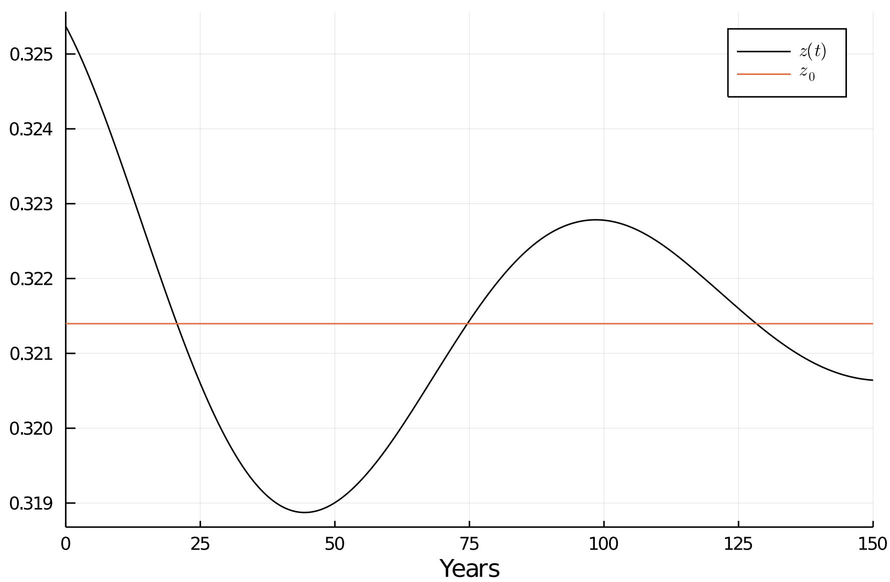
Figure (3) and (4) show the behavior of the two adjusting variables in the long run.

Figure 3: The long-run dynamic of animal spirits



Source: authors' representation

Figure 4: The long-run dynamic of the autonomous demand-capital ratio



Source: authors' representation

An initial 1% divergence between the actual and the normal rate of capacity utilization implies that the initial autonomous demand-capital ratio will be above its steady-state value (Equation 17). Accordingly, the animal spirits proxy variable α_t will increase via Equation (21), thus boosting accumulation. It takes about 15 years for z to approach the equilibrium value, followed by a long period in which $z_t < z^*$.

In a similar fashion as in the Supermultiplier model, it takes about 150 years for the model to stabilize around the steady-state equilibrium. As in the previous model, the simulation yields a similar result: the convergence to the fully-adjusted equilibrium is sluggish. We will see what this implies for the consistency of the theory and for policy analysis in the next conclusive Section.

4 Concluding remarks

The main result of the exercise presented in this paper is that *the long run may be longer than expected.*

As the simulations presented in the previous Section showed, the two models under scrutiny share a very slow pace of adjustment. In other terms, in historical time the adjustment period to the steady-state position may be long enough to be economically meaningless. If so, should we conclude that the long run is a misleading guide not only - with Keynes - to current affairs, but also to economic analysis *as such*? Are we to leave these models to the amusement of few researchers, neglecting the idea of relating them to the real world? Not at all. The main conclusion of the analysis conducted in the paper is that researchers should pay more attention to what happens while reaching a model's steady-state rather than focusing - as they more often do - on the steady-state itself. In other words, the examination of the models' timescale and adjustment period is a fundamental piece of information and a key factor for understanding the relation between the theoretical framework and the real world. Very rarely this information is exploited for economic analysis and policy recommendations, with researchers and policy makers finding themselves more at ease with thinking in logical rather than historical time.

Lastly, it is worth stressing that the goal of this exercise was not to quantify the actual duration of the traverse ("how long is the long run"), but first and foremost to shed light on issues and methods that have not received the deserved attention by growth theorists. On the methodological side, the results presented in the previous Section have been derived by making use of numerical methods of analysis to solve two systems of differential equations that cannot be solved analytically. The mathematical tool is well known by economists and growth theorists, but neglected for the analysis of the traverse and out-of-equilibrium dynamics.

Using more thoroughly these methods and thinking more carefully about the issue of the traverse in steady-state models may result in a significant gain of explanatory power of the models used for the analysis.

