Structural Change and the Wage Share: a Two-Sector Kaleckian Model

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Abstract

In this paper, we look at structural change, and in particular at the shrinking size of manufacturing in favor of the service sector, as one additional source of decline in the wage share. To the purpose, we build on Dutt (1988) to develop a two-sector Kaleckian model of growth and distribution, where the economy consists of the service and manufacturing sectors. The service good is only used for consumption while the manufacturing good is used both for consumption and accumulation of the capital stock. We assume that structural change is exogenous as it arises from a shift in consumers' preferences. We show that, when mark-ups are relatively higher in the service sector, a shift in the sectoral composition of demand in favor of the service sector good generates a rise in the profit share. The unique (non-trivial) steady state is asymptotically stable.

Keywords: structural change, functional income distribution, manufacturing, service

JEL Classification: D33, E11, O14

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1 Introduction

At the onset of modern growth theory, Kaldor (1961) suggested that long-run stability of factors income shares is one of the main 'stylized facts' of market economies. Yet, recent contributions (Jayadev and Rodriguez , 2013; Karabarbounis and Neiman , 2014; OECD , 2015) have shown that the labor share has declined over the past three decades in both developed and developing countries. While the possibility of short- and medium-run fluctuations in factors shares has long been acknowledged (Bentolila and Saint-Paul , 2003; Young , 2004), the prolonged decline in labor share seems to point to either a long-run negative trend in the labor share or a shift to a lower steady state wage share as more plausible descriptions of the evidence.

Several explanations for such a trend have been put forward and investigated both from theoretical and empirical standpoints. Economists working within the neoclassical framework have emphasized the importance of the shape of production function and the nature of technical change in determining factors shares trends. As is well known, a unitary elasticity of substitution (σ) between capital and labor, that is a Cobb-Douglas production function, necessarily implies constant factors shares. There are two possibilities to obtain a fall in the labor share: either capital deepening (in efficiency terms) when labor and capital are substitutes ($\sigma > 1$), or a reduction in the capital-labor ratio when the elasticity of substitution is less than one. Piketty and Zucman (2014) and Karabarbounis and Neiman (2014) support the first mechanism; but while Piketty and Zucman (2014) attributes capital deepening to negative shocks to the (exogenous) growth rate, Karabarbounis and Neiman (2014) find its determinant in the decline of the price of investment goods relative to consumption goods. Acemoglu (2003) analyzes the second possibility in the context of induced technical change, though he only applies it to deviations from the stable steady state wage share.

Other researchers (Berthold et al., 2002; Bental and Demougin , 2010; Checchi and García-Peñalosa , 2010) have investigated the relation between changes in labor market institutions and the labor share trend. Checchi and García-Peñalosa (2010), in particular, show that a reduction in unions' bargaining power might have played a role in reducing the share of income accruing to workers.

Multiple elements of globalization have also been singled out as factors behind the falling labor share. They range from trade (Brock and Dobbelaere , 2006; Doan and Wan , 2017), to offshoring (Elsby et al. , 2013), to capital account openess (Jayadev , 2007).

Finally, economists working withing the Post-Keynesian tradition (Dünhaupt , 2017;

Stockhammer , 2017) have looked at the increasing size of the financial sector as an additional determinant of the decline in the wage share.

Relatively little attention, on the other hand, has been paid to the possible influence of structural change on functional income distribution. De Serres et al. (2001) show that changes in the sectoral structure of the economy help explaining the trend decline in the aggregate wage share observed in five European countries and in the US over the 1980s and 1990s. From a theoretical point of view, a recent paper by Alvarez-Cuadrado et al. (2018) explains the decline in the labor share in a two sector neoclassical growth model, where sectoral differences in productivities growth and factors' elasticities of substitution, and non-homotethic preferences produce an endogenous rise in the service sector relative to manufacturing. Their quantitative analysis shows that within-industries income shares dynamics rather than changes in the sectoral composition of output is mostly responsible for the fall in the aggregate wage share.

