An Attempt at a Reconciliation of the Sraffian and Kaleckian View on Desired Utilization

Reiner Franke*
University of Kiel (GER)

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Abstract

The paper derives the firms' desired rate of utilization from an explicit maximization of a conjectured rate of profit at the micro level. Invoking a strategic complementarity, desired utilization is thus an increasing function of not only the profit share but also of actual utilization. Drawing on recent empirical material and a straightforward functional specification, the model is subsequently numerically calibrated. In particular, this ensures a unique solution for a steady state position in which the actual and the endogenous desired rates of utilization coincide. On the other hand, it turns out that the anticipated losses of the firms by not producing at the desired level are rather marginal. Hence there may be only a weak pressure on them to close a utilization gap in the ordinary way by suitable adjustments in fixed investment. It is indicated that this finding may serve Kaleckian economists as a more rigorous justification for viewing their equilibria as pertaining to the long run, even if they allow actual utilization to deviate persistently from desired utilization.

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*Email: `franke@uni-bremen.de`. I am grateful to Daniele Tavani and Heinz Kurz for clarifying discussions and also Peter Skott for his rather critical remarks on a preliminary draft.

1 Introduction

The Kaleckian modelling framework of distribution and growth has become an influential tool for macroeconomic analysis. This nevertheless does not mean that economists do not have quite different views about its specific design. A central issue that divides them into separate camps is the concept of the firms' desired, or normal, utilization of productive capacity. To begin with authors trained in a Sraffian, Marxist or Classical tradition, in most models they conceive of desired utilization as a given and fixed magnitude determined in the end by some sort of profit maximization, although usually this principle is not made more precise.[1] While generally firms will not utilize their capital stock at the desired level and may under- as well as overutilize their equipment, for an equilibrium to make sense in a long-run perspective there ought to be some mechanism that adjusts the actual utilization rate to the desired rate. Otherwise the firms' rate of capital accumulation cannot remain constant and growth would systematically drift upward or downward, respectively.

Within the confines of current theoretical discussions, a typical implication of this approach is the problem of Harrodian instability, which is a dynamic process of cumulative causation in investment, output and expectation formation that inevitably drives the economy away from steady state growth once it is disturbed from this path. This feature is a big challenge and various ways have been proposed of how to cope with it. Thus either local stability is achieved or a stabilization of the system in the outer regions of the state space, so that the economy would persistently oscillate around the steady state. Many people can accept some of these solutions, at least for the moment being and for a given limited purpose, whereas other economists remain unconvinced by any of them (Hein et al., 2012, p. 140). The latter, then, can be seen as forming a different camp on its own.

These unconvinced economists, who are often labelled Kaleckians or neo-Kaleckians, treat the rate of capacity utilization as an accommodating variable not only in the short run but also in the medium and long term. Specifically, this assumption allows them to maintain two paradoxes that seem particularly dear to them, the paradox of thrift and the paradox of costs. This means that a decrease in the propensity to save or an increase in real wages leads to an increase in output, utilization or growth rates. Consequently, the Kaleckians are also able to reject the famous dictum by Duménil and Lévy (1999) that we are Keynesian in the short term and classical in the long term.

Following the discussion in Hein et al. (2012), Kaleckian economists may in finer detail be subdivided into three groups, although they are not necessarily mutually exclusive. To begin with the second group in the article's ordering, their

[1] This will just be our concern later in the paper.

proponents argue that a normal rate of utilization may be only one of several targets that the firms seek to pursue. This means that, depending on their preferences, the firms trade off one target with another and so are willing to accept certain deviations even if they are persistent.[2]

The third group seems to have gained some particular attention lately. Its adherents abandon the concept of desired utilization as a rate that is exogenously fixed and determine it as an endogenous variable within their models. Most prominent is the drastic approach to reverse the causality of actual and desired utilization altogether. Viewing the desired rate as a moving target influenced by past values, it is the desired rate that adjusts to the actual rate and not the other way around.[3] The approach has been criticized on conceptual grounds (e.g., see Skott, 2012). What makes it even more problematic is that it typically comes as a joint hypothesis. That is, the adjustments of desired utilization are combined with a few other dynamic mechanisms, however in a way such that, technically speaking, they give rise to collinearities in the difference or differential equations. The effect is that instead of a unique equilibrium such a system has a whole continuum of steady states, with a whole continuum of utilization rates. This feature is then praised as hysteresis or path dependence. On the other hand, to an outsider at least, these constructions appear somewhat artificial, built in just for the modeller's special purpose.[4]

The present paper may be mainly of interest to the first group of Kaleckian macroeconomists: they skirt the problem of Harrodian instability by simply denying the firms any need in their investment decisions to react to a possible utilization gap.[5] There are two kinds of justifications for this, which often are not clearly distinguished. One says (quoting Hein et al., 2012, p. 146) that "the notion of 'normal' or 'desired' utilisation should be defined more flexibly as a range of degrees rather than as a single value." According to the other explanation, "firms may be quite content to run their production capacity at rates of utilization that are within an acceptable range of the normal rate of utilization," where "under this interpretation,

[2]Franke (2017b) makes the point that one may not be satisfied too early with such a situation. He brings into consideration the possibility that the agents in a concrete model may have been assumed to be too restrained in their behaviour. Accordingly, upon closer reflection, they would have more flexible or additional ways to react to some of the discrepancies they perceive. The gaps could thus disappear, or new discrepancies might emerge and open up new discussions.

[3]The basic argument refers to behavioural economists who state the managers of the firms are satisficers rather than maximizers. Specifically, if goals are not met a firms readjusts its aspiration level downwards. In this way desired utilization assumes a conventional character (and is better called normal utilization).

[4]By introducing an additional feedback effect at some suitable place in such a model, the collinearities may disappear and uniqueness of a steady state be re-established.

