A monetary theory of endogenous economic growth

Stefan Collignon[1]

Abstract

This paper presents a model where modern credit money is the engine of endogenous growth, because the claims for interest on debt generate liabilities that need to be matched by higher assets and increased income. The argument is based on the utility of money as a means of payment. It requires that interest rates are exogenously determined and not the endogenous equilibrium of saving and investment.

Keynes derived the interest rate from liquidity preference, which made it a function of the exogenous factors of uncertainty, risk averseness and monetary policy. Uncertainty is a fundamental state of not knowing that can be modelled as the (conditional) variance of income spending. Risk averseness reflects the curvature of the utility function of money, which implies that liquidity preference and interest rates are a function of money supply.

The balance between saving and investment is then accomplished by the endogenous adjustment of income spending and not by the interest rate. This implies that the system is driven by the credit market (which prices liquidity preference) to which the other markets (goods, services, and labour) must adjust. Borrowing and lending are determined by liquidity preference today, but they do not necessarily generate higher income tomorrow, although they affect tomorrow's income distribution. Borrowing for consumption purposes impoverishes the borrower in absolute terms but borrowing for investment purposes increases aggregate income so that the borrower is only impoverished in relative terms.

The system hinges on the solvency constraint which ensures that the growing liabilities from debt are serviced by higher income. In a framework of the neoclassical Solow growth model, it is shown that in the steady state (stable capital stock) the economic growth rate is exclusively determined by technological progress, which must equal the interest rate. In the steady state this implies that money supply must grow at the compound interest rate, even though the capital stock is stagnant.

The paper concludes with a re-interpretation of the Malthusian trap, which is conditioned by the inelastic money supply of bullion and coinage in ancient economies.

[1] Scuola Superiore SantAnna, Pisa, and London School of Economics

Money is the most important economic institution, but it plays hardly any role in theories of economic growth. Money is mostly considered to be neutral, meaning that a change in the level of nominal money does not affect real variables, or superneutral which means that the growth of money does not affect the rate of economic growth. In models that find non-neutrality of money, the main mechanism is the effect of inflation on the real interest rate (Orphanides and Solow 1990). In this paper, I suggest a model where money is the engine of growth in a modern credit economy even under conditions of price stability. This effect is obtained by deriving the utility of money from Keynes' liquidity preference. In the quantity theory of money this utility follows from the transaction motive alone, and the Sidrauski model of "money in the utility function" is a hybrid that still yields the inflation effect as the dominant cause of growth.

However, by deriving the utility of money from its function as a means of payment, the motives of holding money for transaction and for security purposes money can be kept separate, which implies that money is exchanged for real assets at the ratio of marginal utilities. That is how exchange in a market economy works.

From this perspective, economic growth depends on the rate at which transaction money increases, but transaction money is a function of liquidity preference and money supply. I interpret liquidity preference not as the ease of making transactions in markets, but as a fundamental psychological condition of seeking safety in an environment of uncertainty. The reason for holding liquid cash is that it protects against uncertainty.[2] Money gives cash holders the power of acquiring assets and making payments *when needed* – and that is uncertain. The utility of money, from which its value is derived, depends therefore not only on its purchasing power but also on the services it provides as a protection against uncertainty.

Keynes (1967 [1936], 166) famously pointed out that the protection and growth of wealth requires two distinct sets of decisions. The first is concerned with *how much of income* does a person wish to consume now versus saving for consumption in the future. The second is about the *form in which a person wishes to hold the command over future consumption* now. Is it in the form of immediate, liquid command, i.e., liquidity preference, or is the person prepared to part and lend money for a specified period? Keynes thought it was a mistake of accepted theories of the interest rate to derive the rate of interest from the first and to neglect the second.

Intertemporal choice models focus on the "how much?" allocation. They assume that a person who saves income today can lend this at a given interest rate and will therefore receive more income and consumption tomorrow. However, one person's lending is another person's borrowing and in aggregate they cancel out. This is the iron law of accounting: at any moment in time total assets equal total liabilities. Defaulting on this law causes the elimination from the market. In a closed economy, net financial wealth is zero. In a stationary economy, this implies that intertemporal choices simply establish distributional shares of income, but not growth in aggregate income.

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[2] Certainty is the highest degree of rational belief that events will turn out the way they are expected. See (Keynes 1973 [1921], 10; 137). When there is "distrust of our own calculations and conventions concerning the future" (see footnote 3), we have fundamental uncertainty, which is a subjective category, contrary to risk which has epistemic objectivity. For the distinction of risk and uncertainty, see (Knight 1921). Nevertheless, we can model (expected) uncertainty as the subjectively perceived probability distribution around an expected value.