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A Appendix: Derivation of the models, Stability and Equilibrium

Open Economy with Government Activity

Let us start from the output equation of an open economy with government activity:

$$Y_t = C_t + I_t + G_t + (X_t - M_t) \tag{27}$$

where the current level of aggregate output (Y_t) is defined as the sum of aggregate consumption (C_t), private investment (I_t), public expenditures (G_t) and net exports ($X_t - M_t$). Consumption, government spending, exports and imports can be modelled as follows:

$$C_t = C_{Yt} + \overline{C_{0t}} = c(1-t)Y_t + \overline{C_{0t}} \tag{28}$$

$$G_t = \overline{G_t} \tag{29}$$

$$X_t = \overline{X_t} \tag{30}$$

$$M_t = mY_t \tag{31}$$

Equation (28) assumes that aggregate consumption is partly induced - via the tax-adjusted propensity to consume $c(1-t)$) - and partly autonomous from the current level of income ($\overline{C_{0t}}$). Autonomous consumption can be understood as 'that part of aggregate consumption financed by credit and, therefore, unrelated to the current level of output resulting from firms' production decisions' (Freitas and Serrano, 2015, p.4). Government spending (Equation 29) and exports (Equation 30) are both treated as autonomous, the first because public consumption and investment depend on the arbitrary decisions of the general government, the second because exports does not depend on the level of national income, but on that of the rest on the world. For the sake of simplicity, imports of goods and services are assumed to be linearly dependent of the level of income, via the propensity to import m (Equation 31).

The modelling choice regarding aggregate investment is what effectively constitutes the main difference between the Sraffian Supermultiplier and the Neo-Kaleckian model, as showed below.

Sraffian Supermultiplier Model

According to the baseline Supermultiplier model, private investment is treated as fully induced (Equation 32), reflecting the simple idea that at the aggregate level firms will invest only as long as there is demand for their products. Therefore, I_t can be model *sic et simpliciter* as the product of the investment share (h_t) times national income.

$$I_t = h_t Y_t \tag{32}$$

Since $\dot{K}_t = I_t - \delta K_t$, the accumulation rate can be derived as follows:

$$g_t^K = \frac{\dot{K}_t}{K_t} - \delta = \frac{I_t}{K_t} - \delta = \frac{h_t Y_t}{K_t} - \delta = h_t \frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \delta = \frac{h_t u_t}{v} - \delta \tag{33}$$

where Y^p is full-capacity output. Let us now solve for the level of output, substituting Equations (28, 29, 30, 31 and 32) in Equation (27):

$$Y_t = \left(\frac{1}{s+m-h_t}\right)\left(\overline{C_{0t}} + \overline{G_t} + \overline{X_t}\right) = \left(\frac{1}{s+m-h_t}\right) Z_t = SM_t Z_t \tag{34}$$

where s denotes the tax-adjusted propensity to save, i.e. $s = 1 - c(1-t)$.

Differentiating Equation (34), we obtain the growth rate of output as the sum of the growth rate of autonomous demand and of the supermultiplier, under the assumption that the investment share behaves in line with Equation (8):

$$g_t^Y = g_t^Z + \frac{h_t \gamma (u_t - u_n)}{s+m-h_t} \tag{35}$$

Lastly, the model closure is given by the assumption of an exogenously given growth rate of autonomous demand:

$$g_t^Z = \overline{g^Z} \tag{36}$$

Let us now analyse the stability of the fully-adjusted equilibrium, whose necessary and sufficient condition is that the determinant of the Jacobian's matrix evaluated at the equilibrium point with $u^* = u_n$ and $h^* = \frac{v}{u_n}(\overline{g^Z} + \delta)$ is positive and its trace is negative:

$$J^* = \begin{bmatrix} \left[\frac{\partial \dot{h}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{h}}{\partial u}\right]_{h^*,u^*} \\ \left[\frac{\partial \dot{u}}{\partial h}\right]_{h^*,u^*} & \left[\frac{\partial \dot{u}}{\partial u}\right]_{h^*,u^*} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\gamma v(\overline{g^Z}+\delta)}{u_n} \\ -\frac{u_n^2}{v} & (\overline{g^Z} + \delta)\left(\frac{\gamma v}{s+m-\frac{v}{u_n}(\overline{g^Z}+\delta)} - 1\right) \end{bmatrix} \tag{37}$$

$$det\ J^* = \gamma u_n(\overline{g^Z} + \delta) \tag{38}$$

$$Tr\ J^* = (\overline{g^Z} + \delta)\left(\frac{\gamma v}{s + m - \frac{v}{u_n}(\overline{g^Z} + \delta)} - 1\right) \tag{39}$$

Since γ, u_n and $\overline{g^Z}$ are assumed to be positive, the determinant is positive *per definitionem*. Similarly to Freitas and Serrano (2015), the stability condition boils down to the sign of the $Tr\ J^*$, which is ensured by the condition:

$$1 - s + m + \gamma v + \frac{v}{u_n}(\overline{g^Z} + \delta) < 1 \tag{40}$$

where $1 - s + m$ may also be interpreted as the tax and imports-adjusted propensity to spend. Equation (40) implies three conditions:

1. The value of the reaction parameter γ should be sufficiently low, implying that induced investment ought not to adjust capacity to demand too fast outside the fully-adjusted position (Freitas and Serrano, 2015). In other terms, the effect of Harrodian instability needs not to be overly strong;
2. The growth rate of autonomous demand $\overline{g^Z}$ cannot be too large;
3. The tax and imports-adjusted propensity to spend $(1 - s + m)$ needs not to be too large and it must be smaller than unity in the entire adjustment process.