[5]Though most often investment still reacts to the level of utilization, which may appear a somewhat mechanical assumption.

the normal rate of capacity utilization is more a conventional norm than a strict target." (Both formulations in Hein et al. in the same paragraph, suggesting that they are essentially considered to be equivalent.)

It follows as a consequence that, in comparing, say, two equilibria before and after a change in one of the parameters in a model, both can be interpreted as long-run equilibrium positions, even though they may exhibit different degrees of utilization. This is legitimate as long as they fall within that "acceptable range". The conclusion rests on the assumption of kind of a corridor of stability, from which Harrodian instability is being excluded. Missing in presentations of this approach are, however, discussions of how wide or narrow such a corridor is and, more importantly, what criteria precisely determine the boundaries beyond which the original conclusions would no longer be valid.

In a first and short description to situate the paper in this literature, our analysis may be said to start out from a Sraffian or Marxist perspective that assumes the existence of a unique desired rate of utilization which is determined on the basis of a profit-maximization principle. One point is that the optimization problem will be made explicit, and that its solution will be state-dependent. On the other hand, we will also emphasize that this notion need not be in full contradiction to the Kaleckian point of view, according to which the firms are willing to accept a certain permanent gap between actual and desired utilization. While, in our framework, the latter is not a conventional and somewhat vague norm but a definite benchmark for the firms' current rate of utilization, a second point is that a possible gap could be regarded as fairly inessential in terms of forfeited profits.

The paper begins by widening the Sraffian perspective on desired utilization, for which it withdraws from the notion of an exogenously given and fixed magnitude. Desired utilization will rather be represented as a function of some macroeconomic variables. This relationship will not just be postulated but derived from a *hypothetical* profit maximization problem which, it may be pointed out, may or may not play a role in the firms' actual decisions about output, prices and investment.[6] It is furthermore important in this concept that the objective function includes some expectational elements regarding imaginary variations of a firm's utilization rate. In particular, these conjectured profits are also influenced by the current state of the economy. This assumption implies that not only the profit share but also actual utilization will have a bearing on the level of desired utilization.[7]

[6]That is, our concept of desired utilization will be compatible with a wide variety of macroeconomic modelling approaches.

[7]A reference to expectational elements is necessary in any case. Otherwise in the macroeconomic standard formulation of profit rates, it would not be clear why firms should not desire to produce at the highest technically and institutionally feasible level, which would then rule out any 'overutilization' in these models.

As a consequence, the Sraffian condition for a long-run equilibrium, which says that actual utilization must equal desired utilization, determines the steady state rate in an endogenous way, namely, as the solution of a fixed-point problem for a real function of the aggregate utilization rate. As it turns out in our specifications, this gives rise to a unique steady state which only changes if the profit share changes (which is here treated as an exogenous variable). On the other hand, changes of a saving propensity, for example, continue to have no long-run effects. So far, our approach to desired utilization remains close to the Sraffian conception, only that it lends more flexibility to it.

The significance of our analysis for the Kaleckian approach can be shown when it comes to numerical issues. It will be very helpful that we can make use of recent empirical estimates of the responsiveness of desired utilization to *ceteris paribus* changes in its determinants (Tavani and Petach, 2018). They allow us to specify our objective function of conjectured profits and calibrate the parameters for its expectational elements. In this way it is found that even larger variations of a firm's utilization rate would affect its profitability only marginally; the effects of other factors (such as the profit share) in the firm's decision on investment will actually be much more important. Hence, as mentioned above, deviations of actual from desired utilization may appear rather inessential to the firms and, therefore, may well persist even over longer periods of time, or vanish only extremely slowly. This result and the ideas from which it is derived are put forward as a proposal of how the Sraffian and Kaleckian view on desired utilization could be reconciled.[8]

The remainder of the paper is organized as follows. Section 2 is a prelude to the paper's main analysis. It first shows that too simple a profit maximization approach to determine desired utilization will not work. Subsequently some treatments in the literature are mentioned that can later be compared to our results. Section 3 introduces the idea of a conjectured rate of profit the maximization of which determines the desired rate of utilization. It begins with a general formulation of the framework and then proposes a convenient functional specification, which permits an explicit computation of the solution to the problem. Section 4 offers a numerical calibration of the concept, while Section 5 contains a more detailed discussion of the methodological points that have just been sketched provisionally. Section 6 concludes.

[8]Some of the ideas or their implications could perhaps be considered to contain some anti-Kaleckian flavour. The paper will not be interested in such labelling attempts. The term "Kaleckian" will only apply to a non-zero utilization gap in a long-run equilibrium, which might now be justified by making reference to our analysis.

2 Preliminary observations

2.1 A simple but futile attempt at desired utilization

Following a standard reference like Kurz (1986), underlying the concept of desired utilization is the principle of cost minimization or, complementarily, profit maximization. Regarding the heterodox macroeconomic literature it is, however, somewhat amazing that there does not seem to be any discussion how this fits in with the standard formulation of the profit rate, which is linearly increasing without bounds in the output-capital ratio.

To introduce profit maximization in a formal way into macroeconomics, it is an immediate idea to abandon the linear specification. Assume instead that at sufficiently high levels of utilization the increments in the profit rate upon a further rise in utilization begin to decline and eventually also the level of the rate itself. Obvious reasons for this nonlinearity are that overtime work or night shifts are more expensive for the firms, and possibly the maintenance costs of the capital stock would then be higher, too. Accordingly, in a succinct manner, the profit rate may be conceived of as a concave function $r = r(u)$ of utilization u that first increases and then decreases. Desired utilization u^d can then be determined as the level of u that maximizes this function, which is characterized by a zero derivative $dr(u^d)/du = 0$.

The problem with this concept is its implication for an IS market clearing equilibrium. Consider the familiar framework of a one-good economy without government and foreign trade where labour is in perfectly elastic supply. Firms pay out all profits to the rentiers households, who save a fraction s of them. Workers do not save, so that total savings normalized by the capital stock are given by $g^s = s\,r(u)$. The firms' benchmark for fixed investment is an expected rate of growth g^e. They plan to increase their capital stock somewhat faster (more slowly) if their current utilization is above (below) their desired utilization. Hence investment is described by a function $g^i = g^e + \gamma(u - u^d)$, where γ is a positive reaction coefficient.