The growth of aggregate income is commonly explained by capital accumulation, although the neoclassical Solow growth model clearly shows the limitations of this approach in the steady state. However, the desire for increasing wealth by capital accumulation is constrained by the desire for security. More is not always better. If we interpret the interest rate as the price for compensating insecurity and risk, liabilities grow at the rate that is exogenous to the system. I will show that it is the solvency constraint that then requires the growth in real assets to match the increasing liabilities caused by interest claims. Economic growth is the endogenous adjustment of assets that assures the solvency constraint.

In the rest of this paper, I will first derive the interest rate from liquidity preference. In the second section, I look at intertemporal effects of saving, lending, and borrowing. Section 3 establishes the warranted growth rate which satisfies the solvency constraint and shows that in the steady state this condition requires technological progress. Section 4 will draw the conclusions for the growth of money supply and monetary policy.

1. Liquidity preference and the utility of money

Neoclassical economics assumes that consumers care only about consumption and do not derive utility from money. Keynes disagreed. "Why, he asked, should anyone outside a lunatic asylum wish to use money as a store of wealth? Because, partly unreasonable and partly on instinctive grounds, our desire to hold money as a store of wealth is a barometer of the degree of our distrust of our own calculations and conventions concerning the future. (…) The possession of actual money lulls our disquietude; and the premium which we require to make us part with money is the measure of the degree of our disquietude. The significance of this characteristic of money has usually been overlooked."[3] I will therefore argue that money has utility which is derived from our preference to lull our disquietude, that is from liquidity preference.

The utility of money is easily perceived when we look at its function as a means of payment. As such money has purchasing power,[4] which means that money is known with certainty to be able, at any time, to discharge debt contracts and other monetary liabilities. There is a set of conditions which must be met for money to fulfil this function, such as scarcity, stable value, etc., that I will not discuss here. The point is that when they are met, money has liquidity value, which is its "advantage of immediacy" (Demsetz 1968). Note that the liquidity value of money is not in terms of commodities against which it is exchanged, but in terms of the utility which is derived from the *security and certainty that it can discharge liabilities.*

There are two kinds of liabilities: commercial liabilities result from purchases of commodities and do not carry interest obligations. Financial liabilities are created by debt contracts that carry interest. Debt contracts are promises of payments - on the spot or in the future. Money as means of payment serves two purposes. First, it enables people to buy commodities. This is the *transaction motive*. Second, by holding money in cash instead of spending it, people protect themselves against the

[3] (Keynes, The General Theory and After. Part II. Defence and Development 1973, 116)
[4] In philosophical terms we should say money has deontic powers, see (Searle 2010). Purchasing power can be *measured* by the inverse of the price level, but money's *purchasing power derives from the fact that it is accepted* as a means of payment.

disquietude generated by uncertainties. This is the precautionary or *liquidity motive*.[5] Because money serves both these purposes, its utility is broader than simply facilitating exchange. A given amount of money will provide utility by satisfying both motives and this utility generates demand for transaction and precautionary money. Transaction money determines economic growth but it is constrained by the supply of total money and the precautionary balances held as protection against uncertainty.

(1) $M = M^{Tr} + M^{L}$

(2) $\mathrm{U(M)} = \mathrm{U}(M^{Tr} + M^{L}) > 0; \quad U'(M) = \frac{\partial U(M)}{\partial M} > 0; \quad U''(M) = \frac{\partial^2 U(M)}{\partial M^2} < 0$

We first determine the demand for transaction money. All exchange involves giving up one good in exchange for another. The marginal rate of substitution between two goods is the price that measures the rate at which the consumer is just willing to substitute one good for the other. This rate is the slope of the indifference curve which reflects the relative marginal utilities of the two goods at which the consumer is indifferent between holding one or the other good. In a monetary economy this logic must apply to money as well.

At the micro-level, the price of a good is the marginal rate of substitution between the commodity and a specified amount of money, and at the margin it reflects the consumer's indifference between holding money or possessing the commodity. The marginal utility of money $U'(M)$ consists of the *certainty* that it *can* discharge additional liabilities at any time upon demand. That is the liquidity value of the monetary unit of account. A buyer evaluates the marginal utilities of commodities $\frac{\partial U(y)}{\partial y}$ in view of the ends to which they serve and that defines their exchange value. If the marginal utility of a commodity is higher than the marginal utility of a stipulated amount of money, a purchase will take place. Otherwise, the buyer will keep his money. We then write the price of a good i:

(3) $p_i = \frac{\partial m_i}{\partial y_i} = \frac{\partial U(y_i)}{\partial U(m_i)}$

If the price of the good is £5, it means that the marginal utility of this good is 5 times as high as the marginal utility of keeping £1.

Because the price is the amount of money per commodity $p_i = m_i/y_i$, we aggregate:

(4) $\Sigma p_i y_i = \Sigma m_i \;\leftrightarrow\; Py = Y = M^{Tr}$.