In this paper, we investigate the relation between structural change and functional income distribution within the Kaleckian theory of growth and distribution. We build on Dutt (1988) and Dutt (1990) to develop a two-sector Kaleckian model of growth and distribution, where the economy consists of the service and manufacturing sectors. The service good is only used for consumption while the manufacturing good is used both for consumption and the accumulation of capital stock. We assume that structural change is exogenous as it arises from shifts in consumers' preferences and in the saving rate. We study two versions of the model, with and without profit rates equalization across sector. Under both specifications we show that, when mark-ups are relatively higher in the service sector, a shift in the sectoral composition of demand in favor of the service sector generates a rise in the steady state profit share. The unique (non-trivial) steady state equilibrium is asymptotically stable. The crucial assumption that mark-ups are relatively higher in the service sector is motivated by recent empirical evidence. In particular, Alvarez-Cuadrado et al. (2018) analyze a set of seventeen industrialized countries between 1970 and 2007, and they show that the labor share in the manufacturing sector is consistently higher than in the service sector.

While seminal contributions by Dutt (1988, 1990); Park (1995); Dutt (1997); Lavoie and Ramirez-Gaston (1997); Franke (2000) laid the foundations of the two-sectors Keynesian-Kaleckian model growth and distribution, recent papers have generalized the model to investigate additional issues. Nishi (2018) analyzes the effects of introducing sectoral endogenous labor productivity growth on cyclical demand, growth and distribution. Fujita (2018) explores how changes in sectoral mark-ups affect sectoral and aggregate capacity utilization

and capital accumulation. Murakami (2018) studies the effect of sectoral interactions on business cycles in a Keynesian model, without focusing on income distribution. None of these recent contributions, however, consider the role that changes in demand composition may produce on income distribution.

The rest of the paper is organized as follows. Section 2 develops the model and states the theoretical results; Section 3 offer some concluding remarks while proofs of the propositions can be found in Section 4.

2 The Model

2.1 Production and technology

The economy consists of the service good (s) and the manufacturing good (m). Output in both sectors (X_i) is produced through a sector-specific Leontief production function:

$$X_i = min[u_i B_i K_i, A_i L_i], i = s, m \tag{1}$$

where B and A are capital and labor productivities, K is the capital stock, L is employment, and u is the degree of capacity utilization. We assume no depreciation of capital. Profit maximization ensures:

$$X_i = u_i B_i K_i = A_i L_i. \tag{2}$$

2.2 Society and preferences

There are two classes in society. Workers supply labor services and receive the wage rate w, uniform across sectors. They consume their whole income. Capitalists earn profits on the capital stock they own. Their propensity to save is $s > 0$.[1] Workers and capitalists share the same preferences, which are defined over the two goods. We assume that individual utility of agent j is:

$$U_j(c_s, c_m) = min[c_s, \alpha c_m],$$

where c_i is consumption of good i, and $\alpha > 0$. The fixed coefficient structure of prefer-

[1] We denote by s both the service sector and the saving rate. Given context no ambiguity should arise.

ences implies $c_s = \alpha c_m$. The same fixed proportion carries over to total demand:

$$C_s \equiv \sum_j c_s = \sum_j \alpha c_m = \alpha \sum_j c_m \equiv \alpha C_m. \tag{3}$$

2.3 Mark-up prices

In line with the original Kaleckian literature, we assume that firms set prices by charging a constant mark-up (z_i) over unit labor cost. Mark-ups are sector specific and our crucial hypothesis is that they are relatively higher in the service sector, as this sector is less open to competition. If we let p_i be the price of good i, and we choose the service sector good as the numerarie we have $p_s = 1 = (1 + z_s)w/A_s$ and $p_m \equiv p = (1 + z_m)w/A_m$, with $z_s > z_m$. Accordingly

$$w = \frac{A_s}{1 + z_s}, \tag{4}$$

$$p = \frac{1 + z_m}{1 + z_s} \frac{A_s}{A_m}. \tag{5}$$

2.4 Value added distribution

In each sector, value added is distributed as wages and profits to labor and capital employed in production. If we let r_i be the interest rate in sector i we have $p_i X_i = wL_i + r_i p_m K_i$, which, after using (2), (4), (5) and rearranging, yields

$$r_s = \frac{z_s}{1 + z_m} \frac{A_m}{A_s} u_s B_s, \tag{6}$$

and

$$r_m = \frac{z_m}{1 + z_m} u_m B_m. \tag{7}$$

2.5 Output uses

The service good is only used for consumption so that $X_s = C_s$. In what follows, it will be useful to distinguish consumption depending on its income source. We denote consumption out of wages as C_i^w, and consumption out of profits as C_i^π, so that