Suitable variations of utilization ensure a balancing of supply and demand, that is, excess demand vanishes, $\mathrm{ED} := g^i - g^s = 0$. Focussing on a a situation where IS utilization happens to coincide with desired utilization, let us ask how $u = u_{\mathrm{IS}}$ changes in response to a rise in the expected growth rate. Using the Implicit Function Theorem, one calculates

$$\frac{\partial u_{\mathrm{IS}}}{\partial g^e} \;=\; \frac{-\,\partial \mathrm{ED}/\partial g^e}{\partial \mathrm{ED}/\partial u} \;=\; \frac{-1}{\gamma} \;<\; 0 \tag{1}$$

owing to the fact that $\partial g^s/\partial u = s\,dr(u^d)/du = 0$ from the tangency condition in the denominator. The result that more optimistic growth expectations cause economic activity to decline would certainly be hard to accept. In addition, $\partial g^s/\partial u = 0$ means

that the Keynesian stability condition cannot possibly be fulfilled. Therefore, as simple and straightforward as this approach to desired utilization is, it just does not work and one has to consider the problem in finer detail.

2.2 Examples of endogenous desired utilization in the literature

Much of economic theory entertains the notion that firms strive to maintain a certain level of excess capital capacity. Two reasons to explain this objective stand out. First, firms operating in imperfect markets keep some capacity idle in order to respond rapidly to unanticipated surges in demand. The other argument is based on the assumption that the amount of excess capacity has a certain influence on the entry of new firms into an industry. The assumption can be justified by an extension of the limit-price argument: the probability of entry may be determined, not by the actual price, but by the price which would prevail if all existing capacity were fully utilized. Excess capacity can then deter entry because it demonstrates commitment to the particular line of business and signals the willingness of the firm to defend its position—should new entry take place—by expanding production and cutting prices (for this and the following, see Skott, 1989, pp. 53ff).

Usually this kind of reasoning is rather informal and quickly leads to the assumption of a given desired, or normal, rate of utilization, with no further hints as to what determines its specific level. In principle the rate may be subject to variations, but this is regarded as an issue outside the model. The discussion by Skott (1989) is a certain exception. He supposes that desired utilization u^d is a decreasing function of expected rates of entry, which in turn are an increasing function of the ratio of pure profits to total revenue. The costs that are to be deducted from the revenue include the wage bill, capital depreciation and the cost of finance. In this way, u^d depends positively on the real wage or the wage share, for that matter (wages$\uparrow \Rightarrow$ profit ratio$\downarrow \Rightarrow$ expected entry rate$\downarrow \Rightarrow$ $u^d\uparrow$.) The depreciation rate and the rate of interest exert a positive influence, too. Interestingly, in the expression for the profit ratio the latter two are divided by the utilization rate of the individual firm. Hence, in addition to the effects just mentioned, desired utilization of the "representative firm" (Skott, 1989, pp. 54) is a decreasing function of the average utilization in the economy. These statements may be later compared to the results implied by our own approach to the determination of desired utilization.[9]

[9]Referring to Skott (1989) and the deterrence mechanism, Palacio-Vera (2009, p. 179) introduces desired utilization directly as a decreasing function of the profit share. Franke (2018, pp. 6f), because of his interest in the effects of monetary policy, focusses on another single component. Likewise referring to Skott, he specifies desired utilization as an increasing function of the (real) rate of interest. By contrast, as already mentioned in the Introduction, Hein et al. (2012, eq. (23) on p. 160) take a

3 The concept of endogenous desired utilization

3.1 The firm's profit maximization problem

Considering a single firm i, let y_i be its output-capital ratio and $\bar{y}$ the maximum value of this ratio, which is the same for all firms and technically given by operating the existing capital stock seven days a week for 24 hours or another institutional constraint.[10] The rate of capacity utilization is $u_i = y_i/\bar{y}$ (hence $u_i \leq 1$). Assuming a uniform rate of capital depreciation δ and a uniform profit share h, which we treat as exogenously given, the firm's current rate of profit is

$$r_i \;=\; h\bar{y}u_i - \delta \tag{2}$$

Evidently, this formulation is unsuitable to derive a desired level of utilization from a maximization argument; with the linear influence of u_i there is simply nothing to maximize in (2). When this paper nevertheless aims at determining desired utilization as the solution of an explicit optimization problem, the underlying objective function has to be reconsidered more carefully.

As it has already been indicated in the Introduction, it should be emphasized at the beginning that such a problem and the utilization rate thus obtained are a hypothetical construct that can stand on its own. Therefore, it has no specific implications for the modelling of the firms' actual decisions about the variables it has under control. Specifically, the firms may continue to be 'satisficers' that do not invoke a formal optimization procedure at all in these decision processes. With our concept of desired utilization we just want to put forward a benchmark with which the firms could compare their present level of utilization, in what particular modelling context so ever.

For example, an investment function may or may not include deviations of actual utilization from this 'new' desired rate. Regarding the determination of output, the concept will also be compatible with both IS market clearing as well as disequilibrium on the goods market, that is, output may adjust to aggregate demand or it may alternatively be predetermined in the short period by some other principle. Likewise, the concept does not anticipate a certain choice regarding the modelling of prices or inflation.

different perspective (a conflicting-claims model of inflation) and derive a negative dependence of u^d on the interest rate.

[10]The analysis accepts the limits of ordinary macroeconomics and neglects different degrees of intensity and also the problem of how to treat different vintages of the capital stock, where dated or 'obsolete' equipment will remain shut down most of the time but occasionally be freed from the dust to earn quasi-rents.