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[5] Keynes has distinguished between the transaction, precaution, and speculative motives for holding money. This can be described (Bofinger, Reischle und Schächter 1996) as an aggregative money demand function like

$$M^{dem} = PL(y,r) = M^{T}\left(\overset{+}{\tilde{Y}}\right) + M^{Prec}\left(\overset{+}{\tilde{Y}}\right) + M^{spec}(\overset{-}{\tilde{r}})$$

Where the plus and minus signs indicate the sign of the first derivative. Because the precautionary motive is about converting money into real assets in the context of uncertainty, it is often fused with the transaction motive, while the speculative motive of buying financial assets is inversely related to the expectation of future interest rate and capital gains. This model is appropriate for a modern economy with efficient financial markets, but in their absence, precautionary money follows a logic that is like speculative money.

which implies that on average the aggregate price level $P = \frac{M^{Tr}}{y}$ reflects all marginal prices.

Transaction money is the aggregate amount of money people plan or expect to spend. The output bought (y) at given prices (p) is the seller's income, so that the total amount of money spent is equal to aggregate income. The stock of transaction money is *proportional* to total expenditure, but to simplify the exposition of my argument I will assume that transaction money is *equal* to aggregate output times the price level, i.e., to aggregate income $(Y = Py = M^{Tr})$. This implies constant velocity of circulation equal to one.[6] To keep things simple, I will also assume price stability, so that money balances are equivalent to real money holdings.[7] Hence:

(5) $M^{Tr} = M(Y) = py$ with $\frac{\partial M^{Tr}}{\partial Y} = 1$, $\frac{\partial M^{Tr}}{\partial y} = p > 0$, $\frac{\partial M^{Tr}}{\partial p} = y > 0$

Equation (5) describes transaction money as the budget constraint of output. When income increases, transaction money M^{Tr}must increase as well (otherwise things do not get sold), and inversely, if money supply falls, income will fall, too.

In a carefree world, risk neutral people would spend all their money on buying goods and services. Yet, for all kinds of reasons people will not be able to buy all commodities when they wish to acquire them.[8] Although they have reasonable *expectations* about what money can buy, actual spending will vary due to stochastic shocks to prices and output. This is the lottery of a market economy and unfulfilled expectations generate disquietude and uncertainty. To protect themselves against this uncertainty, people will reduce their spending budget and hold precautionary liquid money balances M^L, that serve as an insurance against uncertainty. The aggregate amount of money people plan to hold (i.e., not spend) for security purposes is precautionary money. The higher the uncertainty, the more money they will hold back as cash.

We model the uncertainty as the error around the planned or *expected average value of spending on transactions.*

(6) $E(M^{Tr}) = E(Y) = \mu_Y + \varepsilon$

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[6] The Fisher equation, which underlies the quantity theory of money, is $py = M^T V$, where V is the velocity of circulation, i.e., the number of turnovers of the given supply of (transaction) money necessary to sell the output $V = \frac{Y}{M^{Tr}}$. Setting $V = 1$ yields $M^{Tr} = Y$. The alternative formulation of the Cambridge cash-balance equation is $k = \frac{M^{Tr}}{Y}$ comes to the same result, provided we refer only to transaction money balances. Of course, this is no longer the case if we admit that total money balances consist of transaction and precautionary money holding.

[7] It follows by analogy from equation (3) that the aggregate price level represents the ratio of the average utility of output relative to the average utility of transaction money. In other words, it describes the elasticity by which output reacts to a change in transaction money. The inverse of the price level measures the purchasing power of money. With price stability this elasticity and purchasing power are constant.

$$P = \frac{U'(y)}{U'(M^{Tr})} = \frac{U(y)/y}{U(M^{Tr})/M^{Tr}}$$

[8] "Given multilateral credit clearing, a complete set of markets ensuring all contingencies and complete honesty, there would be little need for money. In practise, of course, it is the lack of trust in the counterparties willingness and ability to make promised future payments that makes sellers of spot goods require immediate payment". (Goodhart 1989, 3).

$E(M^{Tr})$ is the amount of money required to carry out the budgeted spending plans in an uncertain world, with μ_Y as the average or planned spending budget. The error term ε has zero mean and the variance σ^2. Thus, our measure of uncertainty regarding people's capacity to buy assets is the error variance σ^2.[9] A high variance means a high probability that actual spending will deviate from budgets and consequently people will keep a larger precautionary balance to stay safe. If money supply is fixed, the budgeted spending on goods and services is negatively related to liquidity holding and there is a trade-off between precautionary M^L and planned transaction money M^{Tr}.

(7) $M^L = \bar{M} - E(M^{Tr}) \quad \rightarrow \quad dE(M^{Tr}) = d\bar{M} - dM^L$

The extent to which people will hold liquidity depends on uncertainty, but also on their risk averseness. Risk averseness implies that the utility function of money is concave, i.e., that it has diminishing marginal utility as in equation (2). Risk neutral persons intend to spend all their money on buying goods and services, but risk averse persons prefer to keep some liquidity as safety. This implies that for a risk averse person the utility of the planned budget is higher than the expected utility of average expenditure and budgeted spending is less than the total amount of money supplied.