Long-run Neo-Kaleckian Model

Neo-Kaleckian models treat capital formation as dependent on the rate of capacity utilization. More specifically, adopting the formulation proposed for the first time by Amadeo (1986), the investment rate will depend on the secular growth rate of sales (α_t) plus discrepancies between the actual and the normal or 'planned' (Steindl, 1952) utilization rates, via the parameter β:

$$I_t = [\alpha_t + \beta(u_t - u_n)]K_t \tag{41}$$

which - under the assumption of a linear depreciation coefficient - implies that the accumulation rate will be equal to:

$$g_t^K = g_t^I - \delta = \frac{I_t}{K_t} - \delta = \alpha_t + \beta(u_t - u_n) - \delta \tag{42}$$

The saving equation in levels is then given by:

$$S_t = Y_t - C_t - G_t - (X_t - M_t) = [1 - c(1-t) + m]Y_t - \left(\overline{C_{0t}} + \overline{G_t} + \overline{X_t}\right) = (s+m)Y_t - Z_t \tag{43}$$

Dividing everything by the capital stock and multiplying/dividing the first term on the right-hand side by full-capacity output, it follows that:

$$g_t^S = \frac{S_t}{K_t} = (s+m)\frac{Y_t}{Y^p}\frac{Y^p}{K_t} - \frac{Z_t}{K_t} = \frac{(s+m)u_t}{v} - z_t \tag{44}$$

Same as for the Supermultiplier model, the model is closed by the assumption of an exogenously given growth rate of autonomous demand - Equation (36) above.

Let us now evaluate the Jacobian matrix in the long-run fully-adjusted equilibrium $\alpha^* = \overline{g_t^Z} + \delta$ and $z^* = \frac{(s+m)u_n}{v} - \overline{g^Z} - \delta$:

$$J^* = \begin{bmatrix} \left[\frac{\partial \dot{\alpha}}{\partial \alpha}\right]_{\alpha^*,z^*} & \left[\frac{\partial \dot{\alpha}}{\partial z}\right]_{\alpha^*,z^*} \\ \left[\frac{\partial \dot{z}}{\partial \alpha}\right]_{\alpha^*,z^*} & \left[\frac{\partial \dot{z}}{\partial z}\right]_{\alpha^*,z^*} \end{bmatrix} = \begin{bmatrix} \frac{\beta v \mu(\overline{g^Z}+\delta)}{s+m-\beta v} & \frac{\beta v \mu(\overline{g^Z}+\delta)}{s+m-\beta v} \\ -\left[\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right]\left(1 + \frac{\beta v}{s+m-\beta v}\right) & \frac{-\beta v}{s+m-\beta v}\left[\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right] \end{bmatrix} \tag{45}$$

$$det\ J^* = \frac{\beta v \mu(\overline{g^Z} + \delta)}{s+m-\beta v}\left(\frac{(s+m)u_n}{v} - \overline{g^Z} - \delta\right) \tag{46}$$

$$Tr\ J^* = \frac{\beta v}{s+m-\beta v}\left[\overline{g^Z}(\mu+1) - \frac{(s+m)u_n}{v}\right] \tag{47}$$

Given that β and v are positive *per definitionem*, the determinant is positive whenever the

Keynesian stability condition holds ($\beta < (s+m)/v$) and the equilibrium autonomous demand-capital ratio z^* is positive, i.e. whenever $\overline{g^Z} < (s+m)u_n/v$). If the Keynesian stability condition holds, then it can be shown that the trace is negative whenever:

$$\mu < \frac{(s+m)u_n}{(\overline{g^Z}+\delta)v} - 1 \tag{48}$$

Taken together, the stability conditions of the long-run Neo-Kaleckian model imply that:

1. The value of the reaction parameter β should be sufficiently low, implying that the reaction of capital formation to discrepancies in utilization rates is not too strong;

2. The growth rate of autonomous demand $\overline{g^Z}$ cannot be too large;

3. Similarly to the Supermultiplier model, demand ought to adjust fairly slowly to capacity, i.e. the Harrodian mechanism need not to be overly strong (Equation 48);

B Appendix: Variables

α_t	Animal spirits (also, expected growth rate of sales)
h_t	Investment share (also, marginal propensity to invest)
g_t^I	Investment rate
g_t^K	Growth rate of the capital stock
g_t^S	Saving rate
g_t^Y	Growth rate of output
g_t^Z	Growth rate of autonomous demand
u_t	Capacity utilization rate
z_t	Autonomous demand-capital ratio

C Online Appendix: Sensitivity Analysis

The interested reader could easily perform a re-parameterization of the two models through the following interactive Web App - created with Shiny R: http://ettoregallo.shinyapps.io/When_is_the_long_run