Starting our discussion of a meaningful profit function, it may be realized that in an environment of imperfect competition (which we presume) a firm i cannot simply assume that it would be able to sell whatever it produces, at least not at the conditions presently prevailing. So, when considering its future revenues under imaginary variations of its utilization u_i, it cannot expect the profit rate to obey the simple relationship (2), not even in the absence of bottlenecks or disproportional cost effects. If for instance the firm considers to increase its production, it may well have to reduce the price somewhat in order to sell the additional output. In fact, this elementary argument corresponds to the concept of a firm's conjectured demand curve that many partial models for a single industry work with. For various reasons the firm might also anticipate higher wages that it would have to pay. A complementary aspect is that higher sales may require higher marketing efforts for the firm or taking other measures to acquire additional customers, which would likewise affect the revenues. Conversely, lower production and thus lower supply could mean higher sales prices or lower unit costs of this kind.

Thinking about variations of its output or utilization rate, respectively, a firm will therefore not maximize an expression like (2) but profits based on expectations about these effects and their quantitative order of magnitude. A straightforward way to take them into account is a *conjectured* rate of profit r_i^c, in which these anticipated losses are subtracted from (2). This rate is concerned with the question what, given the present capital equipment, would be the profits under hypothetical variations of a firm's utilization rate u_i. The question leads us to introduce into (2) a loss function $\ell = \ell(u_i)$ that is increasing in the firm's utilization rate.[11] As usual in cost arguments like this, let us furthermore suppose that the function is convex and twice differentiable, i.e., $\ell'(u_i) > 0$, $\ell''(u_i) > 0$ for all $u_i < 1$ (though the 'losses' themselves could also be negative for low utilization rates, as h or δ might already include normal marketing costs, etc.).

This is a fairly standard reasoning so far. It is, however, supplemented by including another type of the firms' expectations about the future demand for their products. The basic idea and its specific treatment is stimulated by a recent paper by Tavani and Petach (2018). The authors' point is that in deciding on its optimal production, a firm will also take the activity levels of the other firms into account. To this end they refer to a firm's marginal user cost and argue that "no firm wants to be the first to ramp up utilization while everyone else is not, or equivalently that there are strong incentives to wait for other firms to increase utilization first and then do the same" (p. 4). The reason is that the anticipated relative user costs of a single

[11] Anticipated price and/or wage adjustments are also likely to affect a firm's profit share. In order to streamline the results it will, however, be convenient to treat the profit share h as a predetermined variable, which is exogenously given in the following discussion, and assume that possible effects on real wages and labour productivity are all captured by the loss function.

firm are lower when other firms are doing the same (p. 14). On the basis of these considerations the authors postulate that the marginal user cost of an individual firm decreases with average utilization in the economy.

As this short explanation may remain somewhat vague and perhaps also debatable for someone not very familiar with the theoretical conception of user costs, we propose a slightly different and more direct story in this paper. Let us assume that firm i has expectations about the general state of aggregate demand in the near future. They are relevant for the individual firm since it expects that it will be able to profit from a general improvement, too. Accordingly, even if it does not change its utilization, the firm could moderate its marketing activities and/or sell its current flow of output at higher prices. Given the profit share in the expression for the firm's gross value added $h\bar{y}u_i$, the expectations in this case can be captured by multiplying this term by a 'benefit' factor larger than one. Our proposal for a parsimonious modelling of this aspect is *(a)* that the general state of expected demand is proxied by the *aggregate* utilization rate u, and *(b)* that for the firm's conjectured profit rate r_i^c the term $h\bar{y}u_i$ is multiplied by a function that increases in expected demand and thus in u. This function may be called the firm's benefit function and designated $b = b(u)$, with $b'(u) > 0$ for its derivative.[12]

According to the classical argument, on the other hand, firms may also fear new entries in times of generally higher economic activity, which could lure away part of their customers from them. Similar to the reasoning before, a firm's attempt to protect its market share would mean higher costs or lower prices for it. In the present context this idea could be represented by postulating a negative slope for the function $b = b(u)$. Thus, an increasing function amounts to assuming that the threat of new entry is of secondary importance. In the end this is an empirical issue. Fortunately, we can quote evidence in favour of a positive derivative $b'(u) > 0$, which will be a subject in the next subsection.

On the whole, therefore, when a single firm i sets up a benchmark in order to reach an assessment of its current rate rate of utilization, whether the latter is particularly high or low, what it is supposed to maximize by a suitable choice of u_i is the following conjectured rate of profit r_i^c:

$$r_i^c \;=\; b(u)\,h\bar{y}\,u_i \;-\; \delta \;-\; \ell(u_i) \tag{3}$$

[12]The reasoning behind the influence of u is similar to that for an influence of u in the usual macroeconomic investment functions, which likewise in order to make sense contain a little story about expectations that, for simplicity, are condensed in u. Besides, not much would change if the benefit function, instead of attaching it multiplicatively to the profit term $h\bar{y}u_i$, were inversely related to the loss function. Applying a side remark by Flaschel and Skott (2006, p. 318) to the present setting, the benefit function may also, or alternatively, be a function of the economy's current rate of growth. Certainly, in a more comprehensive modelling one could think of additional variables entering the formulation of conjectured profits.

Assuming a sufficiently convex shape of the loss function $\ell = \ell(u_i)$, the profit rate r_i^c will be increasing for low values of u_i and eventually decreasing as u_i approaches unity. A maximum of r_i^c is obtained at a value of u_i where the marginal profit rate $\partial r_i^c / \partial u_i$ is zero. The negative second derivative $\partial^2 r_i^c / \partial u_i^2 = -\ell''(u_i) < 0$ ensures that it is also unique, so that the firm's desired utilization u_i^d is well-defined. As it depends on aggregate utilization and the uniform profit share, this rate is the same for all firms. Hence we can drop the firm index i and write $u^d = u^d(u,h)$. This functional relationship is the paper's proposal for a rigorous and tractable theoretical concept of desired utilization based on profit maximization in a macroeconomic framework.

