Endogenous Beliefs and Social Influence in a Simple Macroeconomic Framework

Roberta Terranova

August 31, 2018

Abstract

We incorporate the notion of animal spirits, that is waves of optimism and pessimism driven by social influence, into an economy composed of islands which trade and among which there is a strategic complementarity. Since the latter is unknown, islands need to form expectations about it. In this regard, agents can be optimistic, neutral or pessimistic and the probability of changing expectation rules depends on expected profits of each type and on social influence, that is, others' beliefs. Studying the dynamics of the fractions of neutral and pessimistic islands, we show that, without social influence, the economy converges to the unique stable equilibrium in which pessimists are at least as much as neutral agents. Moreover, we find that when all islands are neutral, their expectations are rational. This implies that, without social influence, rational expectations are not stable in our model. With a level of social influence which is high enough, however, a qualitative change in the dynamics emerges. In fact, two new equilibria arise, one stable and one unstable, both of which are characterized by a higher number of neutral islands than pessimists. Moreover, we extend the analysis to the two-dimensional dynamics of an economy composed by all three types and find similar results, with the difference that for some levels of social influence, here, the economy is attracted to more optimistic scenarios. Welfare is not monotonically increasing in the number of neutral; yet, under rational expectations, that is when all islands are neutral, welfare is maximized.

JEL codes: D83, D84, E10, E70.
Keywords: animal spirits, heterogeneous expectations, heuristics, uncertainty.

1 Introduction

The idea that our economy is characterized by radical uncertainty in which probabilities of future scenarios are not known was one of the points stressed by Keynes in his General Theory. In such a context, individuals' decisions are not driven by rational expectations but, rather, by “spontaneous optimism” and pessimism, also known as “animal spirits” (Keynes, 1936), which urge people to act in one direction or another and, moreover, partly drive economic fluctuations. Such animal spirits emerge as waves, implying that people’s beliefs are correlated and that the role of social influence has to be taken into account when dealing with animal spirits. One possible reason for this social influence may be found in the example of the beauty contest (Keynes, 1936), which shows that investors have to guess the others’ evaluation of an asset rather than its fundamental value, because it is the former that really drives its value. In general, in an economy in which one’s payoff is affected by others’ actions, it is intuitive that everyone will be influenced in their decisions by what the others do or by the expectation on what the others do. Moreover, social influence plays an even stronger role in environments of uncertainty: in fact, in such contexts, people cannot base their decisions on all the information that they would need, but they have to take decisions that are in part based on instinct. In particular, the decision making under uncertainty has been explored, among others, by Tuckett (2012), which presents the interviews to fifty-two money managers on their decision making under radical uncertainty and information ambiguity. This paper shows, among other things, the relevance of social influence. In fact, a common element in their interviews is that these managers present their past investment decisions as crucially characterized by uncertainty and, moreover, partly driven by the surrounding atmosphere of optimism or pessimism, that is, what we call social influence. The role of uncertainty in influencing individuals’ decisions in an economy has been explored in many economic models that we briefly review in the Literature Review section.

In this paper we develop a formal framework which allows us to analyze the effect of social influence in a macroeconomic setting with uncertainty, heterogeneous beliefs such as optimism or pessimism and an endogenous switching mechanism between them which also depends on the aforementioned social influence. To do so, we present an economy similar to that developed by Angeletos and La’O (2013), in which there are islands that trade in every period in random pairs. Because of the structure of the economy, islands’ output is positively related with the trading partner’s output, giving rise to strategic complementarity among them. However, islands get to know their trading partner’s output only after having made production and employment decisions; therefore, they need to form expectations on the trading partner’s output level and on its higher order beliefs, too. We believe that this simple framework allows us to focus on the role of expectations and social influence.

Islands can have optimistic, pessimistic or neutral beliefs and they can switch their type over time. We consider two versions of the switching mechanism between the different beliefs: the first one is presented in Section 3.1, in this simpler case the switching mechanism is driven only by the difference in expected profits of the types of islands in the economy. Expected profits of each type are public knowledge: this may be due to a newspaper which, every period, publicly announces the value of expected profits. In Section 5 we include the role of social influence, that, following Lux (1995), we model by incorporating an opinion index, which essentially is the difference in the shares of the different types of agents in the economy. The higher is the number of, say, pessimists, the more likely it is that an agent becomes pessimist, too. Given the aforementioned strategic complementarity, social influence acquire a stronger motivation in our macro model.

In the framework just presented, we study how the fractions of the different types of islands evolve over time. For this analysis, first we consider only two types of islands: the neutral and the pessimistic ones. Thus, we have the one dimensional dynamics of the opinion index, i.e. the difference between the shares of neutral and pessimistic islands over the entire population. Our results show that, in the case in which there is no social influence, the economy converges to the unique stable equilibrium in which at least half of the islands are pessimist. We also show that in our model, when all the islands are neutral – i.e. they have neutral beliefs and produce optimally given these beliefs – their expectations are rational, that is, they correspond to the expected output of all islands. As soon as even only one island in the economy is not neutral anymore, nobody in the economy has rational expectations. Therefore, a unique stable equilibrium characterized by at least half agents in the economy implies that the economy converges to a situation in which expectations are not rational. Rational expectations are, therefore, unstable.

Furthermore, we present a welfare analysis through which we find that, while pessimistic islands have on average higher profits than neutral ones, the welfare for the whole economy is highest when expectations are neutral, i.e. rational.

In the second part of our analysis, we study the dynamics of the opinion index when the switching mechanism includes social influence. We find that the latter affects the equilibrium of the dynamics in different ways depending on its level. In fact, if social influence is low, it drives the equilibrium to a number of pessimist islands higher than that emerging without social influence. If it is high enough it creates a qualitative change in the dynamics; in fact two new equilibria arise, one unstable and one stable. Both are characterized by the presence of more neutral islands than pessimists. For some high enough values of social influence, the new stable equilibrium is characterized by all islands being neutral and, therefore, rational. Whether the economy converges to one or the other stable equilibrium depends on the initial shares of types in the economy. In light of the positive relationship between the fraction of neutral islands and welfare, we find that social influence in our model has a positive impact on the economy.

In section 6, we provide an extension of the model in which we explore the dynamics of an economy characterized by three types of beliefs: optimistic, pessimistic and neutral. In particular, we study the two-dimensional dynamics of the optimists' and pessimists' shares in the population and we find a confirmation of the results obtained in the one-dimensional analysis. In fact, also in this extension of the model, social influence has a strong role for reestablishing the rational equilibrium. Here, in addition, social influence creates an area in which the economy is attracted to more optimistic scenarios. However, welfare is again maximized under rational expectations.

1.1 Literature Review

Since the introduction of rational expectations by Muth (1961), the notion and formalization of animal spirits as endogenous waves of optimism and pessimism which can generate economic fluctuations has not received much attention in the standard macroeconomic literature.

However, there have been many attempts to explain the existence and the effects of such animal spirits with models embedding rational expectations. For example, there are papers based on multiple equilibria economies that show how, in such a context, economic fluctuations may be driven by, for example, extraneous uncertainty. This is the case of Azariadis (1981), which shows, with an aggregative model of overlapping generations, that many of the multiple equilibria in such model are characterized by extraneous uncertainty – which in this model can be any kind of uncertainty pertaining to factors that people deem to be relevant, and not only to structural elements of the economy. This raises the possibility that business cycles are set in motion by arbitrary shifts in any factor, however purely subjective, agents see as relevant to economic activity. Another example is Cass and Shell (1983), in which the authors focus on extrinsic uncertainty, or sunspots, i.e. random phenomena that do not affect fundamentals in the economy but do affect the allocation of resources and, thus, some of the equilibria[1].

Among models on unique equilibrium economies, an example that shows the effects of such animal spirits by incorporating rational expectations is provided by Angeletos and La'O (2013), whose model we already described above. In their economy characterized by uncertainty and strategic complementarity, they formalize animal spirits (which they call 'sentiments') as random shocks in the information that every agent receives on the other agents' beliefs – not on fundamentals. When these random shocks are correlated across agents they can generate economic fluctuations.

The works mentioned above formalize animal spirits in models that embed rational expectations. They show that even in standard economic models uncertainty not pertaining to fundamentals may play a role and, thus, self-fulfilling prophecies in multiple equilibria or 'sentiments' can generate economic fluctuations. Animal spirits, therefore, are formalized as random phenomena. Therefore, they do not focus on the endogenous formation of such animal spirits nor on their propagation in the economy.

Since 1950s, alternatives to rationality have been developed, starting, for example, from Herbert Simon who strongly argued for bounded rationality (cfr. Simon (1955)).

[1] Other papers in the framework of multiple equilibria economies are, e.g., Benhabib and Farmer (1994) and Cooper and John (1988).

Within this framework, the idea that agents have heterogeneous expectations played a crucial role. It was initially developed to gain a better understanding of expectation formation in financial markets. One early example is the Santa-Fe Artificial Stock Market (LeBaron, Arthur, Holland, Palmer, and Tayler, 1997), in which traders every period select an expectation rule among a large pool of rules, based on the market conditions that they observe and on a fitness measure of the rules. A simpler approach to formalize heterogeneous expectations is provided by the Brock-Hommes model (Brock and Hommes, 1997, 1998) in which there is a finite set of simple forecasting rules among which agents can choose – e.g. naive and rational expectations. Moreover, they introduce a switching mechanism among forecasting rules, which depends on the rules'(relative) performance measure. This generates a dynamics across predictors which is coupled with the dynamics of endogenous economic variables. Another approach within the field of heterogeneous expectations, known as adaptive learning or statistical learning (e.g. Evans and Honkapohja (2012)), considers agents as using the perceived law of motion of the economy as a forecasting rule and trying to learn the optimal parameters with some learning mechanism, as new realizations become available (e.g. ordinary least squares, OLS, sample autocorrelation).

The heterogeneous expectations approach just mentioned has been used to model animal spirits, intended as waves of optimism and pessimism in the economy, too. For example, in Lux (1995) the author formalizes a financial market where booms and busts are driven by the change in the number of optimist and pessimist investors, which is driven by an explicitly modeled social influence. In particular, there are chartist and fundamentalist investors; the former group is composed by optimists and pessimists. The probability of an optimist becoming pessimist and *viceversa* depends on the difference in the shares of the two types of beliefs. The coefficient of this difference in the probabilities is the social influence parameter.

An example of the notion of animal spirits in a macroeconomic model is developed by De Grauwe (2011); the author presents a standard aggregate demand-aggregate supply model augmented with a Taylor rule. The crucial feature is that agents have heterogeneous expectations on future output and inflation; in particular, when predicting the output, they can be optimistic or pessimistic with respect to the rational expectation benchmark. Furthermore, as in Brock and Hommes (1997), agents continuously evaluate the performance of the expectation rules by looking at their mean squared forecasting errors and this performance measure drives the fraction of agents which choose one rule or another.

Moreover, XXXXXXXXXXX

In Burnside, Eichenbaum, and Rebelo (2016) the authors study the booms and busts in the housing markets by including heterogeneous expectations and social dynamics. In their model, the equilibrium price of houses depends, among other things, on the utility of owning a house, which can change in the future with a small probability. People know this small probability but disagree on the probability distribution of the future utility of owning a house and therefore have heterogeneous expectations on the future price. Moreover, people differ in their degree of confidence in their beliefs. There are three types of agents, two of which hold the same expectations but different confidence levels and one who differs in both dimensions. These agents can 'infect' each other with their expectations, depending on their confidence level: this gives rise to social dynamics, which drives the rise and fall of the number of each agents' types. House prices are determined by the marginal buyer which is identified by sorting agents in declining order of how much they value houses. Thus, depending on the number of the different types of agents, the price dynamics is determined and can be characterized by phenomena of booms and busts or booms not followed by busts, too.

With the present paper, we aim at contributing at this streams of literature with a model built in a simple macro setting which allows us to focus only on the expectation formation process taking into account the role of social influence which, to our knowledge, has been taken into account only in Burnside, Eichenbaum, and Rebelo (2016), in the context of the housing market. Ideally, we would like to use this model as a first simple base to explore more precisely how people form their expectations under uncertainty, taking into account also the findings within the field of psychology.

2 The Model

The model describes an economy composed by a large number n of heterogeneous islands with different productivities. Similarly to Angeletos and La'O (2013), each island is inhabited by a locally owned firm and a single household. The former produces one good employing labor and land; the latter earns a wage by offering his labor to the local firm. Households do not save anything and they want to consume both the local good and the 'foreign' goods; this gives rise to trade among islands. Trade takes place in pairs through random matching: each period, every island meets another island and trades only with it, further, the pairs are randomly chosen every period. Because of the structure of the economy in our model, as we will see in more details below, the price of every island' s local good, and therefore the production, is positively affected by the trading partner's output. This is the source of the strategic complementarity that in our model plays a crucial role.

Every time period t unfolds in two different stages. In the first stage islands do not know the randomly chosen trading pairs and, therefore, they do not know the identity of the island they have been matched with, along with its productivity and its output. Nevertheless, in the first stage islands have to take production and employment decisions and, because of the aforementioned strategic complementarity, they need to form expectations on the trading partner's output level. In the second stage islands actually meet and trade their previously determined output: households choose their consumption of the 'home' good and the 'foreign' good and market-clearing prices are determined.