(8) $U[E(M^{Tr}) > E[U(\mu)]$

Risk averseness reflects the curvature of the concave utility function. Different utility functions will yield different degrees of risk aversion. A steeper curvature of the utility function implies higher risk aversion. When risk averseness declines (the utility curve becomes flatter), and when insecurity diminishes, less precautionary money will be held, and more money will be spent on goods and services.[10] Risk averseness can be measured by the Arrow-Pratt coefficient θ in absolute and relative form.

(9) Absolut risk aversion: $\theta^{abs} = -\frac{U''(\bar{M})}{U'(\bar{M})} > 0$

Relative risk aversion relates the desire for security (i.e., liquidity preference) to the outstanding stock of money.

(10) Relative risk aversion: $\theta^{rel} = -\frac{U''(\bar{M})}{U'(\bar{M})/\bar{M}} = -\frac{U''(\bar{M})}{U'(\bar{M})}\bar{M} = \theta^{abs}\bar{M} > 0$

Because both coefficients are functions of the aggregate money stock, we can write (9) also as

(11) $\theta^{abs} = \rho(\bar{M})$ with $\rho'(\bar{M}) = \frac{\partial\theta^{abs}}{\partial\bar{M}}$

It can be shown that precautionary money holding is roughly proportional to absolute risk-aversion and the measure of uncertainty:[11]

[9] Strictly, the expected uncertainty is the conditional variance, but to keep things simple we use the notation σ^2 for the conditional variance.
[10] For risk neutral individuals, the utility curve is linear. Economists derive risk aversion from easily manageable mathematical functions, which are useful for intertemporal choice problems. They should be seen as approximations of abstract models to reality, but in the end, it is people's subjective perceptions that shape utility and not mathematics.
[11] See (Rao and Jelvis 2021?, chapter 5) for the math.

(12) $M^L = f[\rho(\bar{M}), \sigma^2] \approx \frac{1}{2}\rho(\bar{M})\sigma^2$

With

(13) $\frac{\partial M^L}{\partial \bar{M}} = \frac{1}{2}\rho'(\bar{M})\sigma^2$ and $\frac{\partial M^L}{\partial \sigma^2} = \frac{1}{2}\rho(\bar{M})$

The total change in precautionary money balances is

(14) $dM^L = \frac{\sigma^2}{2}\rho'(\bar{M})d\bar{M} + \frac{1}{2}\rho(\bar{M})d\sigma^2$

which implies that precautionary money holding will unequivocally increase with uncertainty (because $\rho(\bar{M}) > 0$), but the effect of a change in money supply depends on the sign of $\rho'(\bar{M})$. For the utility function of money, it seems reasonable to assume that absolute risk aversion will diminish as money supply increases because people will spend more money on transactions when their desire for security is satisfied. This means $\rho'(\bar{M}) < 0$.

Many textbooks relate risk aversion to non-monetary variables such as consumption and assume isoelastic or logarithmic utility functions, which yield elegant mathematical results. [12] However, a logarithmic utility function for money would yield constant relative risk aversion (CRRA) meaning that people will hold the same percentage of total money in the form of liquidity as money supply increases. This assumption would imply that increasing the supply of money would not affect people's liquidity preference. This does not seem to be reasonable as it would prohibit any form of monetary policy. I will therefore work with the assumption that $\rho'(\bar{M}) < 0$.[13]

From equation (6) and (7) we know that planned transaction money (spending on goods) will be reduced as precautionary money increases. An increase in money supply will raise spending and income (because $\rho'(\bar{M}) < 0$) but an increase in uncertainty will lower it.

(15) $dE(M^{Tr}) = \left(1 - \frac{\sigma^2}{2}\rho'(\bar{M})\right)d\bar{M} - \frac{1}{2}\rho(\bar{M})d\sigma^2$

The trade-off between precautionary and transaction money (equation 7) allows us to define the liquidity premium, which Keynes (1967 [1936], 226) described as the "amount (measured in terms of itself) which [people] are willing to pay for the potential convenience or security given by [the power of disposal over an asset]". Thus, the liquidity premium is the amount of money a person would charge for giving up liquidity. It is therefore also the compensation for taking on financial risk when making financial contracts (loans). Thus, while in commercial spot contracts (cash purchases) the compensation for giving up liquidity is the utility of commodities, in financial contracts it is the utility of obtaining more money in the future measured by the liquidity premium. A borrower obtains the

[12] The isoelastic utility function, which yields constant relative risk aversion (CRRA), is the "canonical form found in all graduate textbooks" (Aghion and Howitt 2009, 37). It has the form

$$U(x) = f(x) = \begin{cases} \frac{x^{1-\theta} - 1}{1-\theta}, & \theta \geq 0, \quad \theta \neq 1 \\ \ln(x), & \theta = 1 \end{cases}$$