$$X_s = C_s = C_s^w + C_s^\pi. \tag{8}$$

Manufacturing output, on the contrary is used both for consumption and for investment (I) in the service and in the manufacturing sectors:

$$X_m = C_m + I. \tag{9}$$

2.6 Balanced growth under alternative closures

The discussion between Park (1995) and Dutt (1997) on the risk of overdetermination in the Kaleckian two sector growth model clarified that there are two possible consistent specifications of the model. In the first one, there is no sectoral capital mobility in the short run, so that K_s and K_m are given; we can specify sectoral growth rates, and profit rates will not be equalized unless by a fluke. The second version of the model assumes that the stock of capital moves between sectors to equate sectoral profit rates in the short run; in this framework, since the sectoral capital stocks are not state variables we can only specify the aggregate growth rate, rather than the sectoral ones.[2] We analyze the two specifications of the model in turn, and we show that the qualitative results on income distribution and structural change are independent of the model closure.

2.6.1 The model without profit rates equalization

Since workers do not save, the whole wage fund is spent as consumption out of wages. Using (3) and (2) we have

$$C_m^w + C_s^w = C_s^w/\alpha + C_s^w = w(L_s + L_m) = w\left(\frac{u_s B_s K_s}{A_s} + \frac{u_m B_m K_m}{A_m}\right).$$

Hence, factorizing C_s^w and substituting for the wage rate from (4) yields

$$C_s^w = \frac{\alpha}{1+\alpha}\frac{A_s}{1+z_s}\left(\frac{u_s B_s K_s}{A_s} + \frac{u_m B_m K_m}{A_m}\right) = \frac{\alpha}{1+\alpha}\frac{1}{1+z_s}\left(u_s B_s K_s + \gamma u_m B_m K_m\right), \tag{10}$$

where $\gamma \equiv A_s/A_m$.

On the other hand, capitalists' propensity to consume out of profits (Π) is $(1-s)$. Accordingly

[2] A variant of this version of the model assumes that profit rates equalization is a slow process. Sectoral capital stocks are given and sectoral investment depends on the profit rates differential. We explore this variant in the stability analysis.

$$C_m^\pi + C_s^\pi = C_s^\pi/\alpha + C_s^\pi = (1-s)\left(r_m p K_m + r_s p K_s\right),$$

which, using (5),(6) and (7) implies

$$C_s^\pi = \frac{\alpha}{1+\alpha}\frac{1-s}{1+z_s}\left(z_s u_s B_s K_s + z_m \gamma u_m B_m K_m\right).$$

Once we know consumption out of wages and profits in the service sector we can use equation (8) to find

$$X_s = \frac{\alpha}{1+\alpha}\frac{1}{1+z_s}\left(u_s B_s K_s(1+(1-s)z_s) + \gamma u_m B_m K_m(1+(1-s)z_m)\right).$$

Define $\delta \equiv K_s/K \in [0,1]$ as the share of the capital stock employed in the service sector, to be determined in equilibrium. Dividing both sides of the previous equation by K and rearranging yields

$$\delta u_s B_s = (1-\delta)u_m B_m \gamma \frac{\alpha(1+(1-s)z_m)}{1+z_s(1+\alpha s)} \equiv (1-\delta)u_m B_m \gamma \Gamma(\alpha, s). \tag{11}$$

It is easy to show that Γ is a positive function of α and negative of s, when $z_s > z_m$. Let us now turn to the equilibrium in the manufacturing sector. If we let g_i be the growth rate of sector i, under the assumption of no sectoral capital mobility, equation (9) becomes $X_m = C_s/\alpha + g_m K_m + g_s K_s = X_s/\alpha + g_m K_m + g_s K_s$, where we used (3). Using factors demand found in (2), and dividing both sides by K, the previous condition becomes

$$u_m B_m(1-\delta) = u_s B_s \delta/\alpha + g_m(1-\delta) + g_s \delta. \tag{12}$$