Our model presents a simple Neoclassical economy, in which firms choose optimally the level of production (which maximizes profits), households choose their consumption maximizing their utility and, finally, market-clearing prices equate goods' marginal utilities. The distinctive feature is the uncertainty under which firms operate: in fact, they make their production and employment decisions before knowing the price at which they will sell their good and, therefore, by forming expectations on it. The way in which they form their expectations is explained later.

2.1 Households' consumption on island i

The household on island i maximizes the following utility function:

$$U_i = \Sigma_{t=0}^{\infty} \beta^t [U(c_{it}, c_{it}^*) - V(n_{it})], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $c_{it} \in \mathbb{R}_+$ and $c_{it}^* \in \mathbb{R}_+$ are the consumptions of, respectively, the 'home' good and the 'foreign' good in time t. $n_{it} \in \mathbb{R}_+$ is the labor supply and $V(n)$ is the disutility of labor. U and V are given by:

$$U(c, c^*) = \left(\frac{c}{1-\eta}\right)^{1-\eta} \left(\frac{c^*}{\eta}\right)^{\eta} \qquad \text{and} \qquad V(n) = \frac{n^{\varepsilon}}{\varepsilon}, \tag{2}$$

where $\eta \in (0,1)$ is the fraction of 'home' expenditure that is spent on the 'foreign' good and $\varepsilon > 1$ is the Frisch elasticity of labor supply. The period t budget constraint for the household's utility maximization on island i is the following:

$$p_{it} c_{it} + p_{it}^* c_{it}^* \leq w_{it} n_{it} + r_{it} K + \pi_{it}, \tag{3}$$

where p_{it} and p_{it}^* are the prices of the 'home' good and the 'foreign' good, respectively. w_{it} is the wage, r_{it} is the rental rate of land and π_{it} are the profits.

The first order conditions of the utility maximization[2], after normalizing the local nominal prices so that the Lagrange multiplier $\lambda_{it} = 1$, are $U_{c_{it}} = p_{it}$ and $U_{c_{it}^*} = p_{it}^*$. Combining them with the trade balance condition, $p_{it}^* c_{it}^* = p_{it}(y_{it} - c_{it})$, where y_{it} is the production of island i, the market clearing condition, $c_{it} + c_{jt}^* = y_{it}$, where c_{jt}^* is the import in island j of the good produced on i, and the corresponding results for i's trading partner, we obtain the following results:

$$c_{it} = (1-\eta) y_{it}, \qquad c_{it}^* = \eta y_{jt} \qquad \text{and} \qquad p_{it} = y_{it}^{-\eta} y_{jt}^{\eta}. \tag{4}$$

[2]See Angeletos and La'O (2013) for the proofs of the results in the present section.

From the above we can observe at the individual level the source of the strategic complementarity in our model. In particular, as shown in equation (4), a rise in y_{jt} increases the import of the 'foreign' good c^*_{it}, which raises the 'home' good's marginal utility and, in turn, its price[3] (equation (4)).

2.2 Production on island i

Firms take production and employment decisions in the first stage of every period, when they still do not know the identity of their trading partner.

Island i's firm produces the local good with the following technology:

$$y_{it} = A_i n_{it}^{\Theta} k_{it}^{1-\Theta}, \tag{5}$$

where A_i is the local total factor productivity, which is formalized as a continuous[4] random variable lognormally distributed: $A_i \sim \log N(0, \sigma_A^2)$, where $\sigma_A > 0$. Further, n_{it} and k_{it} are the labor and land input, respectively; $\Theta \in (0,1)$ parametrizes the income share of labor. All islands are endowed with a fixed and equal amount of land K.

Firms choose n_{it}, k_{it}, the wage w_{it} and the rental rate r_{it} optimally; in particular, as regards the labor market, the optimal amount of labor supply and labor demand are found by equating the wage with, respectively, the household's marginal disutility of working and the marginal revenue of labor. In equilibrium, therefore, we have:

$$V'(n_{it}) = w_{it} = \mathbb{E}_{it}[p_{it}]\Theta \frac{y_{it}}{n_{it}}, \tag{6}$$

where $\mathbb{E}_{it}[p_{it}]$ is the expectation of island i on price p_{it}, which we will discuss in details later. The resulting optimal amount of labor input is given by $n^*_{it} = (\mathbb{E}_{it}[p_{it}]\Theta y_{it})^{\frac{1}{\varepsilon}}$.

As regards land, in equilibrium $k^*_{it} = K$, i.e. firms employ the total amount of land disposable on each island; thus, we set $K = 1$.

By inserting the optimal amount of labor in (5) and recalling that $p_{it} = y_{it}^{-\eta} y_{jt}^{\eta}$ and, thus, $\mathbb{E}_{it}[p_{it}] = y_{it}^{-\eta}\,\mathbb{E}_{it}[y_{jt}^{\eta}]$, we obtain the optimal level of output for island i:

$$y_{it} = K_1^{\alpha} A_i^{\alpha} [\mathbb{E}_{it}(y_{jt}^{\eta})]^{\theta\alpha}, \tag{7}$$

where $K_1 \equiv (\Theta^{\theta})$, $\theta \equiv \frac{\Theta}{\varepsilon} \in (0,1)$ and $\alpha \equiv \frac{1}{1-\theta+\eta\theta}$. It is worth noting that $\eta\theta\alpha \in (0,1)$ represents the degree of the strategic complementarity existing among islands in our model. As already mentioned, it is a positive strategic complementarity, since an higher

[3] As explained in Angeletos and La'O (2013), an increase in the price of island i's good can also be interpreted as an improvement of island i's terms of trade. In fact: $\frac{p_{it}}{p_{jt}} = \frac{y_{jt}}{y_{it}} = p_{it}^{\frac{1}{\eta}}$, which is an increasing function of p_{it}.

[4] We assumed that our economy is composed by a large number n of islands and not by a continuum of them, however, this approximation simplifies some computations in the next sections and, for $n \to \infty$, the approximation error goes to zero.

output of the trading partner j has a positive effect on island i's own output, but the effect is smaller than one. This implies that an increase in the expected production of the trading partner translates in an increase of islands' production of a smaller amount. The degree of the strategic complementarity is increasing in θ and in η. Despite this strategic complementarity, we should not interpret the agents of our economy, the islands, as strategic players of a game: in fact, as pointed out by Angeletos and La'O (2013), they are infinitesimal price takers and the complementarity is a result of competitive market interactions.

3 Endogenous beliefs

So far we have outlined the basic structure of the model, how islands trade and the behavior of households and firms. What we have not defined yet is how agents form their expectations, that is, how we specify $\mathbb{E}_{it}(y_{jt}^{\eta})$ in equation (7). As mentioned in the introduction, we want to formalize the concept of 'animal spirits' with simple behavioral rules that capture the idea that in an economy characterized by uncertainty, in which individuals do not have a complete knowledge of the model underlying the economy, agents' beliefs may be subject to waves of optimism and pessimism. These arise endogenously in the economy and are not driven by exogenous shocks. We formalize this idea with a probability switching model between different types of beliefs, similar to that developed by Lux (1995); in the first formulation that we present, the switching mechanism depends only on the expected profits of each type, which is public knowledge. In every period expected profits of each type are communicated publicly, for instance trough a newspaper. However, we also want to capture the idea that beliefs may be driven by social influence as well, that is, the more a certain type of belief is spread in the economy, the more it is likely that one agent will be 'affected' and become of that type, too. We update the switching mechanism with this idea of social influence in section 5.

In what follows we consider the presence of neutral, optimistic and pessimistic islands. To do so, we need to formally define the 'neutral belief', so that we can use it as a benchmark to define the optimistic and pessimistic beliefs; the benchmark needs to depend only on some exogenous parameters and not on the other islands' expectations, as shown in what follows. To compute it we need to make some assumptions, whose interpretation is given later.

When forming their beliefs on y_{jt}^{η}, islands know that their trading partner j's output, like theirs, is given by $y_{jt} = K_1^{\alpha} A_j^{\alpha} [\mathbb{E}_{jt}(y_{it}^{\eta})]^{\theta\alpha}$. Therefore, they take that into consideration and form their first-order belief:

$$\mathbb{E}_{it}(y_{jt}^{\eta}) = \mathbb{E}_{it}\left(K_1^{\alpha\eta} A_j^{\alpha\eta} [\mathbb{E}_{jt}(y_{it}^{\eta})]^{\theta\alpha\eta}\right). \tag{8}$$

Since K_1^α and $\mathbb{E}_{it}(A_j^{\alpha\eta})$ $\big(= \mathbb{E}(A_j^{\alpha\eta})\big)$ are common knowledge, it turns out that islands' subjective part of the expectation is $\mathbb{E}_{it}[\mathbb{E}_{jt}(y_{it}^\eta)]^{\theta\alpha\eta}$. However, islands know y_{it}, so they can do a step forward in their reasoning and substitute y_{it} in equation (8) with $K_1^\alpha A_i^\alpha[\mathbb{E}_{it}(y_{jt}^\eta)]^{\theta\alpha}$:

$$\mathbb{E}_{it}(y_{jt}^\eta) = K_1^{\alpha\eta(1+\alpha\eta\theta)}\, \mathbb{E}(A_j^{\alpha\eta})^{(1+\alpha\eta\theta)}\, \mathbb{E}_{it}\left(\mathbb{E}_{jt}([\mathbb{E}_{it}(y_{jt}^\eta)]^{\theta\alpha\eta})^{\theta\alpha\eta}\right). \tag{9}$$

The term $\mathbb{E}_{it}\left(\mathbb{E}_{jt}([\mathbb{E}_{it}(y_{jt}^\eta)]^{\theta\alpha\eta})^{\theta\alpha\eta}\right)$ in equation (9) describes what island i believes that island j thinks of the first-order belief of i on j; that is, it corresponds to the third-order belief of i. We define the 'neutral belief' benchmark by assuming that $\mathbb{E}_{it}\left(\mathbb{E}_{jt}[\mathbb{E}_{it}[y_{jt}^\eta]]\right) = \mathbb{E}_{it}(y_{jt}^\eta)$, that is[5], a neutral island i believes that the trading partner j knows its own (i's) belief. With this assumption, we can now actually compute the neutral belief inserting $\mathbb{E}_{it}(y_{jt}^\eta)^{(\theta\alpha\eta)^2}$ on the right hand side of equation (9) and solving for $\mathbb{E}_{it}(y_{jt}^\eta)$. As a result, we obtain the following neutral belief:

$$\mathbb{E}_{nt}(y_{jt}^\eta) = K_1^{\eta\gamma}\, \mathbb{E}(A_i^{\eta\alpha})^{\frac{\gamma}{\alpha}} \tag{10}$$

where the subscript n indicates that it is a neutral belief, $\gamma \equiv \frac{1}{1-\theta}$ and $\alpha \equiv \frac{1}{1-\theta+\eta\theta}$ was already defined in the previous section.

In other words, in our model a neutral island believes that its trading partner knows its own belief and, for this reason, after taking that into account, its (the neutral island's) belief depends only on exogenous parameters and it is independent from the other's belief.

It is now possible to use the aforementioned neutral belief benchmark to define optimism and pessimism in relation with it. In particular, we consider optimism as a positive deviation of $\delta \in (0,1)$ from the neutral belief and, *viceversa*, pessimism as a negative deviation of the same δ from it.

Therefore, the optimistic and pessimistic beliefs are defined as follows:

$$\mathbb{E}_{it}(y_{jt}^\eta) = \begin{cases} K_1^{\eta\gamma}\, \mathbb{E}(A_i^{\eta\alpha})^{\frac{\gamma}{\alpha}}(1+\delta) & \text{if i is optimist, (i=o)} \\ K_1^{\eta\gamma}\, \mathbb{E}(A_i^{\eta\alpha})^{\frac{\gamma}{\alpha}}(1-\delta) & \text{if i is pessimist, (i=p)} \end{cases} \tag{11}$$

where $\delta \in (0,1)$ represents the degree of optimism or pessimism.

The idea behind the deviation of δ from the neutral belief is that an optimistic island i thinks that the trading partner j overestimates its (i's) own belief, so that $\mathbb{E}_{it}\left(\mathbb{E}_{jt}[\mathbb{E}_{it}[y_{jt}^\eta]]\right) > \mathbb{E}_{it}(y_{jt}^\eta)$ and, in particular, $\mathbb{E}_{it}\left(\mathbb{E}_{jt}([\mathbb{E}_{it}(y_{jt}^\eta)]^{\theta\alpha\eta})^{\theta\alpha\eta}\right) = (1+s)^{\theta\alpha\eta}\, \mathbb{E}_{it}(y_{jt}^\eta)^{(\theta\alpha\eta)^2}$, where $(1+s)^{\frac{\theta\alpha\eta}{1-(\theta\alpha\eta)^2}} = (1+\delta)$. In other words, island i believes that the second order belief of j $\mathbb{E}_{jt}([\mathbb{E}_{it}(y_{jt}^\eta)]^{\theta\alpha\eta})$ equals its own actual first-order belief times $(1+s)$, which i will then raise to the power of $\theta\alpha\eta$. In a symmetric way, a pessimistic island i thinks that the trading partner j underestimates i's belief: for this type of island it holds that

[5] Since we are dealing with point beliefs and not beliefs' distribution, this assumption translates directly into $\mathbb{E}_{it}\left(\mathbb{E}_{jt}([\mathbb{E}_{it}(y_{jt}^\eta)]^{\theta\alpha\eta})^{\theta\alpha\eta}\right) = \mathbb{E}_{it}(y_{jt}^\eta)^{(\theta\alpha\eta)^2}$.