[13] It could be argued, however, that when money is coinage made of bullion, as it was in ancient times, money supply depends on the trade balance which is closely correlated with transaction money, so that $\rho'(\bar{M}) = 0$. But of course, in ancient times there was no monetary policy.

advantage of immediacy that the borrowed loan provides and promises to pay the price for obtaining this liquidity by returning more than he has borrowed. The liquidity premium is then:

$$(16)\ \ l = \frac{M^L}{\bar{M}} \approx \frac{\frac{\sigma^2}{2}\rho(\bar{M})}{\bar{M}} \quad \text{with} \quad \frac{\partial l}{\partial M} = \frac{\frac{\sigma^2}{2}\rho'(\bar{M}) - \rho(\bar{M})}{\bar{M}^2} < 0, \qquad \frac{\partial l}{\partial \sigma^2} = \frac{1}{2}\frac{\rho(\bar{M})}{\bar{M}} > 0$$

The liquidity premium raises proportionally to uncertainty and the impact depends on risk aversion. An increase in money supply lowers the liquidity premium, but the effect becomes weaker as the quantity of money increases (this causes Keynes's famous liquidity trap). In the very long run, it may be reasonable to assume that that the liquidity premium remains constant, given that interest rates are stationary in long time series, but in the short run a change in money supply can compensate shocks caused by uncertainty and can therefore stabilise income.

The liquidity premium sets the conditions in the credit market. The interest rate is the price at which money can be borrowed. Making loans means giving up liquidity for a limited period, and the liquidity premium is the exogeneous component in the determination of the interest rate. However, in a monetary credit economy, the interest rate also compensates commercial banks for the cost of borrowing from the central bank (d) and for losses of specific assets (ρ) and other carrying costs (δ).

$$(17)\ r = l + d + \rho + \delta$$

Historically these components have played different roles. When coinage was money and there was no central bank, the discount rate d was zero, although carrying costs δ were substantial. Mediaeval usury laws prohibited charging the liquidity premium l, but they did not consider that charging for the potential risk of loss ρ and carrying costs δ was morally unacceptable and sinful.[14] Usury meant charging for liquidity, because it implied that the lender would receive more than had been given away, and this exchange of something for nothing contradicted the scholastic concept of "just price". The abolition of usury meant that the liquidity premium became the critical value for the determination of interest. The creation of credit money after the foundation of the Bank of England allowed more flexible money supply, which helped to reduce interest rates. However, the critical variable for lending was the liquidity premium. I will therefore continue as if the liquidity premium alone determined the interest rate, which implies that uncertainty will raise and money supply will lower the interest rate.

2. Saving and intertemporal exchange

We have now derived the interest rate in the context of liquidity preference in a spot economy. With this insight we turn to the issue of *how much* of income does a person wish to consume now versus saving for consumption in the future. In a simple intertemporal choice model, we have two time periods. There are N individuals who have an initial endowment of money $\bar{M}_t$. They all generate an equal amount of income Y_1 in the first period but, as we shall see, not necessarily in the second. However, at the end of the second period all income is consumed. We write Y_1 for income and C_1 for consumption in the first (today), and Y_2 and C_2 for the second period (tomorrow). In each period $t = 1,2$ the amount of transaction money is equal to expected income $\mathrm{M}_t^{Tr} = \mathrm{E(Y)}_t$, so that in accordance with (7) aggregate spending in each period is constrained by transaction money, i.e., by

[14] See (Kerridge 1988)

the difference between money supply and liquidity preference. Total planned spending on consumption over the two-period lifetime is then equal to planned expenditure on consumption:

(18) $(\mathrm{M}_1^{Tr} + \mathrm{M}_2^{Tr}) = (\mathrm{Y}_1 + \mathrm{Y}_2) = (C_1 + \mathrm{C}_2)$.

Saving is defined as the positive difference between income and consumption, a negative balance is borrowing. Because all income is consumed at the end of period 2, there are no savings left over. [15]

(19) $S_t = \mathrm{M}_t^{Tr} - \mathrm{C}_t = \mathrm{Y}_t - \mathrm{C}_t$
(20) $(\mathrm{Y}_1 + \mathrm{Y}_2) - (C_1 + \mathrm{C}_2) = S_1 + S_2 = 0$

We now distinguish two groups of people, lenders and borrowers, who have identical utility functions and generate initially equal shares of aggregate income. Saving S by one group is borrowing B by the other group. Borrowers spend the borrowed money today.[16] In a pre-capitalist economy, lenders' saving finance contemporary consumption of borrowers. In a capitalist economy they finance investment, which is the net addition to the capital stock: $S_1^L = B = I_1^B$ and is intended to generate income tomorrow. However, either way *aggregate transaction money is spent on aggregate output of the same period*. The lender reduces her consumption and transfers the saved part of her transaction money to the borrower who then uses this money to buy the saved part of her income. Thus, aggregate output (income) is absorbed ("consumed") by the transaction money of each period.