The reactions of the desired utilization rate to changes in its arguments are easily obtained. Defining the auxiliary function $f = f(u_i,u,h) := \partial r_i^c / \partial u_i = b(u)\,h\,\bar{y} - \ell'(u_i)$, which is zero in $u_i = u^d$, an application of the Implicit Function Theorem yields,

$$\begin{aligned} \frac{\partial u^d}{\partial u} &= \frac{-\partial f/\partial u}{\partial f/\partial u_i} = \frac{h\,\bar{y}\,b'(u)}{\ell''(u_i)} > 0 \\ \frac{\partial u^d}{\partial h} &= \frac{-\partial f/\partial h}{\partial f/\partial u_i} = \frac{b(u)\,\bar{y}}{\ell''(u_i)} > 0 \end{aligned} \tag{4}$$

In particular, desired utilization increases/decreases if actual utilization increases/decreases. However, in contrast to the dynamic adjustments of u^d as they are postulated by a group of Kaleckians mentioned in the Introduction, this rate is here directly linked to u and derived from an explicit maximization argument. The reason for the positive relation is the strategic complementarity established by the benefit function $b = b(u)$ with its positive slope. Accordingly, a rise in aggregate utilization raises the conjectured marginal profits and therefore, as the latter are decreasing in the firm's utilization, the value of u_i where this function cuts the zero line. The mechanism for $\partial u^d / \partial h > 0$ is analogous. Note that the positive reactions of u^d in (4) are the opposite of what Skott (1989, pp. 53–55) concludes from his assumption that firms use excess capacity as a strategic deterrence to new entry (cf. Section 2.2).

The notion of desired utilization should provide for the possibility that it coincides with actual utilization in a state of long-run equilibrium. That is, given the profit share h, the function should have a fixed-point u^o satisfying $u^d(u^o,h) = u^o$, and preferably it should be unique. The latter would be implied if desired utilization increases less than one-to-one with actual utilization in an economically relevant range, which is a feature that one will expect to apply anyway. Rather than formulate some general properties for the benefit and loss functions $b = b(u)$ and

$\ell = \ell(u_i)$ ensuring existence and uniqueness, we will propose a concrete and even numerical specification of them in the next subsection.

3.2 A functional specification of desired utilization

Tavani and Petach (2018) find it convenient to assume constant elasticities for their adjustment cost function, which serves to capture the paper's user cost of capital. Applying this obvious idea to the present setting, let us specify the loss and benefit functions as follows,

$$\begin{aligned} \ell(u_i) &= \theta_{\ell u}\, u_i^{\lambda} \;-\; \theta_{\ell o}\;, & \lambda > 1 \\ b(u) &= \theta_{bu}\, u^{\beta}\;, & \beta > 0 \end{aligned} \tag{5}$$

where of course $\theta_{\ell u}, \theta_{bu}$ are positive, too. The formulation with the three coefficients $\theta_{\ell o}$, $\theta_{\ell u}$, θ_{bu}, which presently might appear a bit clumsy, will make it easier for us to set up a position where the output-capital ratio attains a reasonable value and the actual and conjectured profit rates coincide. Moreover, using (5) in the profit maximization condition $\partial r_i^c / \partial u_i = 0$ gives rise to a meaningful closed-form solution for the desired utilization rate. Solving the equation for $u = u^d$ and computing the partial derivative with respect to actual utilization, we obtain,

$$u^d \;=\; u^d(u,h) \;=\; \mu\, h^{1/(\lambda-1)}\, u^{\beta/(\lambda-1)}\;, \qquad \mu := \left[\frac{\bar{y}\,\theta_{bu}}{\lambda\,\theta_{\ell u}}\right]^{1/(\lambda-1)} \;>\; 0 \tag{6}$$

$$\frac{\partial u^d}{\partial u} \;=\; \text{const.} \cdot u^{\beta/(\lambda-1)-1} \;>\; 0 \tag{7}$$

Certainly, given u and h a suitable choice of μ can ensure $0 < u^d < 1$. Desired utilization is a concave function of actual utilization if (and only if) the exponent in $\partial u^d / \partial u$ is negative. In order for the reactions of u^d to be not too strong one feels (and this will be justified a little below) that this property should indeed prevail. This requires the exponent of the benefit function to be bounded relative to the exponent of the loss function, while it is still positive. Hence, in precise terms, the following inequality should additionally be satisfied in (5),

$$0 \;<\; \beta \;<\; \lambda - 1 \tag{8}$$

Apart from the multiplicative factor μ, Tavani and Petach's (2018) so-called best-response function has the same form as (6), $u^d = h^{\eta}\, u^{\alpha}$ (maintaining our symbols). It is a great merit of their paper that they put this specification to empirical data on

(annual) state-by-industry output and labour shares for the United States.[13] After taking logarithms, the elasticities η and α can be estimated by simple linear regressions. The robustness of the results is checked by a number of variations in the construction of the data. The happy end of this endeavour is that the exponents come out highly significant, with the correct sign and the condition for concavity in u fulfilled, $\alpha < 1$.[14]

We make use of this work by choosing $\eta = 0.33$ and $\alpha = 0.70$ from somewhere in the middle of the range of estimates reported in the study. From this, our model's exponents β and λ result like $\beta = 2.10$ and $\lambda = 4.00$. With reference to the regression analysis in Tavani and Petach (2018) we can thus conclude that our approach to desired utilization and its specification by (6) has a certain empirical support, which also includes the validity of inequality (8).[15]