$\mathbb{E}_{it}\left(\mathbb{E}_{jt}([\mathbb{E}_{it}(y_{jt}^{\eta})]^{\theta\alpha\eta})^{\theta\alpha\eta}\right) = (1-s)^{\theta\alpha\eta}\,\mathbb{E}_{it}(y_{jt}^{\eta})^{(\theta\alpha\eta)^2}$, where $(1-s)^{\frac{\theta\alpha\eta}{1-(\theta\alpha\eta)^2}} = (1-\delta)$

However, for our analysis it is not so important how exactly island i implements its optimistic (pessimistic) belief. In fact, what matters is that there is a positive (negative) deviation from the neutral benchmark which implies a belief that the trading partner, on average, will produce more (or less). This, because of the strategic complementarity, will translate into the optimal choice of producing (on average) more than the neutral islands.

Output for the three types of islands Islands' optimal output depends, among other factors, on their belief on the trading partner's production. By substituting the beliefs into equation (7), we obtain the optimal output for the different types of island. They are:

$$y_{it} = \begin{cases} K_1^{\gamma} A_i^{\alpha}\,\mathbb{E}[A_j^{\eta\alpha}]^{\theta\gamma} & \text{if i is neutral, (i=n)} \\ K_1^{\gamma} A_i^{\alpha}\,\mathbb{E}[A_j^{\eta\alpha}]^{\theta\gamma}(1+\delta)^{\theta\alpha} & \text{if i is optimist, (i=o)} \\ K_1^{\gamma} A_i^{\alpha}\,\mathbb{E}[A_j^{\eta\alpha}]^{\theta\gamma}(1-\delta)^{\theta\alpha} & \text{if i is pessimist, (i=p)} \end{cases} \tag{12}$$

We can observe that also with respect to production, optimists' and pessimists' output are positive and negative deviations, respectively, from the neutral output. The size of this deviation is $(1+\delta)^{\theta\alpha}$ for the optimists and $(1-\delta)^{\theta\alpha}$ for the pessimists.

It is interesting to note the following proposition about the neutral belief.

Proposition 1 *The neutral belief about the expected output of neutral islands is correct. Therefore, when every island in the economy is neutral, we can consider neutral islands as having rational expectations.*

$$\mathbb{E}_{nt}(y_{jt}^{\eta}) = K_1^{\eta\gamma}\,\mathbb{E}(A_i^{\eta\alpha})^{\frac{\gamma}{\alpha}} = \int_0^{+\infty} K_1^{\eta\gamma} A_i^{\eta\alpha}\,\mathbb{E}(A_i^{\eta\alpha})^{\eta\gamma\theta} dF(A_i) = \mathbb{E}(y_{nt}^{\eta}), \tag{13}$$

where on the left hand side there is the neutral belief and on the right hand side there is the expected value of the neutral islands' production.

Proposition 1 does not hold for the other types of islands; we show this by taking the expectation of the production functions (raised to the power of η) of equation (12). To do so, we recall, through Proposition 1, that $\mathbb{E}(K_1^{\gamma\eta} A_i^{\alpha\eta}\,\mathbb{E}[A_j^{\eta\alpha}]^{\theta\gamma\eta} = \mathbb{E}(y_{nt}^{\eta})$, therefore we obtain the optimists' and pessimists' production:

$$\mathbb{E}(y_{it}^{\eta}) = \begin{cases} \mathbb{E}(y_{nt}^{\eta})(1+\delta)^{\theta\alpha\eta} & \text{if i is optimist, (i=o)} \\ \mathbb{E}(y_{nt}^{\eta})(1-\delta)^{\theta\alpha\eta} & \text{if i is pessimist, (i=p).} \end{cases} \tag{14}$$

If we compare equation (14) with the beliefs in equation (11), considering that $K_1^{\eta\gamma}\,\mathbb{E}(A_i^{\eta\alpha})^{\frac{\gamma}{\alpha}} = \mathbb{E}(y_{nt}^{\eta})$, we note that in the latter there are the terms $(1+\delta)$ and $(1-\delta)$, while in the former the same terms are raised to the power of $\theta\alpha\eta \in (0,1)$. As a result, given that $1+\delta > 1 \rightarrow 1+\delta > (1+\delta)^{\theta\alpha\eta}$, that is, the optimist belief systematically overestimates the optimists' expected production. On the other hand, $1-\delta<1 \rightarrow 1-\delta>(1-\delta)^{\theta\alpha\eta}$, that is, pessimist belief systematically underestimates pessimists' expected production.

Profits for the three types of islands The profit function of an island i trading with an island j in period t is given by[6]:

$$\pi_{ij,t} = y_{it}^{1-\eta}(y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta})) \tag{15}$$

and therefore depends also on the type of i and j. When i is a neutral island, its expected profits are given by

$$\begin{aligned}\mathbb{E}(\pi_{n,t}) = & \left(\frac{n_n}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)\left[\mathbb{E}(y_{nt}^{\eta}) - \mathbb{E}(y_{nt}^{\eta})\right] + \\ & + \left(\frac{n_o}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)\left[\mathbb{E}(y_{nt}^{\eta})(1+\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})\right] + \\ & + \left(\frac{n_p}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)\left[\mathbb{E}(y_{nt}^{\eta})(1-\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})\right],\end{aligned} \tag{16}$$

where the share of each type of island is multiplied by what the neutral island earn when it meets that type. In the same way, we find the expected profits of optimists and pessimists. For optimists:

$$\begin{aligned}\mathbb{E}(\pi_{o,t}) = & \left(\frac{n_n}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)(1+\delta)^{\theta(1-\eta)\alpha}\left[\mathbb{E}(y_{nt}^{\eta}) - \mathbb{E}(y_{nt}^{\eta})(1+\delta)\right] + \\ & + \left(\frac{n_o}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)(1+\delta)^{\theta(1-\eta)\alpha}\left[\mathbb{E}(y_{nt}^{\eta})(1+\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})(1+\delta)\right] + \\ & + \left(\frac{n_p}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)(1+\delta)^{\theta(1-\eta)\alpha}\left[\mathbb{E}(y_{nt}^{\eta})(1-\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})(1+\delta)\right].\end{aligned} \tag{17}$$

For pessimists:

$$\begin{aligned}\mathbb{E}(\pi_{p,t}) = & \left(\frac{n_n}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)(1-\delta)^{\theta(1-\eta)\alpha}\left[\mathbb{E}(y_{nt}^{\eta}) - \mathbb{E}(y_{nt}^{\eta})(1-\delta)\right] + \\ & + \left(\frac{n_o}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)(1-\delta)^{\theta(1-\eta)\alpha}\left[\mathbb{E}(y_{nt}^{\eta})(1+\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})(1-\delta)\right] + \\ & + \left(\frac{n_p}{n}\right)\mathbb{E}\left(y_{nt}^{(1-\eta)}\right)(1-\delta)^{\theta(1-\eta)\alpha}\left[\mathbb{E}(y_{nt}^{\eta})(1-\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})(1-\delta)\right].\end{aligned} \tag{18}$$

Profits play a crucial role in the switching mechanism presented in section 3.1; in fact, it is driven by the difference in each type's expected profits. We begin our analysis of the dynamics by considering an economy composed only by neutral and pessimistic islands. In this scenario, the switching mechanism depends on the difference between the expected profits made by neutral islands and the expected profits made by the pessimist ones in every period t, and, in an extended version presented later, also on social influence. This difference is indicated with $\bar{\pi}_{np}(x)$, it is a linear function of x and therefore changes over

[6]Its derivation is shown in the Appendix.

time. Notice that $\frac{x+1}{2} = \frac{n_n}{n}$ and $\frac{1-x}{2} = \frac{n_p}{n}$; we have that

$$\begin{aligned}\bar{\pi}_{np,t}(x) = & \int_0^{+\infty}\int_0^{+\infty} \frac{x+1}{2}[\pi_{n,n}(A_i, A_j)] + \frac{1-x}{2}[\pi_{n,p}(A_i, A_j)]dF(A_i)dF(A_j) + \\ & - \int_0^{+\infty}\int_0^{+\infty} \frac{x+1}{2}[\pi_{p,n}(A_i, A_j)] + \frac{1-x}{2}[\pi_{p,p}(A_i, A_j)]dF(A_i)dF(A_j),\end{aligned} \tag{19}$$

where $\pi_{n,n}$ is the profits made by a neutral island meeting another neutral island, $\pi_{n,p}$ is the profits made by a neutral island meeting a pessimist one and so on.

Proposition 2 In the absence of social influence, pessimists always earn higher profits than neutral islands.

$$\bar{\pi}_{np}(x) < 0, \qquad \text{for any } x. \tag{20}$$

In fact, if we recall the profit function $y_{it}^{1-\eta}(y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta}))$ and we focus on the difference between j's expected production and i's belief on j, we can see that, in order for the profits to be positive, the belief should be lower than j's expected output. Consider the case in which island i is pessimistic and trades with j which is neutral: we have that $\mathbb{E}_{nt}(y_{jt}^{\eta}) > \mathbb{E}_{nt}(y_{jt}^{\eta})(1-\delta)$. This means that pessimists on average are better off then neutral islands because they underestimate neutral islands' production. Interestingly, the same happens also when a pessimist trades with another pessimist: $\mathbb{E}_{nt}(y_{jt}^{\eta})(1-\delta)^{\alpha\theta\eta} > \mathbb{E}_{nt}(y_{jt}^{\eta})(1-\delta)$, which is always true because the exponent $\eta\theta\alpha$, that is the strategic complementarity, is smaller than one. In other words, the latter result implies that pessimistic islands, in our model, are 'too pessimistic', since they systematically underestimate even the pessimists' production. This gives them an advantage through a reduction of their costs and a gain from the improved terms of trade. Nevertheless, as already mentioned, islands in this economy are not strategic agents since they are infinitesimal price takers; they behave optimally given their belief. This holds true also for neutral islands, which, given their neutral belief, choose their profit maximizing production.

3.1 Switching mechanism between beliefs, without social influence

As mentioned, in our model the composition of the economy in terms of beliefs – i.e. the number of islands that are optimistic, pessimistic or neutral – in each time period is driven by switching probabilities between those beliefs, in a similar way to that developed by Lux (1995). We begin our analysis by considering an economy composed by only two types of belief, i.e. the neutral and the pessimistic. Such economy can be described by an 'opinion indexes', $x = \frac{n_n - n_p}{n}$, where n_n and n_p are the numbers of neutral and pessimistic islands, respectively. The dynamics of x is defined by the aforementioned switching probabilities.

From the definition of x, it follows that $x \in [-1, 1]$; therefore, if $x = 0$, the economy is a balanced situation, while if $x > 0$ the economy is characterized by more optimist islands. On the other hand, $x < 0$ implies that more islands are pessimist. The extreme cases are $x = -1$ where all islands are pessimistic and $x = 1$ where all islands are neutral. Moreover, from Proposition 1, $x = 1$ implies that the economy is under rational expectations.

Our basic model, as specified by Angeletos and La'O (2013), unfolds in discrete time and it is characterized by a specific succession of events, such as the production, the employment decisions and the actual trading. However, in order to study the dynamics of the economy, we will treat time as continuous and build a dynamic model similar to that developed by Lux (1995). The reason for this choice is that whereas switching models in discrete time may offer a slightly more diverse dynamics, in this particular model this advantage does not seem to be worth the loss of simplicity. In what follows we briefly show how we derive a continuous dynamics from a model originally expressed in discrete time. In the latter, the opinion index x in time $t + \varepsilon$ is given by:

$$x(t+\varepsilon) = \frac{n_n(t+\varepsilon) - n_p(t+\varepsilon)}{n}, \tag{21}$$

where

$$\begin{aligned} n_n(t+\varepsilon) &= n_n(t) - n_n(t)p_{np}(\varepsilon, \bar{\pi}_{np,t}(x)) + n_p(t)p_{pn}(\varepsilon, \bar{\pi}_{np,t}(x)) \\ n_p(t+\varepsilon) &= n_p(t) - n_p(t)p_{pn}(\varepsilon, \bar{\pi}_{np,t}(x)) + n_n(t)p_{np}(\varepsilon, \bar{\pi}_{np,t}(x)). \end{aligned} \tag{22}$$

In words, $n_n(t+\varepsilon)$ is given by the number of islands that were neutral in time t, minus those of them which became pessimistic $(n_n(t)p_{np}(\varepsilon, \bar{\pi}_{np,t}(x)))$, plus those which were pessimistic in time t and became neutral $(n_p(t)p_{pn}(\varepsilon, \bar{\pi}_{np,t}(x)))$. p_{np} and p_{pn} are the probability of switching from neutral belief to pessimism and *viceversa*, respectively. They depend on the time interval ε and on $\bar{\pi}_{np,t}(x)$, which is the difference between the expected profits of neutral and pessimistic islands in t. Substituting (22) into (21), we can then compute the change over time and, for $\varepsilon \to 0$, we get $\dot{x}$, i.e. the change of x in continuous time:

$$\dot{x} = \lim_{\varepsilon \to 0} \frac{x(t+\varepsilon) - x(t)}{\varepsilon} = \frac{2n_p(t)q_{pn}}{n} - \frac{2n_n(t)q_{np}}{n} \tag{23}$$

where $q_{np} = \lim\limits_{\varepsilon \to 0} \frac{p_{np}}{\varepsilon}$ and $q_{pn} = \lim\limits_{\varepsilon \to 0} \frac{p_{pn}}{\varepsilon}$ are the transition rates from neutral to pessimism and *viceversa* of the system in continuous time.