(21) $\mathrm{M}_1^{Tr,Le} = \mathrm{Y}_1^{Le} - S_1^{Le} = C_1^{Le}$ lender
(22) $\mathrm{M}_1^{Tr,B} = \mathrm{Y}_1^B + S_1^{Le} = \mathrm{Y}_1^B + B = \mathrm{Y}_1^B + I_1^B$ borrower

Hence:

(23) $S_1^{Le} - I_1^B = 0$ and
(24) $\mathrm{M}_1^{Tr} = \mathrm{M}_1^{Tr,Le} + M_1^{Tr,B} = \mathrm{E}(\mathrm{Y}_1)$

We also define the savings ratios:

(25) $s_1' = \frac{S_1^{Le}}{Y_1^{Le} + Y_1^B} = \frac{S_1}{Y_1}$ and $s_2' = \frac{S_2}{Y_2}$

History provides three models how the two periods are related. First, for an *autonomous household* with a given amount of income, there is no exchange and therefore no money.[17] Intertemporal transfer of wealth means, the household can decide how much of its real income it will save now to consume it tomorrow. If saving means not having dinner today,[18] this implies that the household will eat the leftovers tomorrow. This is a way of transferring income from period 1 to period 2, but aggregate income over the two periods remains unchanged. If there are carrying costs that

[15] For ease of presentation, I drop the expectation term. I also assume real income as a continuum of goods as in (Dornbusch, Fischer and Samuelson 1977), so that the non-consumed income of this period is not necessarily left over for consumption in the next period.
[16] I model the relation between households and non-financial corporations as described in the flow of funds literature. This is not an overlapping generations model because lenders and borrowers spent their income in the same period.
[17] This model also applies to non-monetary palatial societies, where reserve stocks were kept in the palaces. Examples are ancient Mesopotamia, Egypt, Crete, the Incas in Peru or Angkor in Cambodia.
[18] (Keynes 1967 [1936], 210)

depreciate the saved output, the rate of depreciation would be the discount rate, although I will ignore this case for now.

Second, in a *precapitalist exchange economy*, money exists in the form of coinage and people lend to and borrow from each other, but they do not charge interest. We may call this *charity lending*. The lender will consume less than the borrower. The borrower's additional consumption is the lender's saving. In period 2 the relation is inversed. The borrower will repay the lender by saving from his income in period 2. Without interest the logic is "I share my food with you today and you share yours tomorrow".[19] Charity lending works because the motivation for lending does not follow economic but moral criteria.

Third, in a *capitalist economy*, credit contracts carry interest obligations. The saver lends her savings S_1^{Le} to the borrower if she gets compensated for her liquidity preference. The borrower issues a bond that promises to pay back principal B and interest at the rate r in period 2. The lender "buys" the bond and therefore transfers her money to the borrower. In exchange, she receives the promise to get this money back *plus the interest*, and the borrower has a liability in period 2 that exceeds the value of the principal by the amount of interest: $B(1+r)$.

The credit contract distorts the equality of income in the model, because the lender will receive a larger amount of money and income in the future. She has given away her savings $S_1^{Le} = B$, but she receives back the borrowed money plus interest $B(1+r)$. The borrower is therefore enabled to spend more than his income today, while the saver will be able to spend *even more* tomorrow. Contrary to the previous case of charitable lending, in the capitalist economy the lender's spending capacity in period 2 is not only supplemented by her previous savings, but she receives additional money income in the form of interest from the debtor. Assuming aggregate income has not changed, the borrower must therefore reduce consumption more than if he had not spent the borrowed money in the previous period. Hence:

(26) $S_1^{Le} = B;\ S_2^B = (1+r)B$

From this we get the *solvency constraint*

(27) $S_1^{Le} = \frac{S_2^B}{1+r} = \frac{Y_2^B - C_2^B}{1+r}$

By referring to the definition of the saving ratios (25) and the solvency constraint (27) we get

(28) $s_2' Y_2 = (1+r) s_1' Y_1$

Which yields the intertemporal rate of transformation of present into future consumption

(29) $\frac{C_2}{C_1} = \frac{s_2' Y_2}{s_1' Y_1} = (1+r)$

[19] The prevalent form for providing subsistence security in Ancient Greece was through interest-free *eranos*-loans, which were seen as a contribution to the community. In the pre-monetary economy described by Homer, ἔρανος meant *meal to which each contributed his share, picnic.* See (Millett 1991, 39; 154-55).