4 Calibration of the loss and benefit functions

4.1 Setting a steady state position

With the above estimate of the exponent of u in (6) and the resulting concavity of u^d, we are now sure that a salient condition for the existence of a long-run equilibrium value of utilization is satisfied. To be more precise, we think of a fully-adjusted steady state, as it is often called, which here reads $u^d(u^o,h) = u^o$. In fact, the function $u \mapsto u^d(u,h)$ is defined for all $u \geq 0$, with $u^d = 0$ for $u = 0$ and an infinite slope at this point. Given the profit share h, we have thus $u^d > u$ for low values of u. Since the slope steadily diminishes as u increases, it will eventually be less than one and subsequently so small that the function will cut the 45° degree line at some point $u = u^o$, which establishes the consistency condition $u^d(u^o,h) = u^o$. It

[13] The sample they construct consists of 19 years of observations over 20 industries and 50 federal states. After dropping a few meaningless state-by-industry pairs, they obtain a sample of size $N = 18,943$. To be precise, the estimations are on actual rather than 'desired' utilization, assuming that it is only idiosyncratic shocks that drive a wedge between the two. Apart from other estimation details to validate the results, general sceptical remarks may perhaps, to be begin with, be countered with the question, why should one find a clear relationship for actual utilization but would have to reject it (if such data could be constructed) for desired utilization? And why should actual utilization exhibit such a regularity at all?

[14] It is a remarkable detail of the estimations that even in non-tradable producing sectors, the demand for which is by definition only local, capacity utilization responds to changes in the level of average utilization (Tavani and Petach, 2018, p. 15). Generally, this finding is validated by using the interactive fixed coefficients estimator to control for unobserved common trends (private communication with Daniele Tavani).

[15] There is also quite some range of values η and α that would leave this inequality intact.

has, however, also to be checked that this utilization rate is feasible and falls short of unity.[16]

For such a numerical analysis we reverse the chain of reasoning. That is, we start with a numerical value of u^o that we would like to constitute a steady state. Quite arbitrarily, let us set it at $u^o = 0.30$. For the profit share we employ a rather familiar value of $h = 0.30$. The condition $u^d(u^o, h) = u^o = 0.30$ is then brought about by substituting these values and the exponents β and λ from above in (6) and solving the equation for the multiplicative constant μ. Table 1 summarizes our settings and the result. Demonstrating the uniqueness of this steady state position (apart from the meaningless solution $u = 0$), Figure 1(a) plots the function $u \mapsto u^d(u, h)$ over a wide range of utilization rates (the dashed line is the 45° line).

u^o	h	β	λ	$\longrightarrow$	μ
0.30	0.30	2.10	4.00		1.041

Table 1: Calibration of a steady state position.

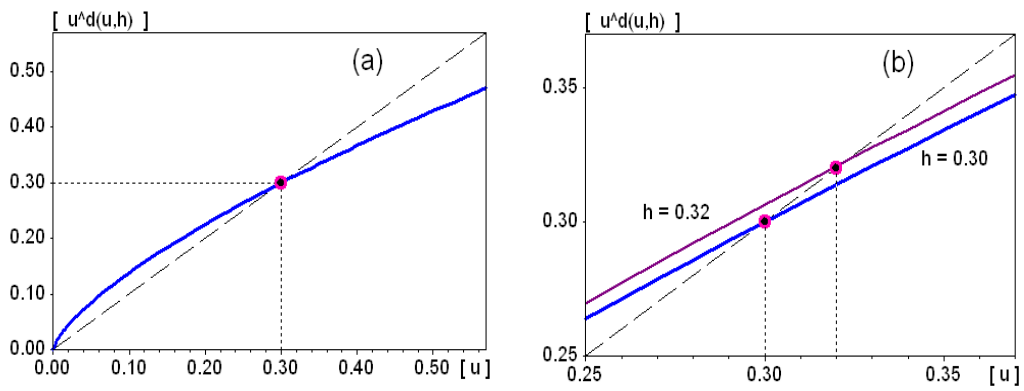

Figure 1: The calibrated function of desired utilization $u \mapsto u^d(u, h)$.

Panel (b) in Figure 1 depicts the graphs of the desired utilization function locally around $u = 0.30$ for two values of the profit share. In this detail the functions

[16] Tavani and Petach (2018) view the condition as a decentralized Nash equilibrium for the firms' best-response functions. They contrast its utilization rate with the rate obtained from an optimal growth problem and its socially coordinated choice of utilization. Finding that the former is always less than the latter, it is this difference that they interpret as the economy's spare capacity.

appear almost linear. The lower (blue) line has the original profit share $h = 0.30$ from panel (a) underlying, while the upper (purple) line is based on a higher share $h = 0.32$. It is already known from (7) that this change shifts the function upward. Therefore, as the utilization function is increasing in u and cuts the 45° line from above, its point of intersection with the latter shifts to the right.

It follows that independently of whether aggregate demand is supposed to be wage-led or profit-led in some sense, an improvement for the workers in the income distribution conflict, i.e. a lower profit share, causes utilization in a fully-adjusted steady state not to increase, but to decrease. Though the conclusion might not necessarily be welcomed in general, it should come as no surprise. The reason for it is, of course, the profit maximization on which the notion of desired utilization is based; this principle does not only apply to a given value of aggregate utilization u, i.e. $u^d(u,h^1) > u^d(u,h^o)$ for $h^1 > h^o$, but also to a comparison of two endogenously determined long-run equilibrium positions, where $u^1 = u^d(u^1,h^1) > u^d(u^o,h^o) = u^o$.

Numerically it can also be seen from Figure 1(b) that a rise in the profit share from 30% to 32% increases the utilization rate from 0.30 to approximately 0.32. Given that over the business cycle the macroeconomic output-capital ratio fluctuates by less than ± 5 per cent around its trend (i.e., u would fluctuate in a range 0.300 ± 0.015), the order of magnitude of this effect is certainly not negligible.