Such transition rates are defined following Lux (1995). In the absence of social influence, they need to satisfy the following requirements. In particular, they have to be positive and q_{pn} must be positively related with the difference of expected profits made by neutral and pessimistic islands; in the same way, q_{np} must be negatively related with the said difference. Furthermore, the transition rates include a parameter for the speed of switching, v, which guarantees that some changes in the beliefs happen even when the difference between expected profits of the two types equals zero: $q_{pn}(x) = q_{np}(x) = v > 0$. These changes can be interpreted as due to personal circumstances not taken into account by the model. An example of transition rates satisfying our criteria are the following:

$$\begin{aligned} q_{pn}(x) &= v \exp(a_0 \bar{\pi}_{np,t}(x)) \quad \text{for switching from being pessimist to neutral;} \\ q_{np}(x) &= v \exp(-a_0 \bar{\pi}_{np,t}(x)) \quad \text{for switching from being neutral to pessimist.} \end{aligned} \tag{24}$$

The coefficient a_0 represents the strength of the impact of the difference between the two types' expected profits on the transition. When $a_0 = 0$, the transition rates in both directions equal v, which means that they are determined only by the speed of switching. For higher levels of a_0, instead, islands become more sensitive to the difference in expected profits and transition with a higher speed to the belief characterized by higher expected profits.

From equation (24) it follows that an example of switching probabilities satisfying $q_{np} = \lim_{\varepsilon \to 0} \frac{p_{np}}{\varepsilon}$ and $q_{pn} = \lim_{\varepsilon \to 0} \frac{p_{pn}}{\varepsilon}$ may be

$$p_{np} = e^{-(a_0 \pi(x))} \varepsilon \qquad \text{and} \qquad p_{pn} = e^{(a_0 \pi(x))} \varepsilon. \tag{25}$$

In fact, $\bar{\pi}_{np,t}(x)$ is bounded from above and, thus, also q_{np} and q_{pn}; therefore it is possible to choose an ε small enough such that p_{pn} and p_{np} are smaller or equal than one.

In this first formulation of the switching mechanism, $\bar{\pi}_{np,t}(x)$ is what drives the process, so it is assumed that islands know it; as already mentioned, we assume it is public knowledge.

4 Dynamics of the economy, without social influence

We begin the dynamic analysis by considering a benchmark scenario in which all islands are neutral – and as shown in Proposition 1, rational – and do not have the possibility to switch belief. Of course in this scenario constituted of just one type of belief there is no dynamics in the composition of the population. It is interesting to note that when all islands in the economy are neutral, the expected profits earned are zero:

$$\bar{\pi}_{n,n} = \int_0^{+\infty} \int_0^{+\infty} \pi_{n,n}(A_i, A_j) dF(A_i) dF(A_j) = 0. \tag{26}$$

As mentioned before, in our model islands' profits are given by $y_{it}^{1-\eta}(y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta}))$. This implies that, when an island has correct expectations on the trading partner's output, it makes zero profits: this is the case for neutral islands trading in an economy composed only by neutral islands. Therefore, in this case, expected profits of neutral islands are zero.

The reason is that our economy is perfectly competitive and, thus, firms will produce until their marginal cost equals the price – that, under rational expectations, is known – which, in turn, equals firms' average costs, driving profits to zero.

Recalling what we already mentioned above, $\frac{x+1}{2} = \frac{n_n}{n}$ and $\frac{1-x}{2} = \frac{n_p}{n}$, we can rewrite (23) as

$$\dot{x} = (1-x)q_{pn} - (1+x)q_{np} \tag{27}$$

If we plug (24) in (27), knowing that $\exp(y) - \exp(-y) = \sinh(y)$, $\exp(y) + \exp(-y) = \cosh(y)$ and that $\frac{\sinh(y)}{\cosh(y)} = \tanh(y)$, we obtain the differential equation characterizing the dynamics of our model without social influence:

$$\dot{x} = 2v[\tanh(a_0\bar{\pi}_{np}(x)) - x]\cosh(a_0\bar{\pi}_{np}(x)). \tag{28}$$

This function is illustrated in figure 4.1.

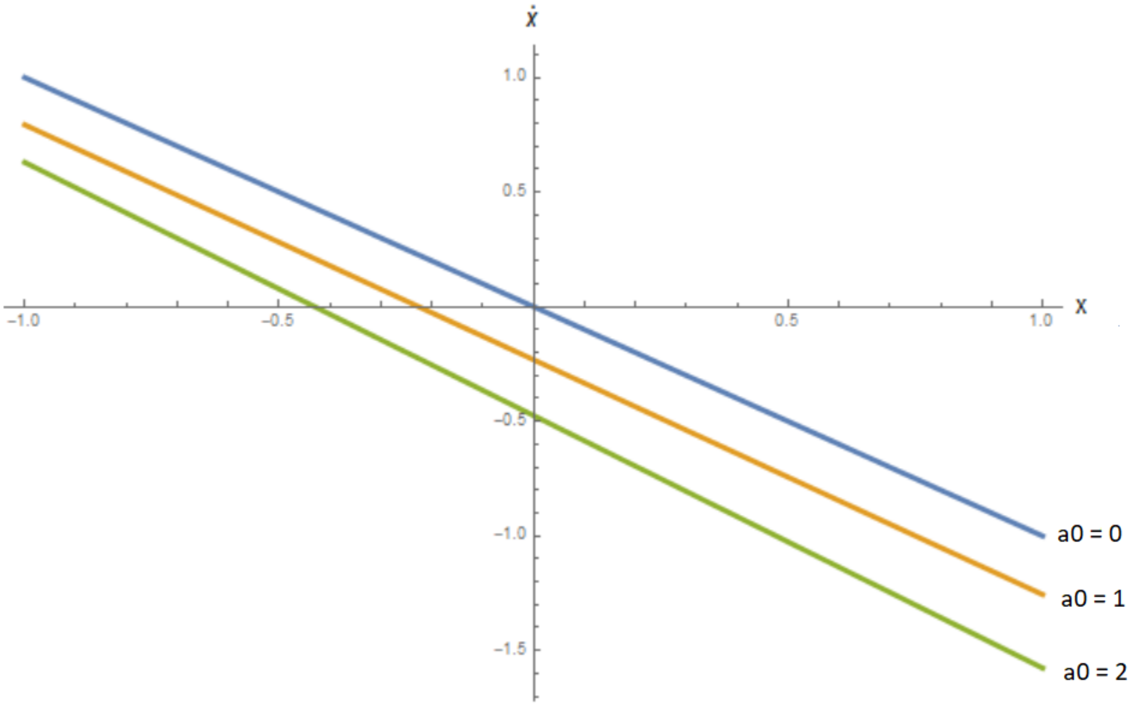

Figure 4.1: Dynamics of the economy

The differential equation (28) is continuous and monotonically decreasing, in $x = -1$ it is positive and in $x = 1$ it is negative, therefore the solution exists, it is unique and it is globally stable. In particular, the fixed point is given by:

$$\tanh(a_0\bar{\pi}_{np}(x)) = x, \tag{29}$$

which implies either $a_0 = 0$ and $x = 0$ or, if $a_0 \neq 0$, it must hold that $\frac{e^{a_0 \bar{\pi}np(x)} - e^{-(a_0 \bar{\pi}np(x))}}{e^{a_0 \bar{\pi}np(x)} + e^{-(a_0 \bar{\pi}np(x))}} = x$, which has no analytical solution. However, we can see from figure 4.1 that the higher a_0, the more islands tend to become pessimistic abandoning neutral beliefs. In fact, assuming $a_0 \geq 0$, the number of pessimists will in any case overcome the number of neutral islands, except when $a_0 = 0$, in which the economy is in a balanced situation. The reason for this shift away from $x = 1$ lies in the fact that pessimists' expected profits are always higher than those earned by neutral islands. Recalling that $x = 1$ implies rational expectations, we can state that in our model rational expectations (i.e. every island having neutral beliefs) are not stable. In fact, from an initial situation in which islands are assumed to be only neutral, if we then introduce a new type of belief, such as the pessimist one, in the absence of social influence, a certain number of islands (with $a_0 \geq 0$ at least half of them) become pessimist.

4.1 Welfare analysis with different compositions of the population

In the previous section we discussed the instability of rational expectations caused by the negative difference between the expected profits of neutral and pessimistic islands. However, we need to study how the welfare changes with x in order to assess whether an economy dominated by pessimists is necessarily worse than one dominated by neutral islands.

In the present model, utility on island i is given by

$$U_i = \Sigma_{t=0}^{\infty} \beta^t \left[\left(\frac{c_{it}}{1-\eta} \right)^{1-\eta} \left(\frac{c_{it}^*}{\eta} \right)^{\eta} - \frac{n_{it}^{\varepsilon}}{\varepsilon} \right]. \tag{30}$$

In equilibrium, $c_{it} = (1-\eta)y_{it}$ and $c_{it}^* = \eta y_{jt}$. Moreover, $n_{it} = (\mathbb{E}_{it}[p_{it}]\Theta y_{it})^{\frac{1}{\varepsilon}} = (\mathbb{E}_{it}[y_{jt}^{\eta}]\Theta y_{it}^{1-\eta})^{\frac{1}{\varepsilon}}$, so we can rewrite the utility as

$$U_i = \Sigma_{t=0}^{\infty} \beta^t \left[(y_{it})^{1-\eta} \left((y_{jt})^{\eta} - \mathbb{E}_{it}[y_{jt}^{\eta}] \frac{\Theta}{\varepsilon} \right) \right], \tag{31}$$

where we already defined $\frac{\Theta}{\varepsilon} = \theta$. Welfare is computed as the expected utility of the whole economy and it is given by:

$$\begin{aligned} \mathbb{E}(U) = \Sigma_{t=0}^{\infty} \beta^t \Bigg\{ & \left(\frac{x+1}{2} \right) \int_0^{+\infty} \int_0^{+\infty} \frac{x+1}{2} U_{n,n}(A_i, A_j) + \frac{1-x}{2} U_{n,p}(A_i, A_j) dFA_i dFA_j + \\ & + \left(\frac{1-x}{2} \right) \int_0^{+\infty} \int_0^{+\infty} \frac{x+1}{2} U_{p,n}(A_i, A_j) + \frac{1-x}{2} U_{p,p}(A_i, A_j) dFA_i dFA_j \Bigg\}, \end{aligned} \tag{32}$$

where $U_{n,n}(A_i, A_j)$ is the utility earned by a neutral island i, with productivity A_i trading with another neutral island j, with productivity A_j. $U_{n,p}(A_i, A_j)$ is the utility obtained by a neutral island trading with a pessimistic island and so on.

Figure 4.2 shows the expected utility of the economy with $\varepsilon = 2$ and $\sigma = 0.038$, which are the values set by Angeletos and La'O (2013) in their variant of the RBC model[7]. Moreover, in the graph on the left, $\eta = 0.5$, $\theta = 0.325$ and δ varies. In the graph on the right, $\eta = 0.5$ and $\theta = 0.9$. We observe that the expected utility is monotonically increasing in x when $\eta = 0.5$ and $\theta = 0.325$. In the other case, it is not; in order to understand why it can happen that the expected utility is not monotonically increasing in x, we observe what follows[8]. We can express equation (32) – for a single period of time – as follows, noting that A_i and A_j are independently distributed:

$$
\begin{aligned}
\mathbb{E}(U_i) = & \left(\frac{x+1}{2}\right)^2 \mathbb{E}\left(y_{nt}^{(1-\eta)}\right) \left[\mathbb{E}(y_{nt}^{\eta}) - \mathbb{E}(y_{nt}^{\eta})\theta\right] + \\
& + \left(\frac{1-x^2}{4}\right) \mathbb{E}\left(y_{nt}^{(1-\eta)}\right) \left[\mathbb{E}(y_{nt}^{\eta})(1-\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})\theta\right] + \\
& + \left(\frac{1-x^2}{4}\right) \mathbb{E}\left(y_{nt}^{(1-\eta)}\right) (1-\delta)^{\theta(1-\eta)\alpha} \left[\mathbb{E}(y_{nt}^{\eta}) - \mathbb{E}(y_{nt}^{\eta})(1-\delta)\theta\right] + \\
& + \left(\frac{1-x}{2}\right)^2 \mathbb{E}\left(y_{nt}^{(1-\eta)}\right) (1-\delta)^{\theta(1-\eta)\alpha} \left[\mathbb{E}(y_{nt}^{\eta})(1-\delta)^{\theta\eta\alpha} - \mathbb{E}(y_{nt}^{\eta})(1-\delta)\theta\right].
\end{aligned}
\tag{33}
$$

The first line of equation (33) shows the amount of expected utility generated by a neutral island trading with another neutral: it is always positive since $\theta < 1$ and it increases with x. The second line refers to the expected utility of neutral islands meeting pessimists; if the expression in the square brackets is positive, the utility deriving from neutral meeting pessimists is increasing in x for $x < 0$ and decreasing for $x > 0$. Alternatively, if the expression in the square brackets is negative, the aforementioned utility is increasing in x for positive x's and decreasing for negative x's. The expression under consideration is positive if $\delta < 1 - \theta^{\frac{1}{\theta\eta\alpha}}$, where we recall that $\alpha \equiv \frac{1}{1-\theta+\eta\theta}$, and negative if the opposite is true. In other words, in order for the utility of neutral islands trading with pessimists to be positive, if θ increases, then δ, that is the degree of pessimism and the deviation from the neutral expected production, needs to decrease. We recall that $\theta \equiv \frac{\Theta}{\varepsilon}$, where Θ is the labor share of income and ε is the Frisch elasticity of labor supply: they are, respectively, positively and negatively related with the optimal amount of labor employed in the production. Thus, we can say that when neutral islands trading with pessimists obtain a negative expected utility, the reason is that they are working too much with respect to the expected production of the pessimists, which negatively depends on δ.