The Kaleckian tradition posits that investment depends on utilization of capacity as a measure of aggregate demand. In our case, the actual growth rate of capital in each sector (g_i) is a function of the sector's degree of capacity utilization:

$$g_m = g_m(u_m), \tag{13}$$

and

$$g_s = g_s(u_s). \tag{14}$$

Finally, balanced growth requires that sectoral growth rates be equalized

$$g_m = g_s. \tag{15}$$

We have a consistent system of five equations, (11),(12),(13),(14) and (15), in the five unknowns $\delta, u_m, u_s, g_m, g_s$. Our focus is on income distribution. The profit share π is the ratio between the value of total profits and value added. We can use (2), (5), (6), (7), (11) to calculate its equilibrium value

$$\pi^* = \frac{r_s p K_s + r_m p K_m}{X_s + p X_m} = p \frac{r_s \delta + r_m (1-\delta)}{\delta u_s B_s + (1-\delta) p u_m B_m} =$$

$$\frac{p}{1+z_m} \frac{u_s \delta/\gamma + z_m u_m (1-\delta)}{\delta u_s B_s + (1-\delta) p u_m B_m} = \frac{z_s \Gamma(\alpha, s) + z_m}{(1+z_s)\Gamma(\alpha, s) + (1+z_m)}. \tag{16}$$

Inspection of (16) shows that π^* is economically meaningful being bounded between zero and one. It is a function of the sectoral mark-ups, consumers' preferences between the two consumption goods, and the saving rate.

We are now in a position to state:

Proposition 1. *an increase in consumption demand of the service good relative to the manufacturing good (a rise in α) raises the equilibrium profit share.*

Proof. See the Appendix. □

Proposition 2. *a decrease in the saving rate raises the equilibrium profit share.*

Proof. See the Appendix. □

In both Proposition 1 and 2 the rise in the profit share follows the increase in Γ due to shocks to α and s. In order to understand the economic meaning of an increase in Γ we can re-write equation (11) as $X_s/(X_m \gamma) = L_s/L_m = \Gamma(\alpha, s)$. When Γ rises, employment in the service sector rises relative to the manufacturing one. Given labor producitivities and sectoral mark-ups, the change in relative employment carries over into relative sectoral value added. Therefore, the increase in the profit share depends on the change in the composition of production in favor of the sector with higher mark-up, which can be caused either by a change in consumers' preferences or by a reduction in the saving rate. Contrary to the standard one sector Kaleckian growth model, the profit share depends on savings.

2.6.2 The model with profit rates equalization

In the second version of the model, sectoral capital stocks are not state variables since capital adjusts in the short run to ensure profit rates equalization. Accordingly, there are no sectoral growth rates and we need to replace equation (12) with

$$u_m B_m (1-\delta) = u_s B_s \delta/\alpha + g. \tag{17}$$

Equation (11) is not affected by the new closure, whereas we need to drop (13) and (14) and replace them with a single equation for the growth rate of capital. We assume it depends on the degree of capacity utilization in both sectors:

$$g = g(u_{s,} u_m). \tag{18}$$

Next, we impose the equalization of profit rates across sectors, so that $r_s = r_u$. Using (6) and (7), the equalization yields:

$$u_s = \frac{A_s}{A_m}\frac{z_m}{z_s}\frac{B_m}{B_s}u_m. \tag{19}$$

We now have a consistent system of four equations, (11),(17),(18) and (19), in the four unknowns δ, u_m, u_s, g. In particular, use (19) into (11) to find

$$\delta^* = \frac{\Gamma(\alpha, s)}{\Gamma(\alpha, s) + z_m/z_s}. \tag{20}$$

Let us now turn to the profit share:

$$\pi^* = \frac{r_s p K_s + r_m p K_m}{X_s + p X_m} = \frac{rp}{\delta u_s B_s + (1-\delta) p u_m B_m} =$$

$$= \frac{z_m}{(1-\delta)\left((1+z_s)\Gamma(\alpha, s) + 1 + z_m\right)} = \frac{z_s \Gamma(\alpha, s) + z_m}{(1+z_s)\Gamma(\alpha, s) + (1+z_m)}, \tag{21}$$

where we used the equalization of profit rates, (2), (5), (6), (7), (11), and (20). Equations (13) and (21) show that the final expression for the profit share is the same irrespective of the model closure; therefore, a shift of consumers' preferences in favor of the service sector and a decrease in the saving rate bring about an increase in the profit share, whether we assume profit rates equalization or not.