Optimal consumption smoothing over the two periods implies that the marginal rate of substitution between C_1 and C_2, which is the ratio of the two respective marginal utilities, must be equal to the rate of transformation.

$$(30)\quad \frac{U'(C_2)}{U'(C_1)} = (1+r) \qquad \leftrightarrow \qquad U'(C_1) = \frac{U'(C_2)}{(1+r)}$$

Equation (30) is the Euler equation that characterises the first-order condition of intertemporal optimal choice which equates the marginal cost of giving up consumption today for the marginal benefits of receiving higher income and consumption tomorrow. It implies that the utility of present consumption must be equal to the discounted utility of future consumption, with the liquidity premium as the discount rate (assuming $r = l$).[20]

The solvency constraint in equation (27) makes clear that in the capitalist economy the borrower has two strategies to service his debt in period 2. He can reduce consumption or increase income. If the reduction in consumption hits the subsistence limit, this is not a sustainable strategy for poor people.[21]

3. The warranted growth rate and technological progress

We now give up the assumption that income is constant over time. How much does income have to grow to satisfy the solvency constraint? We define income growth as

$$(31)\quad \frac{Y_2}{Y_1} = 1 + g$$

Using equation (28) and (31) we have

$$(32)\quad s_2' = \frac{S_2}{Y_2} = \frac{(1+r)S_1}{Y_2} = \frac{(1+r)}{(1+g)} s_1' \leftrightarrow \frac{s_2'}{s_1'} \equiv \left(1 + \frac{\Delta s'}{s_1'}\right) = \frac{(1+r)}{(1+g)}$$

We define the growth rate warranted by the solvency constraint as the *warranted growth rate*:

$$(33)\quad g_w = \left(\frac{1}{1+\frac{\Delta s'}{s_1'}}\right)\left(r - \frac{\Delta s'}{s_1'}\right)$$

Thus, for constant saving rates $\frac{\Delta s'}{s_1'} = 0$ we get

$$(34)\quad g_w = r$$

Equation (34) tell us that, given the solvency constraint, the savings ratios can only remain constant if the economic growth rate is equal to the interest rate. Most economic growth models assume a constant saving rate and empirical evidence points to long run stationarity in this rate. Changes in the saving rate are then reflecting stochastic shocks, but its mean is constant. Hence, in the long run the warranted growth rate must equal the interest rate. However, *if the interest rate is exogenously*

[20] Because medieval usury laws did not permit charging the liquidity premium, the marginal utilities of consumption remained constant in pre-market economies.

[21] After the liberalisation of interest taking in the 16th century by Elizabeth I, Puritanism developed rapidly in England which focused on both increasing income and reducing consumption, which, as Max Weber (1958 [1922]) has argued, initiated modern capitalism.

determined, then the mean economic growth rate must endogenously adjust to this rate. Otherwise, the solvency constraint is violated and borrowers will default.

This raises the question what will ensure that income growth satisfies the solvency constraint. I will discuss that below. First, however, a remark on the controversial relation between the interest and the growth rate. According to capital market equilibrium theory, long-term real interest rates are determined by long-term trends in saving and investment. Real interest rates fall when there is an excess of saving over investment. Alternatively, a low real interest rate may reflect a lack of productive investment opportunities in the economy. However, as Keynes has shown, saving and investment depend on aggregate income, and it is income and not the interest rate that ensures the equality of saving and investment. In the long run, dynamic efficiency should ensure that the interest rate is equal to the growth rate.[22] Monetary policy is supposed to have only short-term effects but does not determine long run interest rates. This view is not compatible with the Keynesian view that the interest rate is exogenously determined by liquidity preference which reflects uncertainty. However, we must keep in mind that we have two exogenous factors determining the interest rate in the Keynesian model: one is uncertainty, the other is money supply which is controlled by monetary authorities.[23] Uncertainty pushes up the liquidity premium but increasing money supply will lower the interest rate because it reduces risk averseness, as we have seen above.

Now, what is required for actual growth to match the warranted rate? We can explain the growth rate of aggregate income by using a standard neoclassical growth model with technological change.[24] I do not wish to explain technological change as such, but simply show that if the interest rate is exogenously determined by uncertainty, technological progress will ensure that the solvency condition is satisfied even in the steady state.[25]

We start with a standard Cobb-Douglas production function. All variables are indexed on time t, although I do not show this explicitly to keep readability simple.

(35) $Y = (AL)^{1-\alpha} K^{\alpha}$

Where K is the capital stock, A is a productivity indicator and AL is the "effective supply of labour", which grows at the rate of population growth $\hat{n}$ and the growth rate of productivity $\hat{a}$. Technological *progress* $\frac{dA}{dt}/A = \hat{a}$ reflects all factors, such as technological improvements, better skills, more efficient institutions, etc., that allow increasing output without having to save and invest more or using more resources. Capital per efficiency unit is

(36) $k = \frac{K}{AL}$ with the growth rate $\hat{k} = \frac{dk/dt}{k}$

And output per efficiency unit is

[22] At least this is true for abstract models (Blanchard and Fisher 1989). Empirical verification is more difficult (Abel, et al. 1989).