[7] These values are in turn taken from King and Rebelo (2000).

[8] It is worth noting that the parameters considered in Angeletos and La'O (2013) are $\eta = 0.5$ and $\theta = 0.325$, which are shown in the left graph of figure 4.2, that shows an expected utility monotonically increasing in x.

Furthermore, the third line refers to the expected utility of pessimistic islands meeting neutral islands. It is decreasing in x when x is positive and increasing when x is negative. Finally, the expression in the square brackets of the last line of equation (33) is always positive, given the strategic complementarity $\theta\eta\alpha < 1$ already mentioned. Therefore the expected utility of a pessimist meeting a pessimist, as intuitively clear, is decreasing in x.

To sum up, the overall effect of a change in x on the expected utility depends on the compositions of all the partial effects mentioned above. Thus, it may happen that, when $x < 0$, an increase in x produces an increase in the utility generated by neutral and pessimistic islands trading with neutral (first and third line of equation (33)), but at the same time it may produce a larger loss of utility for neutral islands meeting pessimists – because of excessive work – and, obviously, for pessimists meeting pessimists (second and last line of equation (33)).

An important result of our welfare analysis is the following:

Proposition 3 Welfare is maximized under rational expectations, that is, when $x = 1$ and every island has neutral beliefs.

This result is easily proven by observing that, for $\delta = 0$, all the islands produce the same amount of output of neutral islands and the welfare of the economy is the same, too. Formally, for $\delta = 0$, the economy's welfare is given by the first line of equation (33) repeated four times. For $\delta > 0$ we are back in the economy made of neutral and pessimistic islands; given that the derivative of the sum of the squared brackets in equation (33) with respect to δ is negative, it results that the welfare is maximized under rational expectations (i.e. all islands being neutral).

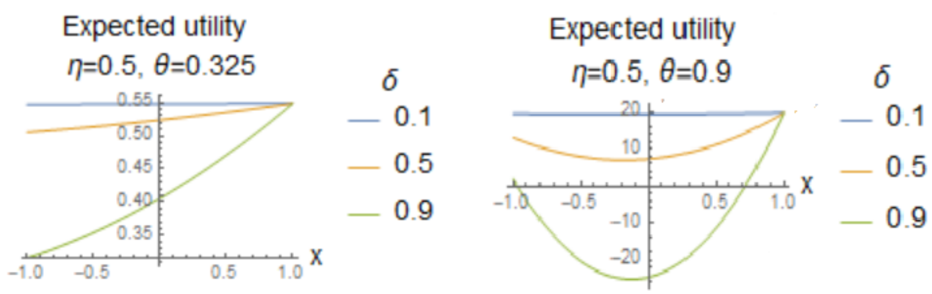

Figure 4.2: Expected Utility

5 Social Influence

So far we considered a switching mechanism between beliefs which only depends on the difference between the expected profits of the two types. In the present section we want to extend this formulation by taking into account that the belief of each agent may also be affected by the beliefs of the other agents in the economy. For example, if in the economy the majority of people are pessimists, it will be more likely that a neutral agent becomes pessimist, too, and *viceversa*. In our model islands do not know with which island they will trade, i.e. they do not know its productivity and its second-order belief. Thus, their production and employment decisions are the result of their guess, which takes into account their knowledge of the model but also leaves room for deviations from the neutral benchmark. In such a context, it is reasonable that islands' beliefs are driven by the expected profits of each type but also by the composition of beliefs in the economy; in fact, if there are many neutral islands, it will be more likely that they will meet a neutral island and, thus, it will be more likely that they become neutral too, since the belief is defined as a third-order belief.

In order to take into account the social influence just described, we follow Lux (1995) and consider that the opinion index x influences the transition rates in the switching mechanism by having a positive impact on the switching from pessimistic to neutral belief – the higher x, the more neutral islands are in the economy and the more likely it is for an island to become neutral – and, *viceversa*, a negative impact on the switching from neutral to pessimistic belief – the more neutral islands there are in the economy, the less likely it is for an island to become pessimistic. Therefore, the updated transition rates are:

$$\begin{aligned} q_{pn}(x) &= v\exp(a_0\bar{\pi}_{np}(x) + a_1 x) \quad \text{for switching from being pessimistic to neutral;} \\ q_{np}(x) &= v\exp(-a_0\bar{\pi}_{np}(x) - a_1 x) \quad \text{for switching from being neutral to pessimistic.} \end{aligned} \tag{34}$$

a_1 represents the social influence, which has a positive impact on q_{pn} and a negative impact on q_{np}. The introduction of social influence leads to a different dynamics in the economy, which is described by the following differential equation:

$$\dot{x} = 2v[\tanh(a_0\bar{\pi}_{np}(x) + a_1 x) - x]\cosh(a_0\bar{\pi}_{np}(x) + a_1 x). \tag{35}$$

Equation (35) is shown in figure 5.1, in which we set $v = 2$, $a_0 = 1$, $\varepsilon = 2$, $\eta = 0.5$ and $\theta = 0.325$.

The dynamics shown in equation (35) is a continuous function of x, which takes positive values for $x = -1$ and negative values for $x = 1$; therefore, it has at least one solution. It is not analytically solvable whether and for which values the dynamics is monotonically decreasing. satisfying We can study its behavior by analyzing its representation in figure

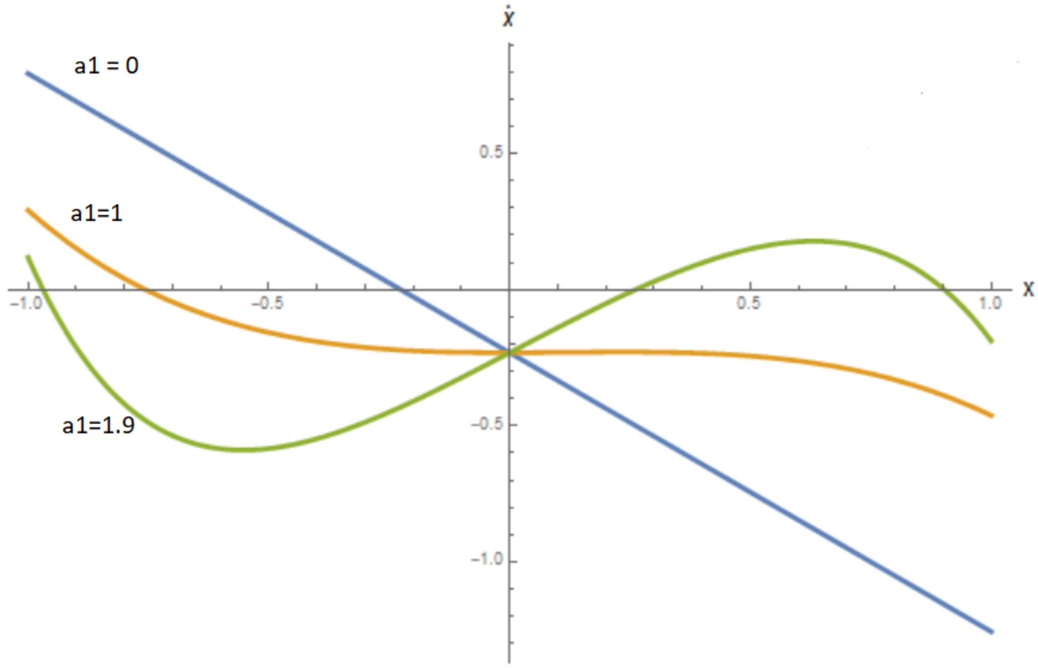

Figure 5.1: Dynamics of the economy

5.1, where $a_0 = 1$ and a_1 varies as shown. The blue line represents the case in which there is no social influence; here the solution is unique and the fixed point is globally stable. If we include in the dynamics the parameter of social influence $a_1 = 1$, the unique stable equilibrium shifts to the left, leading even more islands to become pessimists and therefore decreasing the whole economy production – and for these parameters also welfare is diminshed. Social influence, in this case, has a negative effect on the economy and only amplifies the number of pessimistic islands. However, if we increase the social influence to 1.9, as green line shows, the negative stable equilibrium shifts even more to the left and we observe a qualitative change in the dynamics – a fold bifurcation or a catastrophe –, where two new equilibria arise, one locally stable and one locally unstable. Therefore, both the lowest fixed point and the highest one are stable, each with its own basin of attraction, whereas the intermediate unstable fixed point separates the basins.

In other words, this dynamics implies that if there is a certain minimum number of neutral islands in the economy, the economy will converge to the positive stable equilibrium in which there are more neutral islands than pessimistic ones and, therefore, the economy production will be larger.

This can be observed more clearly in the bifurcation diagram in figure 5.2 – which is drawn for the same parameters as figure 5.1 – where the black line in the lower part of the graph shows how the negative stable equilibrium changes with the social influence parameter a_1. As already mentioned, it decreases until it reaches $x^* = -1$, i.e. where the economy is composed only of pessimistic islands. However, the red vertical line shows the critical value of a_1, around 1.53 for the parameters considered here, where two new equilibria arise in the upper part of the graph. The one represented by a black continuous line is the locally stable positive equilibrium, which increases with a_1 until it reaches $x^* = 1$, a situation in which every island is neutral – and therefore rational. The dotted blue line, instead, shows the locally unstable positive equilibrium. In other words, if social influence is high enough and if the initial composition of the economy is good enough (with more neutral islands than pessimistic ones), rational expectations is indeed a situation to which islands can converge. Therefore, social influence plays a crucial role in reestablishing the rational expectations equilibrium in the economy.

Moreover, as mentioned, figure 5.2 shows the bifurcation diagram for some parameters; in particular, $\varepsilon = 2$, $\eta = 0.5$, $\theta = 0.325$, $\upsilon = 2$ and $a_0 = 1$. Therefore, it corresponds to the graph on the left of figure 4.2, in which the expected utility of the whole economy is monotonically increasing in x. Recalling that when all islands are neutral their expectations are rational and that under rational expectations welfare is maximizied, we find that social influence has a positive impact on the economy's welfare.

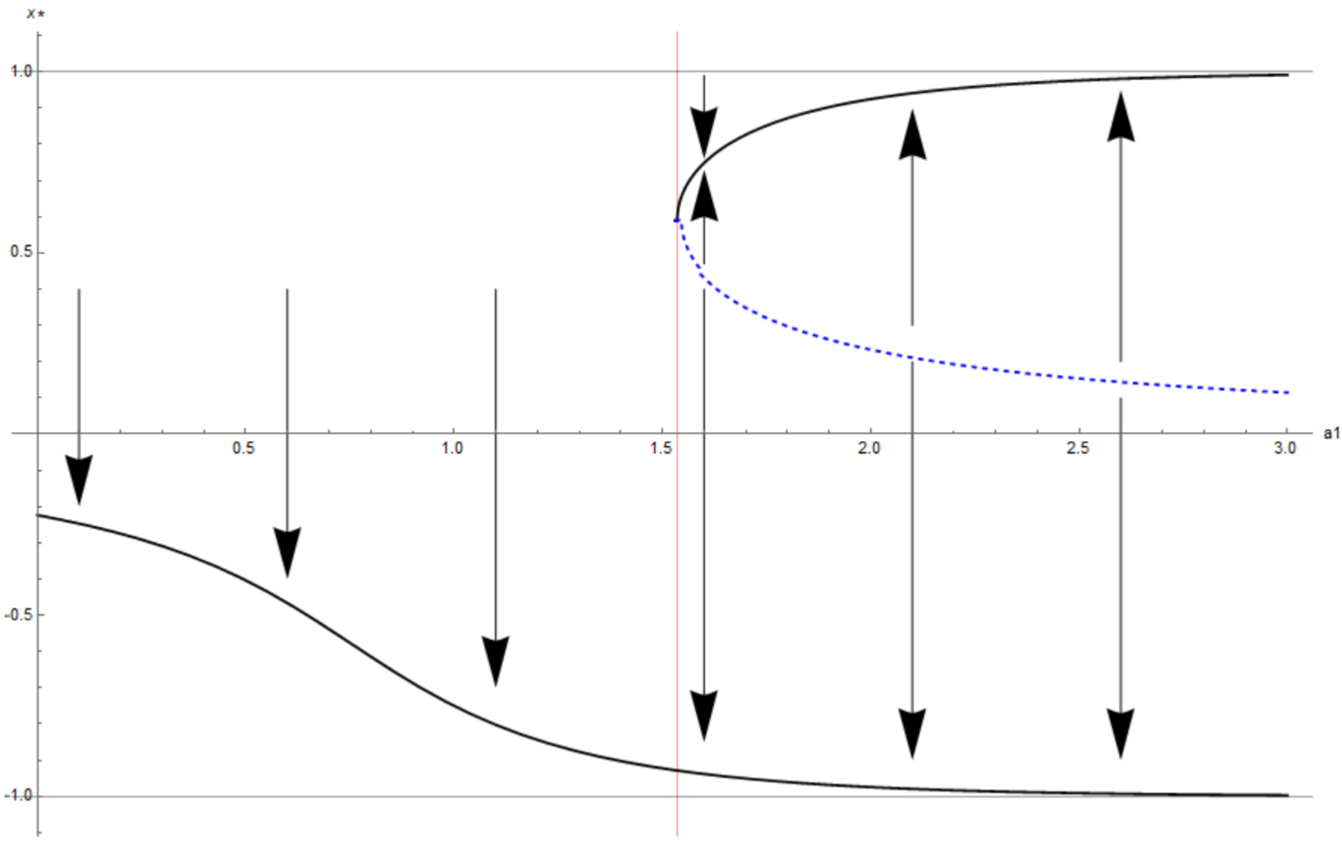

Figure 5.2: Bifurcation Diagram

6 An economy with three types of beliefs: optimistic, pessimistic and neutral ones

In the present section we want to extend our analysis on endogenous beliefs and social influence to an economy which is composed not only by pessimistic and neutral islands but which also includes optimistic islands. The dynamics of such economy can be represented by a two-dimensional system in which the two variables are the shares of two types of belief – the third one is then univocally determined. In particuar, we focus on the shares of optimistic and pessimistic islands on the total population, which are n_o and n_p, respectively. As in the previous one-dimensional analysis, first we study the case without social influence and then we include it, in order to study its impact on the dynamics. Without social influence, the transition rates are modified in the following way:

$$\begin{aligned}
q_{pn}(n_p, n_o) &= v\exp(a_0\bar{\pi}_{np,t}(n_p, n_o)) \quad \text{pessimistic} \rightarrow \text{neutral} \\
q_{np}(n_p, n_o) &= v\exp(-a_0\bar{\pi}_{np,t}(n_p, n_o)) \quad \text{neutral} \rightarrow \text{pessimistic} \\
q_{on}(n_p, n_o) &= v\exp(-a_0\bar{\pi}_{on,t}(n_p, n_o)) \quad \text{optimistic} \rightarrow \text{neutral} \\
q_{no}(n_p, n_o) &= v\exp(a_0\bar{\pi}_{on,t}(n_p, n_o)) \quad \text{neutral} \rightarrow \text{optimistic} \\
q_{po}(n_p, n_o) &= v\exp(a_0\bar{\pi}_{op,t}(n_p, n_o)) \quad \text{pessimistic} \rightarrow \text{optimistic} \\
q_{op}(n_p, n_o) &= v\exp(-a_0\bar{\pi}_{op,t}(n_p, n_o)) \quad \text{optimistic} \rightarrow \text{pessimistic.}
\end{aligned} \tag{36}$$

In equation (36) we introduce the transition rates from the optimistic to the neutral belief and *viceversa* and the transition rates from the pessimistic to the optimistic belief and *viceversa*. These depend on the shares of the different types of agents on the total population (through the difference in expected profits), rather than on x, as in the previous analysis. In fact, in the present extension x does not univocally define the economy anymore. Moreover, the transition rates from one belief to another (and *viceversa*) now depend on the difference in expected profits of the respective types. Therefore, for instance, the transition rate from the optimistic to the pessimistic beliefs depend on the difference between the expect profits of optimists and those of pessimists. Every type of island reacts with the same way intensity (a_0) to the expected profits' difference. The intuition behind these transition rates is similar to the one-dimensional case: expected profits of each type are public knowledge, thus they are publicly announced every period to all islands, and they influence the transition rates.

With the transition rates just defined we can study the dynamics of n_o and n_p:

$$\dot{n_p} = (n - n_o - n_p)q_{np} + n_o q_{op} - n_p q_{pn} - n_p q_{po}, \tag{37}$$

$$\dot{n_o} = (n - n_o - n_p)q_{no} + n_p q_{po} - n_o q_{on} - n_o q_{op}. \tag{38}$$

The idea is that the share of pessimistic islands in one period is given by the neutral ones that switched to the pessimistic belief minus the pessimistic islands that switched to the neutral belief; in a simmetric way the share of optimistic islands is obtained.

If we plug in the transition rates of equation (36), we get the dynamics for the share of pessimists

$$\begin{aligned}\dot{n_p} &= (n - n_o)v \exp(-a_0\bar{\pi}_{np,t}(n_p, n_o)) \\ &- 2vn_p(\cosh(a_0\bar{\pi}_{np,t}(n_p, n_o)) + n_o v \exp(-a_0\bar{\pi}_{op,t}(n_p, n_o)) - n_p v \exp(a_0\bar{\pi}_{op,t}(n_p, n_o))\end{aligned} \tag{39}$$

and the dynamics for the share of optimists:

$$\begin{aligned}\dot{n_o} &= (n - n_p)v \exp(a_0\bar{\pi}_{on,t}(n_p, n_o)) \\ &+ n_p v \exp(a_0\bar{\pi}_{op,t}(n_p, n_o)) \\ &- 2n_o v \cosh(a_0\bar{\pi}_{on,t}(n_p, n_o)) - n_o v \exp(-a_0\bar{\pi}_{op,t}(n_p, n_o)).\end{aligned} \tag{40}$$

Figure 6.1 shows the isoclines of the two-dimensional system without social influence and with $a_0 = 1$ – all the figures in Section 6 are drawn for $\Theta = 0.65$, $\theta = 0.325$, $\varepsilon = 2$, $\sigma = 0.038$, $\eta = 0.5$, $\delta = 0.3$, $v = 0.5$. As we can see, there is a unique equilibrium $A = (0.4975, 0.1887)$ and it is a stable node. In fact, the eigenvalues of the Jacobian matrix for equilibrium A are -1.8063 and -1.3615 of which the latter – which has a smaller absolute value – is represented by the double arrows in figure 6.1.

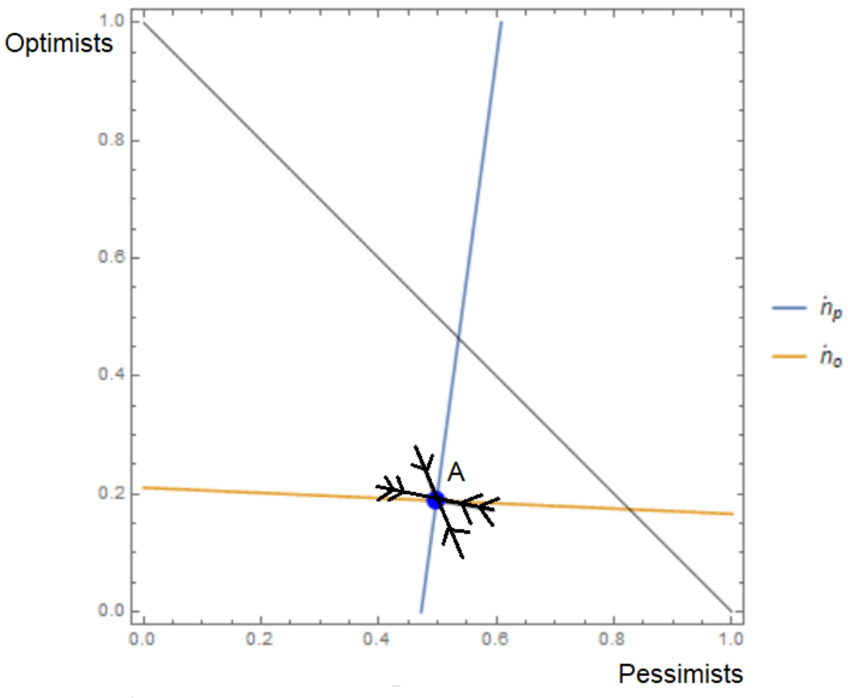

Figure 6.1: Phase portrait, no social influence and $a_0 = 0$

Equilibrium A is thus characterized by 49.75% of pessimists, 18.87% of optimists and a remaining 31.38% of neutral islands. If we increase the parameter a_0, which represents the sensitivity of islands to the different types' expected profits, we observe a shift of the equilibrium towards a situation characterized by more pessimists. For example, with $a_0 = 2$ we have a new equilibrium point at (0.6494, 0.0932)[9].

6.1 Social influence

In order to include social influence in the two-dimensional system presented above, we have to modify it such that the transition rates depend also on the other islands' beliefs. We do so by keeping the same logic that we introduced in the one-dimensional system, that is, the transition rate between two types depends on the difference of their shares in the economy. For example, the transition rate from the optimistic to the pessimistic belief depends positively on the difference between optimists' and pessimists' shares; *viceversa*, the transition rate in the opposite direction depends negatively on the same difference. The difference with the formalization of social influence in the one-dimensional system is just a matter of definitions: in the present extension we do not mention anymore the variable x but consider explicitly optimists' and pessimists' shares. The share of neutral islands is defined as $n_n = 1 - n_o - n_p$. Therefore the updated transition rates are:

$$\begin{aligned}
q_{pn}(n_p, n_o) &= v\exp(a_0\bar{\pi}_{np,t}(n_p, n_o) + a_1(1 - n_o - 2n_p)) \quad \text{pessimist} \rightarrow \text{neutral} \\
q_{np}(n_p, n_o) &= v\exp(-a_0\bar{\pi}_{np,t}(n_p, n_o) - a_1(1 - n_o - 2n_p)) \quad \text{neutral} \rightarrow \text{pessimist} \\
q_{on}(n_p, n_o) &= v\exp(-a_0\bar{\pi}_{on,t}(n_p, n_o) - a_1(2n_o - 1 + n_p)) \quad \text{optimist} \rightarrow \text{neutral} \\
q_{no}(n_p, n_o) &= v\exp(a_0\bar{\pi}_{on,t}(n_p, n_o) + a_1(2n_o - 1 + n_p)) \quad \text{neutral} \rightarrow \text{optimist} \\
q_{po}(n_p, n_o) &= v\exp(a_0\bar{\pi}_{op,t}(n_p, n_o) + a1(n_o - n_p)) \quad \text{pessimist} \rightarrow \text{optimist} \\
q_{op}(n_p, n_o) &= v\exp(-a_0\bar{\pi}_{op,t}(n_p, n_o) - a1(n_o - n_p)) \quad \text{optimist} \rightarrow \text{pessimist.}
\end{aligned} \tag{41}$$

The three types of island have the same sensitivity to the difference in expected profits (a_0) and the same social influence parameter (a_1).

The dynamics of n_p and n_o are:

$$\begin{aligned}
\dot{n_p} &= (n - n_o)v\exp(-a_0\bar{\pi}_{np,t}(n_p, n_o) - a_1(1 - n_o - 2n_p)) \\
&- 2vn_p(\cosh(a_0\bar{\pi}_{np,t}(n_p, n_o)) + n_o v\exp(-a_0\bar{\pi}_{op,t}(n_p, n_o)) - n_p v\exp(a_0\bar{\pi}_{op,t}(n_p, n_o))
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
\dot{n_o} &= (n - n_p)v\exp(a_0\bar{\pi}_{on,t}(x, z)) \\
&+ n_p v\exp(a_0\bar{\pi}_{op,t}(n_p, n_o)) \\
&- 2n_o v\cosh(a_0\bar{\pi}_{on,t}(n_p, n_o)) - n_o v\exp(-a_0\bar{\pi}_{op,t}(n_p, n_o)).
\end{aligned} \tag{43}$$

[9]See figure 8.1 in the Appendix.

We study how different levels of social influence affect the dynamics of the population in our model, keeping $a_0 = 1$. First, we consider $a_1 = 1$; the phase plane is shown in figure 6.2. We observe a unique stable equilibrium at $A2 = (0.7895, 0.0619)$, characterized by 78.95% pessimists and 6.19% optimists. It is stable because the eigenvalues are both negative: -2.6347, -0.7890, as in the previous cases the less negative eigenvalue corresponds to the eigenvector represented in the figure with double arrows.