4.2 Implications for the conjectured rate of profit

The other coefficients θ_{bu}, $\theta_{\ell u}$, $\theta_{\ell o}$ in the loss and benefit functions in (5) can be determined once an anchor is provided for the latter. Let us to this end assume that the conjectured profits coincide with the realized profits in our steady state, that is, $b(u^o) = 1$ and $\ell(u_i^o) = \ell(u^o) = 0$, which is essentially just a matter of scale. The first equality can be directly solved for the value of θ_{bu}. Next, from the fixed-point equation $u^d(u,h) = u$ the auxiliary coefficient μ in (6) can be alternatively expressed as $\mu = u/[h^{1/(1-\lambda)}\, u^{\beta/(1-\lambda)}]$. Putting $u = u^o = 0.30$ and equating this μ to the term in (6) allows us to determine $\theta_{\ell u}$. The maximum value $\bar{y}$ of the output-capital ratio appearing in this equation is obtained from choosing a steady state value of the ratio, designated y^o, and then solving the relationship $u^o = y^o/\bar{y}$ for $\bar{y}$. For y^o itself we draw on Franke (2017a) and settle down on $y^o = 1.09$. Lastly, $\ell(u^o) = 0$ can be solved for the remaining coefficient $\theta_{\ell o}$. The results are documented in Table 2.

After these preparations we are able to plot the conjectured rate of profit r_i^c as a function of u_i, for which we fix the profit share and average utilization at their steady state values. The depreciation rate is set at $\delta = 8\%$ (likewise inferred from Franke, 2017a). The outcome is shown in Figure 2. The profit rate is a smooth

y^o	$\longrightarrow$	$\bar{y}$	θ_{bu}	$\theta_{\ell u}$	$\theta_{\ell o}$
1.09		3.633	12.533	10.093	0.082

Table 2: Remaining calibration of the loss and benefit function.

and well-behaved function, although it is not perfectly symmetric. By construction, it is maximized at $u_i = u_i^o = 0.30$, which gives rise to a conjectured rate of profit $r_i^c = 24.70\%$ (this is indicated by the solid vertical line in the diagram).

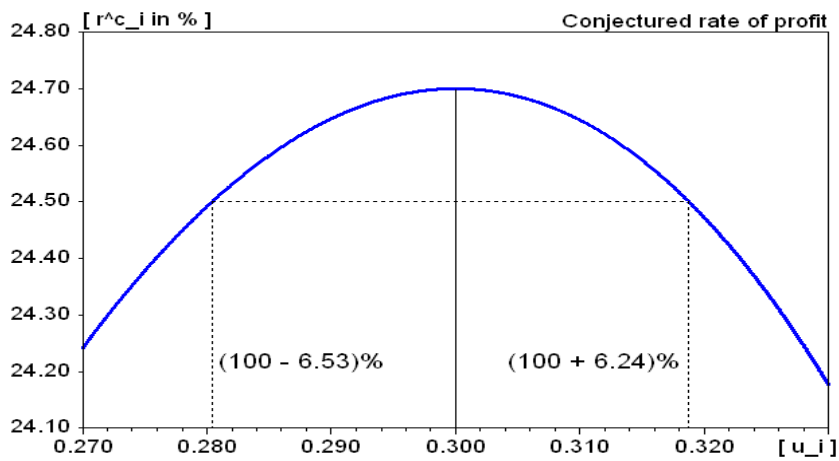

Figure 2: The conjectured rate of profit $u_i \mapsto r_i^c(u_i; u^o, h)$; $u^o = 0.30, h = 0.30$, $\delta = 0.08$.

Apart from the nice qualitative feature of the conjectured profits, we should have a look at the numerical details. The remarkable point to note is the range of u_i and r_i^c under study. Given that, as already mentioned, the aggregate output-capital ratio fluctuates by less than ± 5 per cent around its trend over the business cycle, the extent of the utilization rates shown in the diagram has to be regarded as rather wide. By contrast, the range of the resulting changes in the profit rate is extremely small. The diagram accentuates this by the two vertical dotted lines. If the firm considers to utilize its productive capacity at a rate within this interval, it expects a loss in its profit rate relative to its maximum of less than 0.20 percentage points; that is, r_i^c would still exceed 24.50%. The boundaries of the utilization interval are, however, distinctly beyond the macroeconomic business cycle variations of this variable. To

the left, we have $u_i = u^o - 6.53\%$ of u^o, and to the right $u_i = u^o + 6.24\%$ of u^o. It can thus be summarized that the forfeit in profits of a firm by not producing at its optimal level tends to be of a marginal order of magnitude.

5 A short methodological discussion

What could the observation of the only minor changes in the conjectured rate of profit mean for Kaleckian macroeconomic analysis? We are thinking of the studies that, especially when comparing two equilibria before and after a change in one of a model's parameters, tend to regard them as pertaining to the long term, even if actual and desired utilization differ in the second state. Sraffian or Marxist theorists (to assign a rough and ready label to them) strongly refute this interpretation. They hold that there will be agents somewhere in the model who would attempt to close this gap and, therefore, their behaviour would necessarily deviate from what so far constitutes the alleged equilibrium.

As a typical reaction in a case where $u \neq u^d$, so the general discussion, the firms would change their fixed investment. That is, they would not increase their capital stock at the current or expected rate of growth but rather expand it at a higher (lower) speed if their capacities are overutilized (or underutilized), the argument being that refraining from these adjustments would be detrimental to the firms in one way or another. Quite often, the kind of this "detriment" is not very clearly discussed in the literature. With the present notion of conjectured profits, however, it can be given a concrete meaning. Accordingly, an economy could only be considered to be in a state of rest if firms have their conjectured profits maximized; in other words, a state where $u \neq u^d$ would violate the profit maximization principle.