2.7 Stability

We now turn to the stability analysis of the balanced growth path. In the model with profit rate equalization, however, the adjustment to the balanced growth equilibrium is instantaneous and there is no transitional dynamics. In order to introduce a dynamic adjustment in this version of the model, we assume that sectoral profit rates are different in the short run, but changes in sectoral investment bring about profit rates equalization in the long run. This is the process known as 'classical competition'. After this modification, the dynamics of the economy in both models is described by the slow adjustment in the the allocation of capital between sectors. To the purpose, we derive a differential equation for δ, the share of capital employed in the service sector. Given the definition of δ, taking time derivative and rearranging yields

$$\dot{\delta} = \delta(1-\delta)(g_s - g_m). \tag{22}$$

2.7.1 The model without profit rate equalization

In order to study the dynamic behavior of δ, we start by assuming explicit functional forms for sectoral growth rates. Equations (13) and (14) become

$$g_s = \vartheta_0 + \vartheta_1 u_s \tag{23}$$

and

$$g_m = \beta_0 + \beta_1 u_m. \tag{24}$$

We can use the two previous equations together with (11) and (12) to solve for utilization rates as functions of δ :

$$u_s(\delta) = \frac{\gamma\Gamma(\alpha, s)}{B_s \Theta}\left(\beta_0 \frac{1-\delta}{\delta} + \vartheta_0\right) \tag{25}$$

and

$$u_m(\delta) = \left(\beta_0 + \vartheta_0 \frac{\delta}{1-\delta}\right)/\Theta, \tag{26}$$

where $\Theta = [1 - (\beta_1 + \gamma\Gamma(\alpha, s)(\vartheta_1/B_s + 1/\alpha))]$. Notice that economically meaningful (positive) solutions for u_m and u_s require $\Theta > 0$, that is $(\beta_1 + \gamma\Gamma(\alpha, s)(\vartheta_1/B_s + 1/\alpha)) < 1$.

This condition is the equivalent of the standard Keynesian 'stability' condition in one-sector Kaleckain growth models, which states that investment need be less responsive than saving to economic activity. β_1 and ϑ_1 represent how sectoral invesment reacts to capacity utilization; the role of saving is captured by $\Gamma(\alpha, s)$, which is a negative function of the saving rate.

We can rewrite (22) as

$$\dot{\delta} = \delta(1-\delta)\left[g_s(\delta) - g_m(\delta)\right],$$

and state

Proposition 3. *The system has two locally unstable trivial steady states at $\delta = 0$ and $\delta = 1$. The system has one non-trivial steady state δ^*, which is asymptotically stable for $\delta \in (0, 1)$.*

Proof. see the Appendix. □

Proposition 3 shows that if the intial condition of the system is such that both sector exist, the economy will converge towards the non- trivial steady state. If, on the other hand, the economy consists of only one sector at the beginning of time, the two-sector structure will never appear. Notice, however, that $\delta = 1$ does not have economic meaning because there cannot be accumulation of capital without production of the manufacturing good. When $\delta = 0$, we are back to the standard one-sector model, where the only output is used for both consumption and investment.

2.7.2 The model with profit rates equalization

In order to introduce a dynamic adjustment in this version of the model, we assume that profit rates equalization is not instantaneous. Sectoral profit rates are different in the short run, but changes in sectoral investment bring about profit rates equalization in the long run. We follow Dutt (1997) in assuming that the difference in sectoral growth rates depends on the profit rates differential

$$g_s - g_m = \lambda(r_s - r_m), \lambda > 0. \tag{27}$$

On the other hand, firms choose the total rate of invesment based on the average degree of capacity utilization in the economy $\bar{u}$. Assuming a linear form for the investment function we have:

$$g = g(\bar{u}) = \mu_0 + \mu_1 \bar{u}, \tag{28}$$

where

$$\bar{u} = \frac{X_s + pX_m}{pK} = \frac{1+z_m}{1+z_s}\frac{u_s \delta B_s}{\gamma} + u_m(1-\delta)B_m.$$