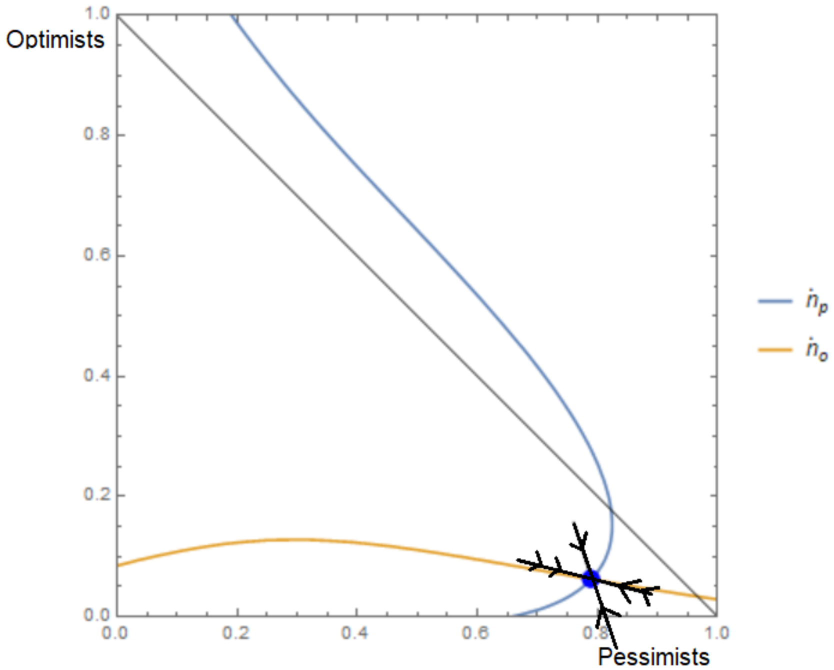

Figure 6.2: Phase plane, social influence of 1

If we increase social influence a_1 to 2, we find four equilibria, as shown in figure 6.3 (a). They are: point $A3 = (0.9786, 0.005)$, which has a similar position of point $A2$ in figure 6.2, but shifted down and to the right. The eigenvalues of the Jacobian Matrix evaluated at $A3$ are both negative (-9.8292 and -3.9022), thus $A3$ is a stable node. Point $B = (0.3217, 0.6780)$ is slightly inside the line delimiting the triangle of existence of the types' shares. It is a saddle node – eigenvalues: 2.9399 and -1.865. Point $C = (0.0443, 0.0220)$ is characterized by almost all islands being neutral and therefore this point is close to rational expectations. It is a stable node, characterized by negative eigenvalues (in particular, -3.2866 and -1.5050). Finally, point $D = (0.3157, 0.0588)$ is a saddle node whose Jacobian Matrix' eigenvalues are -1.8017 and 0.8264. In order to understand the dynamics of the system with social influence $a_1 = 2$, it may be useful to look at its stream plot in figure 6.3 (c). There, we understand what happens to the system when it starts from any point. We observe that the stable eigenvector of the saddle node D separates the basins of attraction of the stable nodes C and $A3$. Moreover, the stable eigenvector of point B separates the areas of points that are attracted towards C, D and $A3$ with those that are attracted towards more optimistic scenarios, even though there are no equilibria in that area.

The next scenario that we study is characterized by a social influence of 5. The equilibria and their nature are shown in figure 6.3 (b), where we observe again four equilibria but they are in different positions. In particular, we have equilibrium point $E = (0.4668, 0.0031)$, which is a saddle node with eigenvalues -8.8318 and 4.0493. Point $F = (0.2641, 0.4034)$ is a repelling node with eigenvalues 4.3860 and 3.1009. Point $G = (0.0064, 0.6343)$ is a saddle node with eigenvalues 19.9942 and -8.5572. Ultimately, point H = (0.00007, 0.00004) is a stable node with eigenvalues -60.1513 and -58.9383. To sum up, the dynamics with a social influence of 5 presents four equilibria, three of which are unstable (either repelling nodes or saddle points) and one is a stable node. The latter is characterized by almost zero pessimists and zero optimists, which implies that almost every island is neutral and thus it is a rational expectations equilibrium. As we can see from the stream plot in figure 6.3 (d), points below F are attracted either to the the rational expectations equilibrium or towards a situation characterized by all islands being optimistic, even though there is no equilibrium there. Instead, points above F are attracted towards scenarios characterized by more optimistic islands.

To sum up, in the two-dimensional analysis presented above, we observe the role of social influence in an economy composed by optimistic, pessimistic and neutral islands. In particular, we observe that such economy, without social influence, presents a unique stable equilibrium point, characterized by a certain percentage of pessimistic islands which is around 50% if $a_0 = 0$ and increases if $a_0 > 0$. This result is similar to that obtained in the one-dimensional analysis with no social influence: there the equilibrium was characterized by half of the islands being pessimistic for $a_0 = 0$ and a higher portion of pessimistic islands for higher values of a_0. When we consider a positive but small ($a_1 = 1$) social influence, the equilibrium is still unique and stable – this result is also similar to the one-dimensional analysis. However, when we consider higher values of social influence three new equilibria arise: for $a_1 = 2$ we observe two stable equilibria characterized by the majority of islands being neutral and the majority of islands being pessimistic. Moreover, the other two equilibria are saddle nodes. This situation is also somewhat similar to the one-dimensional dynamics with social influence. In fact, in both cases social influence is able to increase the number of pessimistic islands in the already pessimistic equilibrium and to create a new equilibrium in which almost all islands are neutral and thus have rational expectations. The analysis that includes also the optimistic islands, however, presents one saddle node characterized by optimistic and pessimistic islands. Moreover, in this case there is the possibility for the economy to be attracted to a situation characterized by more optimistic

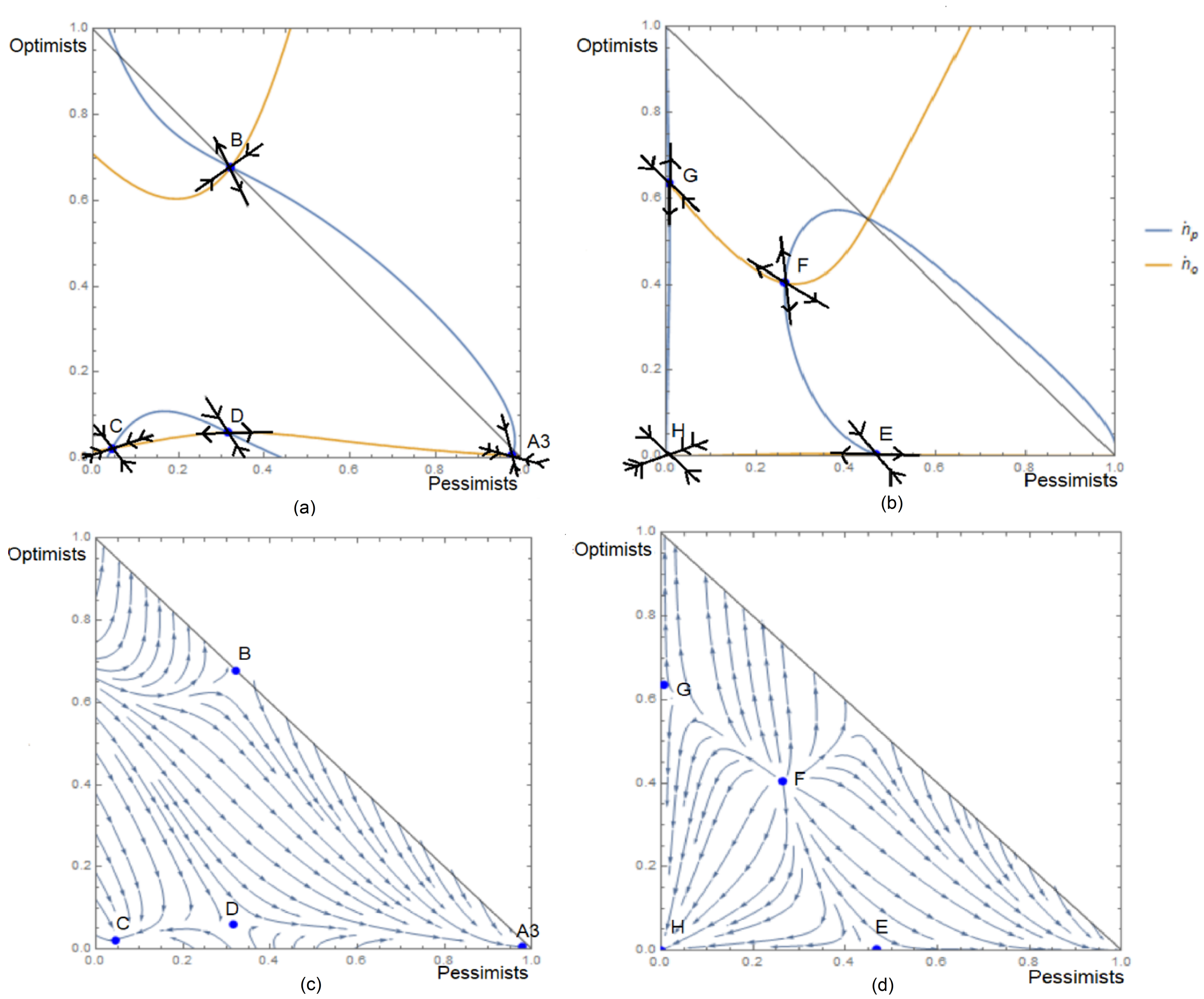

Figure 6.3: Equilibrium points and stream plot under social influence of 2 ((a), (c)) and social influence of 5 ((b) and (d)).

islands – in the area above the stable eigenvector of point B. When we increase social influence to $a_1 = 5$, the pessimistic equilibrium disappears, even though it is still there the area in which the economy is attracted to a more pessimistic composition. A new repelling node (F) arises and the saddle node G is characterized by optimistic and neutral islands. Furthermore, the area attracted to more optimistic scenarios becomes larger.

To conclude, including the optimistic islands in our analysis confirmed the result that social influence creates the possibility for a rational expectations equilibrium and, also, lets the economy be attracted to more optimistic scenarios.

6.2 Welfare analysis for an economy with optimistic, neutral and pessimistic islands

Welfare is defined as the expected utility of the whole economy, as in section 4.1:

$$\begin{aligned}\mathbb{E}(U) = \Sigma_{t=0}^{\infty}\beta^t \Big\{ & (n_n)\int_0^{+\infty}\int_0^{+\infty} n_n U_{n,n}(A_i,A_j) + n_p U_{n,p}(A_i,A_j) + n_o U_{n,o}(A_i,A_j) dFA_i dFA_j + \\ & + (n_p)\int_0^{+\infty}\int_0^{+\infty} n_n U_{p,n}(A_i,A_j) + n_p U_{p,p}(A_i,A_j) + n_o U_{n,o}(A_i,A_j) dFA_i dFA_j + \\ & + (n_o)\int_0^{+\infty}\int_0^{+\infty} n_n U_{p,n}(A_i,A_j) + n_p U_{p,p}(A_i,A_j) dFA_i dFA_j + n_o U_{n,o}(A_i,A_j) \Big\}.\end{aligned} \tag{44}$$

As in section 4.1, we can decompose the welfare into its different parts generated by the various types of islands – optimistic, pessimistic and neutral – meeting each other and producing a certain amount of utility. This decomposition is shown in equation (45).

$$\begin{aligned}\mathbb{E}(U_i) = & \left(\mathbb{E}(y_{nt}^{(1-\eta)})\,\mathbb{E}(y_{nt}^{\eta})\right)(n_n)^2\,[1-\theta] + \\ & + (n_n n_p)\left[(1-\delta)^{\theta\eta\alpha} - \theta\right] \\ & + (n_n n_o)\left[(1+\delta)^{\theta\eta\alpha} - \theta\right] \\ & + (n_p n_n)\,(1-\delta)^{\theta(1-\eta)\alpha}\,[1-(1-\delta)\theta] \\ & + (n_p)^2\,(1-\delta)^{\theta(1-\eta)\alpha}\left[(1-\delta)^{\theta\eta\alpha} - (1-\delta)\theta\right] \\ & + (n_p n_o)\,(1-\delta)^{\theta(1-\eta)\alpha}\left[(1+\delta)^{\theta\eta\alpha} - (1-\delta)\theta\right] \\ & + (n_o n_n)\,(1+\delta)^{\theta(1-\eta)\alpha}\,[1-(1+\delta)\theta] \\ & + (n_o n_p)\,(1+\delta)^{\theta(1-\eta)\alpha}\left[(1-\delta)^{\theta\eta\alpha} - (1+\delta)\theta\right] \\ & + (n_o)^2\,(1+\delta)^{\theta(1-\eta)\alpha}\left[(1+\delta)^{\theta\eta\alpha} - (1+\delta)\theta\right].\end{aligned} \tag{45}$$

The interpretation is the following: the first line is the utility generated by a neutral island meeting another neutral island, times the product of their shares, the second line is the utility generated by a neutral island trading with a pessimistic island times the product of their shares and so on. This representation of welfare may be useful to assess the impact of a change in the composition of the economy on welfare. In particular, we observe that when $\delta = 0$, all islands believe and produce like they were neutral. Thus, we can study the derivative of equation (45) with respect to δ: we find (numerically) that it is negative, meaning that an economy composed by neutral, pessimistic and optimistic islands generates less utility than one where all islands are neutral and thus have rational expectations. The same is true for the case in which we consider only neutral and optimistic islands. The reason for this is that neutral and pessimistic islands trading with optimists generate a positive utility, however, optimistic islands trading with both the other types lose utility

– because they produce too much with respect to them – and the loss is greater than the gain. Therefore, welfare is maximized under rational expectations even when we include optimistic islands. In other words, increasing the number of optimistic or pessimistic islands, from a situation in which all islands are neutral, leads to a decrease in welfare. However, there are other initial situations – different from the rational expectations case – for which, under certain parameters, increasing the number of pessimistic or optimistic islands has a positive impact on welfare. For example, if $\theta > \frac{1}{1+\delta}$, optimists trading with neutral islands get a negative utility; thus, when there are many optimists, it is better for welfare that neutral islands decrease even more. We observed a similar logic for an economy composed only by neutral and pessimists, in section 4.1. The intuition, both in the two-types and in the three-types economy, is that θ is positively related to the amount of work, therefore, if it is too with respect to the others' production, it implies that islands are working too much and losing utility.

Social influence, thus, has a positive impact on the economy's welfare as far as it creates the rational expectations equilibrium, but has a negative impact on it in those cases in which the economy is attracted to a situation characterized by all islands being optimistic. However, in some cases, even the attraction to more optimistic scenarios can be positive for welfare.