[23] Because we model a closed economy, we can ignore balance of payment movements.

[24] See (Aghion and Howitt 2009, 27-29)

[25] Solow (2000, 119; 149) has pointed out that exogenous growth depends only on technological and demographic parameters, while "endogenous growth means output should be growing faster than the exogenous factors alone can make it grow". In my model, the exogeneous factors are the liquidity premium (uncertainty) and population growth, while capital accumulation is a parameter in the steady state.

(37) $\varphi = \frac{Y}{AL} = k^{\alpha}$

Saving raises the k-ratio, while depreciation lowers it. Investment is the net increase in capital with δ the rate of depreciation.

(38) $I = \frac{dK}{dt} = sY - \delta K.$

Because of diminishing marginal productivity of capital, growth in the number of efficiency units at the rate of $\hat{n} + \hat{a}$, causes k to fall. Therefore, the rate of change in k is

(39) $\frac{dk}{dt} = sk^{\alpha} - (\hat{n} + \hat{a} + \delta)k$

In the long run, k will approach a unique steady state value k^* and φ approaches the steady state $\varphi^* = (k^*)^{\alpha}$. Although output per efficiency unit does not grow in the long run, the same is not true for output per person.

(40) $\frac{Y}{L} = A\varphi = \mathrm{A}k^{\alpha}$

The growth rate of output per person is

(41) $g - \hat{n} = \hat{a} + \alpha\hat{k}$

Given that in the long run k approaches the steady state $k^*$where it no longer changes, the growth rate $\hat{k}$ tends to zero, so that in the steady state per capita income will grow at the exogeneous rate of technological change and total income grows at the rate of technical progress plus population growth.

Combining (34) with (41) we get the equilibrium condition

(42) $g_w = r = \hat{n} + \hat{a}$

In order to satisfy the solvency constraint, the economy must grow at the exogenously determined rate of interest which requires that a larger population generates the income necessary to service the debt and that technological progress will increase aggregate income. In a modern credit economy, firms that borrow money to invest are under pressure to increase their productivity to avoid going bankrupt, and this is a major driver of growth. Technological progress is the consequence of the monetary credit economy.

4. The supply of money and monetary policy

We have now established that in a credit economy the solvency constraint will force economic growth through technological progress. In the steady state, economic growth will equal the interest rate. The critical component determining the interest rate is the liquidity premium which reflects risk averseness and uncertainty, but varies with money supply. The growth of income is constrained by the amount of transaction money which reflects the difference between total money supply and the precautionary money holding. Thus, if we assume that prices, total money supply, uncertainty and risk averseness remain constant, income cannot grow at the warranted rate. The system is too rigidly constrained if the money supply is fixed.

This analysis offers a new interpretation of the mediaeval Malthusian equilibrium. Malthus thought that the growth of the means of production was linear, while population increased exponentially. Lack of food caused shrinking populations. It is often argued that this Malthusian trap was overcome by industrialisation, which - like technological progress - is treated as an exogenous *ad hoc* variable. Our model emphasises money. In the Middle Ages, the curse of "bullion famines" (Spufford 1988) determined the inelastic money supply. Bullion supply increased little, and countries without silver mines had to generate trade surpluses.[26] Mercantilist trade policies assumed the role of monetary policy. Transaction money varied largely with the uncertainty of stochastic shocks to the economy, mainly caused by weather and wars. Given that a large part of the population lived close to the subsistence level, the Malthusian equilibrium would emerge when an increase in precautionary money holding reduced transaction money and aggregate income, thereby pushing the poorest of society into deprivation, increasing mortality rates, and lowering population growth. Prohibiting extravagance and luxury of lenders was the purpose of sumptuary laws for centuries. In the long run, economic growth was stationary, but it fluctuated in the short and medium term. Only increasing income turned out to be a viable long run strategy consistent with population growth. If we abandoned the assumption of stable prices, an increase in uncertainty and liquidity preference would reduce demand for output and prices would fall. This made exports more competitive and foreign currency (bullion) would have flown into the country until the higher supply of money restored the previous price level. This was the famous specie-flow mechanism. However once prices had returned to their previous level, transaction money and aggregate income could no longer grow.

The prospect of economic growth changed when credit money became an accepted means of payment. The innovation was fractional deposit banking in England and the issue of bank notes by the Bank of England after 1694. It now became possible to expand money supply and generate the growth dynamics which characterise modern economies. The increase in credit and paper money allowed lowering the liquidity premium and therefore also precautionary money holding. Hence, transaction money was able to expand, and this generated additional spending and economic growth. However, when credit became money, the interest claim became a constitutive factor of the market economy. It meant that output had to grow at the compound rate of interest. Hence, modern credit was the engine of capitalism and monetary policy became the steering wheel that prevented capitalism from destroying itself.

[26] This redistribution of the existing stock of bullion was the core idea underlying the theories of mercantilism.

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