We can now use (11), (17), (6), (7) and (28) to solve for sectoral profit rates as functions of δ :

$$r_s(\delta) = \frac{z_s}{1+z_m}\frac{1}{\gamma}B_s u_s(\delta) = \frac{z_s}{1+z_m}\frac{\mu_0}{\Psi}\frac{1}{\delta}, \tag{29}$$

and

$$r_m(\delta) = \frac{z_m}{1+z_m}B_m u_m(\delta) = \frac{z_m}{1+z_m}\frac{\mu_0}{\Psi}\frac{1}{1-\delta}, \tag{30}$$

where $\Psi = \left[1 - \gamma\Gamma(\alpha,s)/\alpha - \mu_1\left(1 + \frac{1+z_m}{1+z_s}\Gamma(\alpha,s)\right)\right]$. Economically meaningful (positive) solutions for r_m and r_s require $\Psi > 0$. Similarly to the previous case, we can interpret it as the equivalent of the standard Keynesian 'stability' condition in one-sector Kaleckian growth models. Using the latest results and (27) in (22) we find

$$\dot{\delta} = \lambda\delta(1-\delta)\left[r_s(\delta) - r_m(\delta).\right] \tag{31}$$

We can state

Proposition 4. *The system has two locally unstable trivial steady states at $\delta = 0$ and $\delta = 1$. The non trivial steady state $\delta^* = \frac{\Gamma(\alpha,s)}{\Gamma(\alpha,s)+z_m/z_s}$ is asymptotically stable over $\delta \in (0,1)$.*

Proof. See the Appendix.

Similarly to the comparative dynamics results found in Proposition 1 and 2, the comparison between Proposition 3 and 4 show that the stability properties of the model are independent of the model closure. □

3 Conclusions

Evidence on the process of structural change shows that the share of the service sector in the total economy tends to rise as countries become richer (Herrendorf et al. , 2014). Since, as documented in Alvarez-Cuadrado et al. (2018), the wage share in the service sector is relatively low, growth and structural tranformation in mature economies necessarily bring about a reduction in the aggregate wage share, absent mitigating factors. As a consequence, changes in the composition of output and employment across sectors should be taken into ac-

count when investigating the ongoing negative trend in the wage share in most industrialized countries.

This paper has adopted the Kaleckian two-sector growth model of growth and distribution to analyze this process. We have shown that there is one (non trivial) unique asymptotically stable balanced growth path. The steady state functional income distribution depends on the composition of output, which, in turn, changes with consumers' preferences over the two consumption goods and with the saving rate. When the size of the service sector rises the profit share increases.

4 Appendix

4.1 Proof of Proposition 1

$$\frac{d\pi^*}{d\alpha} = \frac{(z_s - z_m)\Gamma'_\alpha(\alpha, s)}{((1+z_s)\Gamma(\alpha, s) + (1+z_m)\gamma)^2} > 0,$$

since $z_s > z$ and $\Gamma'_\alpha(\alpha, s) = \frac{\alpha(1+(1-s)z_m)(1+z_s)}{(1+z_s(1+\alpha s))^2} > 0$.

4.2 Proof of Proposition 2

$$\frac{d\pi^*}{ds} = \frac{(z_s - z_m)\Gamma'_s(\alpha, s)}{((1+z_s)\Gamma(\alpha, s) + (1+z_m)\gamma)^2} < 0,$$

since $z_s > z$ and $\Gamma'_s(\alpha, s) = \frac{-\alpha(z_m+\alpha z_s+z_m z_s(1+\alpha))}{(1+z_s(1+\alpha s))^2} < 0$.

4.3 Proof of Proposition 3

Let us start with the two trivial steady state. Inspection of (22) shows that $\dot{\delta} = 0$ at $\delta = 0$, and $\delta = 1$. Turning to stability, we have $\frac{\dot{d\delta}}{d\delta} = (1-\delta)\left[g_s(\delta) - g_m(\delta)\right] - \delta\left[g_s(\delta) - g_m(\delta)\right] + \delta(1-\delta)\left[g'_s(\delta) - g'_m(\delta)\right]$. At $\delta = 0$, $g_s(0)$ is not defined but $lim_{\delta\to 0}\frac{\dot{d\delta}}{d\delta} = lim_{\delta\to 0}\left[g_s(\delta) - g_m(\delta)\right] \to \infty > 0$, so that the first trivial steady state is locally unstable. At $\delta = 1$, $g_m(1)$ is not defined but $lim_{\delta\to 1}\frac{\dot{d\delta}}{d\delta} = lim_{\delta\to 1} - \left[g_s(\delta) - g_m(\delta)\right] \to \infty > 0$, so that the second trivial steady state is locally unstable.Let us now to turn to prove the existence and stability of the non-trivial steady state. Plug 25 and (26) into (22) to find