7 Conclusions

In the present paper, our aim is to formalize in a simple way the formation of animal spirits under uncertainty. Animal spirits are waves of optimism and pessimism that partly drive the business cycle and that are also influenced by real economic outcomes. They are correlated across individuals because individuals' beliefs and expectations are influenced by the beliefs of others; therefore, social influence plays a crucial role.

We incorporate such animal spirits in a macro model similar to that developed by Angeletos and La'O (2013), in which the economy is composed of islands that trade in random pairs and take production and employment decisions in a context of uncertainty, that is, before knowing their terms of trade. In such a model, therefore, islands have to base their decisions on their beliefs about the terms of trade: we assume that they can be optimistic, pessimistic or neutral. Islands are heterogeneous in their productivity, which is log-normally distributed across them. Therefore, islands' beliefs influence their production, meaning that the expected output of optimists is higher than that of neutral islands that, in turn, is higher than the expected output of pessimistic ones. Islands can switch belief over time, the switching mechanism is similar to that presented in Lux (1995). In particular, we develop two versions of it: in the first one, the transition rate from one type to the other depends only on the real economic outcomes, that is, on the difference of the expected profits earned by those two types. In the second version we include also social influence: the higher the number of one type of islands in the economy, the higher the transition rate towards that type.

In order to study the dynamics of our economy, we first focus on an economy composed by only two types of islands: the neutral and the pessimistic ones. We find that, without social influence, the dynamics has a unique globally stable equilibrium which is characterized by at least half of the islands being pessimistic. Therefore, without social influence, the economy tends to be dominated by more pessimistic beliefs. By including social influence beyond a certain threshold, we find that, the dynamics of the economy presents a qualitative change. In particular, two new equilibria arise, one unstable and one stable, both characterized by more islands being neutral. Moreover, with a certain level of social influence, the result is that actually the stable equilibrium is characterized by all islands being neutral, which is an interesting scenario, given that when all islands are neutral, we find that they have rational expectations. Moreover, under rational expectations, welfare is maximized. Therefore, social influence emerges to have a positive influence one the dynamics of the economy: it allows the economy to converge to the rational expectations equilibrium increasing welfare.

We extend our analysis to an economy composed of three types of islands: optimists, pessimists and neutral ones. We study the two-dimensional dynamics of such an economy and observe that, without social influence, even in the three-types economy, the equilibrium is stable, unique and characterized by the majority of islands being pessimistic, some neutral and very few optimistic. By including a social influence beyond a certain threshold, equilibria rise to four; in particular, one stable equilibrium is characterized by almost all islands being neutral and another, instead, by almost all islands being pessimistic. Furthermore, there is an area where the economy is attracted to being composed of more optimistic islands. As regards welfare, even taking into account optimistic islands, it is maximized under rational expectations.

To conclude, we find that social influence plays a crucial role in reestablishing the rational expectations equilibrium – which otherwise is unstable – increasing welfare of the whole economy. An important future step regarding the formation of animal spirits may be to formalize precisely how individuals form their expectations in an economy characterized by uncertainty.

8 Appendix

Proof of proposition 1 The average output of neutral islands is given by:

$$\int_0^{+\infty} \left\{ K_1^{\eta\gamma} A_i^{\eta\alpha} \, \mathbb{E}(A_i^{\eta\alpha})^{\eta\gamma\theta} \right\} dF(A_i), \tag{46}$$

which can be rewritten as $K_1^{\eta\gamma}\,\mathbb{E}(A_i^{\eta\alpha})^{\eta\gamma\theta+1}$. Given that $\gamma \equiv \frac{1}{1-\theta}$ and $\alpha \equiv \frac{1}{1-\theta+\eta\theta}$, we have that $\eta\gamma\theta+1 = \frac{\gamma}{\alpha}$ and therefore the average output of neutral islands equals the neutral belief $K_1^{\eta\gamma}\,\mathbb{E}(A_i^{\eta\alpha})^{\frac{\gamma}{\alpha}}$.

Profits Profits are given by the difference of revenues and costs: $\pi_i = p_i y_i - w_i n_i - r_i k_i$. Costs are obtained by considering the optimal quantities and prices of labor and land. As for labor, we have that wage must be equal to the marginal disutility of working and to the expected marginal utility of labor:
$\frac{\delta y_{it}}{\delta n_{it}} = \frac{\delta(A_i(n_{it})^{\Theta}(k_{it})^{1-\Theta})}{\delta n_{it}} = \mathbb{E}_{it}[p_{it}]\Theta\frac{y_{it}}{n_{it}}$, therefore we have that $n^{\varepsilon-1} = \mathbb{E}_{it}[p_{it}]\Theta\frac{y_{it}}{n_{it}}$. From the latter we can find the optimal level of labor $n_{it} = (\mathbb{E}_{it}[p_{it}]\Theta y_{it})^{\frac{1}{\varepsilon}}$. Since $w_{it} = n^{\varepsilon} - 1$, therefore the optimal $w_{it}n_{it} = n^{\varepsilon} = \mathbb{E}_{it}[p_{it}]\Theta y_{it}$.

As for land, the optimal r_{it} must be equal to the marginal utility of k_{it} which is given by $\frac{\delta y_{it}}{\delta k_{it}} = \frac{\delta(A_i(n_{it})^{\Theta}(k_{it})^{1-\Theta})}{\delta k_{it}} = \mathbb{E}_{it}[p_{it}](1-\Theta)\frac{y_{it}}{k_{it}}$, k_{it} in equilibrium is equal to K which is the fixed endowment of land and ti is normalized to one, therefore the optimal $r_{it}k_{it} = \mathbb{E}_{it}[p_{it}](1-\Theta)y_{it}$.

We already know that $p_{it} = y_{it}^{-\eta}y_{jt}^{\eta}$, therefore $\mathbb{E}_{it}[p_{it}] = y_{it}^{-\eta}\,\mathbb{E}_{it}[y_{jt}^{\eta}]$. Therefore, profits are given by $p_{it}y_{it} - w_{it}n_{it} - k_{it}r_{it} = y_{it}^{1-\eta}y_{jt}^{\eta} - \mathbb{E}_{it}[p_{it}]\Theta y_{it} - \mathbb{E}_{it}[p_{it}](1-\Theta)y_{it} = y_{it}^{1-\eta}y_{jt}^{\eta} - \mathbb{E}_{it}[y_{jt}^{\eta}]\Theta y_{it}^{1-\eta} - \mathbb{E}_{it}[y_{jt}^{\eta}](1-\Theta)y_{it}^{1-\eta} = y_{it}^{1-\eta}y_{jt}^{\eta} - y_{it}^{1-\eta}\,\mathbb{E}_{it}[y_{jt}^{\eta}]$, or:

$$y_{it}^{1-\eta}(y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta})). \tag{47}$$

Profits of neutral islands, when everybody in the economy is neutral We showed before that neutral islands, in an economy without social influence and composed only by neutral islands, know the expected output of the trading partner. Therefore, $y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta}) = 0$ and profits are zero.

Profits of optimist islands when all islands are optimist Given what we wrote above, we can easily show that profits of optimist islands are negative when all islands are optimist. Profits are negative only when $(y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta})) < 0$, since $y_{it}^{1-\eta} > 0$. We already showed that this is the case for optimists.

In the case in which there are also other types of islands – neutral or pessimists –, given that $y_{it}^{\eta} < y_{jt}^{\eta}$, where i is neutral or pessimist and j is optimist, it is obvious that $(y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta}) < 0$ is even more negative.

Profits of pessimist islands when all islands are pessimist Similarly, we already know that $y_{jt}^{\eta} - \mathbb{E}_{it}(y_{jt}^{\eta}) > 0$, therefore on average pessimists earn positive profits when trading with pessimists and even more so when they trade with other types of islands.

Transition rates for optimists and pessimists If we plug in the transition rates we get the dynamics for the share of pessimists

$$\begin{aligned}\dot{n_p} &= (n - n_o - n_p)v\exp(-a_0\bar{\pi}_{np,t}(n_p, n_o)) \\ &+ n_o v\exp(-a_0\bar{\pi}_{op,t}(n_p, n_o)) - n_p v\exp(a_0\bar{\pi}_{np,t}(n_p, n_o)) \\ &- n_p v\exp(a_0\bar{\pi}_{op,t}(n_p, n_o)),\end{aligned} \tag{48}$$

and the dynamics of the share of optimists:

$$\begin{aligned}\dot{n_p} &= (n - n_o)v\exp(-a_0\bar{\pi}_{np,t}(n_p, n_o)) \\ &+ n_o v\exp(-a_0\bar{\pi}_{op,t}(x, z)) - 2n_p v\cosh(a_0\bar{\pi}_{np,t}(n_p, n_o)) \\ &- n_p v\exp(a_0\bar{\pi}_{op,t}(n_p, n_o)).\end{aligned} \tag{49}$$

Three-types economy without social influence Figure 8.1 shows the two-dimensional system without social influence and with $a_0 = 2$. As for $a_0 = 1$, we observe a unique equilibrium at $A1 = (0.6494, 0.0932)$ with two negative eigenvalues: -2.3757 and -1.3454 – of which the smaller in absolute value is represented by the double arrows. This is therefore a stable node.

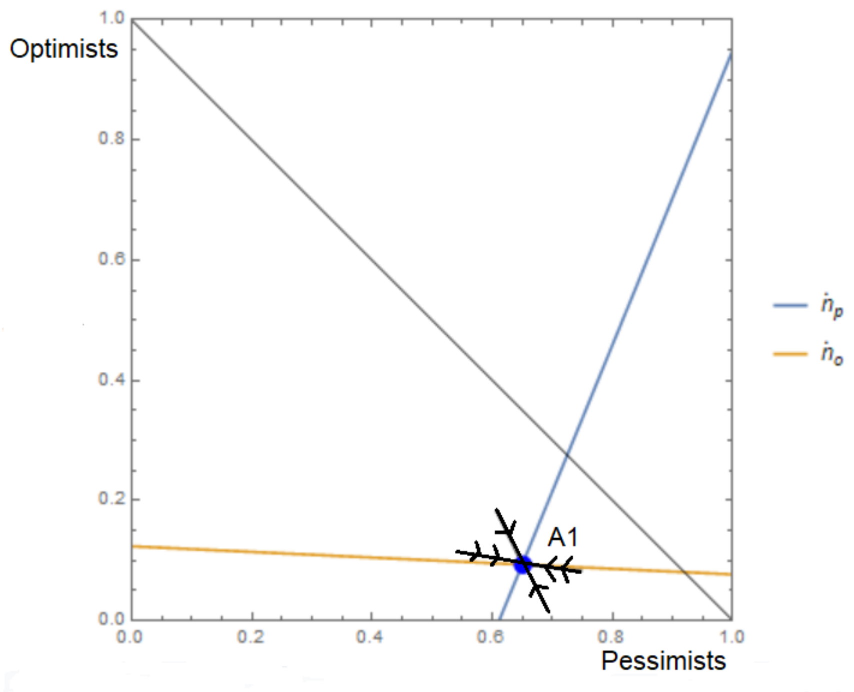

Figure 8.1: Phase portrait, no social influence and $a_0 = 1$

References

Angeletos, G.-M., and J. La'O (2013): "Sentiments," *Econometrica*, 81(2), 739–779.

Azariadis, C. (1981): "Self-fulfilling prophecies," *Journal of Economic Theory*, 25(3), 380–396.

Benhabib, J., and R. E. Farmer (1994): "Indeterminacy and increasing returns," *Journal of Economic Theory*, 63(1), 19–41.

Brock, W. A., and C. H. Hommes (1997): "A rational route to randomness," *Econometrica: Journal of the Econometric Society*, pp. 1059–1095.

——— (1998): "Heterogeneous beliefs and routes to chaos in a simple asset pricing model," *Journal of Economic dynamics and Control*, 22(8-9), 1235–1274.

Burnside, C., M. Eichenbaum, and S. Rebelo (2016): "Understanding booms and busts in housing markets," *Journal of Political Economy*, 124(4), 1088–1147.

Cass, D., and K. Shell (1983): "Do sunspots matter?," *Journal of political economy*, 91(2), 193–227.

Cooper, R., and A. John (1988): "Coordinating coordination failures in Keynesian models," *The Quarterly Journal of Economics*, 103(3), 441–463.

De Grauwe, P. (2011): "Animal spirits and monetary policy," *Economic theory*, 47(2-3), 423–457.

Evans, G. W., and S. Honkapohja (2012): *Learning and expectations in macroeconomics*. Princeton University Press.

Keynes, J. M. (1936): *The General Theory of Employment Terest and Money*. Macmillan and Company.

King, R., and S. Rebelo (2000): "Resuscitating Real Business Cycle, Handbook of Macroeconomics,(Woodford, M., and Taylor, J., Eds.), Vol. IB," .

LeBaron, B., W. B. Arthur, J. H. Holland, R. Palmer, and P. Tayler (1997): "Asset pricing under endogenous expectations in an artificial stock market," .

Lux, T. (1995): "Herd behaviour, bubbles and crashes," *The economic journal*, pp. 881–896.

Muth, J. F. (1961): "Rational expectations and the theory of price movements," *Econometrica: Journal of the Econometric Society*, pp. 315–335.

Simon, H. A. (1955): "A behavioral model of rational choice," *The quarterly journal of economics*, 69(1), 99–118.

Tuckett, D. (2012): "Financial markets are markets in stories: Some possible advantages of using interviews to supplement existing economic data sources," *Journal of Economic Dynamics and Control*, 36(8), 1077–1087.