$$\begin{aligned}\dot{\delta} &= \delta(1-\delta)\left[\vartheta_0 + \vartheta_1\frac{\gamma\Gamma(\alpha,s)}{B_s\Theta}\left(\beta_0\frac{1-\delta}{\delta} + \vartheta_0\right) - \beta_0 - \beta_1\left(\beta_0 + \vartheta_0\frac{\delta}{1-\delta}\right)/\Theta\right] \\ &= \delta(1-\delta)\left[\vartheta_0 - \beta_0 + \vartheta_1\frac{\gamma\Gamma(\alpha,s)}{B_s\Theta}\vartheta_0 - \beta_1\beta_0/\Theta\right] + (1-\delta)^2\vartheta_1\frac{\gamma\Gamma(\alpha,s)}{B_s\Theta}\beta_0 - \delta^2\beta_1\vartheta_0/\Theta\end{aligned}$$

$= \delta^2(K_2 - K_1 - K_3) + \delta(K_1 - 2K_2) + K_2$, where $K_1 = \vartheta_0 - \beta_0 + \vartheta_1 \frac{\gamma\Gamma(\alpha,s)}{B_s\Theta}\vartheta_0 - \beta_1\beta_0/\Theta$, $K_2 = \vartheta_1 \frac{\gamma\Gamma(\alpha,s)}{B_s\Theta}\beta_0$, and $K_3 = \beta_1\vartheta_0/\Theta$. Therefore $\dot{\delta}(\delta)$ is a quadratic function. As a first step, notice that $\dot{\delta}(0) = K_2 > 0$, and $\dot{\delta}(1) = -K_3 < 0$. Since $\dot{\delta}$ is continuous, over the domain $\delta \in [0,1]$, it must cross the horizontal axis from above at least once in order to move from positive to negative values, according to Bolzano's theorem for continuous functions defined over a compact set. In principle, there could be a second root since the function is quadratic, but that cannot be the case or there would need to be a third real root for the function to approach a negative value as $\delta \to 1$. Therefore, for $\delta \in (0,1)$ there can only be one steady state δ^*. It is asymptoticall stable as $\dot{\delta} < 0$ for $\delta > \delta^*$ and $\dot{\delta} > 0$ for $\delta < \delta^*$.

4.4 Proof of proposition 4

The analysis of two trivial steady states is analogous to the proof of proposition 3. Inspection of (31) shows that $\dot{\delta} = 0$ at $\delta = 0$ and $\delta = 1$. Turning to stability, we have $\frac{d\dot{\delta}}{d\delta} = \lambda(1-\delta)\left[r_s(\delta) - r_m(\delta)\right] - \lambda\delta\left[r_s(\delta) - r_m(\delta)\right] + \lambda\delta(1-\delta)\left[r_s'(\delta) - r_m'(\delta)\right]$. At $\delta = 0$, $r_s(0)$ is not defined but $lim_{\delta\to 0}\frac{d\dot{\delta}}{d\delta} = lim_{\delta\to 0}\left[r_s(\delta) - r_m(\delta)\right] \to \infty > 0$, so that the first trivial steady state is locally unstable. At $\delta = 1$, $r_m(1)$ is not defined but $lim_{\delta\to 1}\frac{d\dot{\delta}}{d\delta} = lim_{\delta\to 1} - \left[r_s(\delta) - r_m(\delta)\right] \to \infty > 0$, so that the second trivial steady state is locally unstable.

Let us now to turn to prove stability of the non-trivial steady state. $\forall\delta \in (0,1)$ we have $d\dot{\delta}/d\delta = -\frac{\lambda}{1+z_m}\frac{\mu_0}{\Psi}\delta(1-\delta)\left(z_s\frac{\Gamma(\alpha,s)}{\alpha}\frac{1}{\delta^2} - +z\frac{1}{(1-\delta)^2}\right) < 0$. Hence $\delta^* = \frac{\Gamma(\alpha,s)}{\Gamma(\alpha,s)+z_m/z_s}$ is asymptotically stable over $\forall\delta \in (0,1)$.

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