Clearly, this statement would so far describe the Sraffian position within our framework. It is, however, remarkable, that Kaleckian economists would not necessarily have to contradict this view. Instead, they might formulate their disagreement at another level, where one asks for the practical relevance of this hypothetical profit maximization. Reasonably, it may be assumed, the firms will react more strongly to a given utilization gap the stronger the pressure on them, which means the greater their losses for not doing so. Now, given the marginal losses in the conjectured profit rates that we observed in Figure 2 above, there should be only a weak pressure on the firms' investment if their utilization is not on target. Idiosyncratic shocks to the individual firms are quite likely to affect their profits more heavily than changes in their utilization rates. Furthermore, in many models there will be quite certainly other variables that have a decidedly stronger impact on profits. The firms may pay greater attention to them and investment may react much stronger to their varia-

tions. One example that was already hinted at in Figure 1(b) are changes in the profit share.

Recognizing that there are factors in the determination of the firm's investment that are far more important than the utilization gap, one could simplify things and even argue that the other factors are so dominant in the firms' investment decisions that here the utilization argument might be omitted altogether, at least as long as the economy does not move too far away from its steady state. Therefore, if a utilization gap does not play a significant role elsewhere in a model, it would also make economic sense to ascribe explanatory power—even in a long-run context—to an equilibrium in which firms may not operate at their optimal level. Its features would be all the more informative if (explicitly or by way of a background story) this equilibrium is considered to be locally unstable and it only serves as a point of reference around which the economy fluctuates. In such a dynamic setting the single firms will hardly ever achieve an exact profit maximization anyway, whether $u^o = u^d(u^o, h)$ were to prevail in a theoretical steady state or not.

If the assumption that investment does not react at all to a utilization gap is held to be too radical, the reasoning may be a bit more differentiated. It may at that level of the argument be admitted that the firms do respond to a utilization gap, although relatively weakly so, and that a state exhibiting such a gap could not be regarded as a long-run equilibrium in a strict sense. However, the analysis may not end with this assessment. Given the weak investment reactions, it could be argued that a 'long-run' perspective within which eventually any utilization gap would be closed—persistently or approximately as a time average—may be considered to be too long as to ever become relevant. In the meantime several structural changes outside a given model's explanation will occur in the economy that may dominate the original effects in a comparison of two equilibria. Therefore, another state of rest of the economy, one that that does not demand the elimination of a possible utilization gap, may indeed be economically more significant as a point of reference than, say, the fully-adjusted long-run equilibrium in Figure 1, where we have $u^o = u^d(u^o, h)$. To assign an extra label to such a point of reference, it might be called a 'near long-run equilibrium'.

Whether the simpler or the methodologically more refined version, Kaleckian economists may use this kind of reasoning to justify their approach to the analysis of long-run equilibria, for which they do not particularly seem to care whether actual utilization persistently deviates from desired utilization. In the author's view, the present notion of a conjectured rate of profit and its weak sensitivity in the firms' profit maximization problem could offer the Kaleckians another and more concrete line of defence, a line that Sraffian economists would have to take more seriously than the rather informal statements in the literature about a practically irrelevant

utilization gap, which we have alluded to in the Introduction.[17]

6 Conclusion

The paper has advanced the notion of a conjectured rate of profit for the individual firms, proposing that its maximization determines the benchmark rate at which the firms would desire to utilize their capital stock. It was emphasized that this is a hypothetical optimization procedure with no implications for the modelling of the firms' actual decisions about output, prices and investment. As the conjectured profits also depend positively on the aggregate activity of the other firms, a feature that constitutes a strategic complementarity, desired utilization proves to be an increasing function of not only the profit share but also of average utilization in the economy. It was very convenient that we could draw on the functional specifications and the regression estimates in Tavani and Petach (2018). This information made it possible to complement our analysis with a sensible numerical calibration of the model.

According to a Sraffian or Marxist point of view, a steady state position requires actual utilization to be equal to desired utilization. Given the profit share, this amounts to a fixed-point equation in utilization, $u = u^d(u,h)$. The numerical treatment was able to ensure a unique positive solution $u = u^o$ to this problem. It was also seen that the profit share has a sizeable impact on this equilibrium rate. Perhaps not every economist might be enthusiastic about the result that this effect is positive, $h\uparrow \Rightarrow u^o\uparrow$, but it is a direct consequence of the profit maximization principle in a one-good economy in conjunction with the model's strategic complementarity.

It was indicated that nevertheless this notion of a long-run equilibrium need not be completely rejected by Kaleckian economists, who continue to treat the utilization rate as an accommodating variable even in a long-run perspective. They may accept the present framework but argue in terms of practical relevance. Their point can be the only marginal sensitivity of the conjectured rate of profit to the hypothetical changes in the firms' utilization, which we have found in the numerical analysis. Specifically, deviations of the actual utilization rates from their optimal level by definitely more than their variations over the business cycle cause the conjectured profit rate to decrease from its maximum level of 24.70% by less than 0.20 percentage points.

It follows that the firms would only feel a weak necessity to adjust their fixed investment in order to close a utilization gap, that is, to expand their capital stock

[17] In an explicitly dynamic setting, for example, one might distinguish between a 'long-run' version of the model, where the utilization gap enters the investment function, and a 'near long-run' version, where this effect is omitted.

at a higher or lower speed than the current growth rate. Accordingly, if the firms' investment responds at all to a gap, the adjustments may be rather slow. Supposing stability, it would presumably take quite some time until, after a *ceteris paribus* change of a model's parameter, say, an equality of actual and desired utilization will be finally achieved. In the absence of stability, an equality of the two time averages, which could serve as a proxy for their equilibrium values, would even take longer. As in the meantime quite different things may happen and the economy may also undergo further structural changes, it could make good economic sense to neglect the properties of a perfect long-run equilibrium with $u^o = u^d$ and concentrate on another state of the economy with balanced growth but persistent deviations of u from u^d.

These considerations are a proposal of a certain reconciliation of a Kaleckian and Sraffian point of view regarding long-run analysis. Instead of a short-run *versus* long-run perspective, the difference between the two approaches could be identified as a difference between the, let us say, 'long-run' and the 'very long-run', a distinction that is less radical than the first. The present paper has provided a rigorous basis for such a conclusion. At the same time, readers from both camps may have their doubts about the paper's functional specifications and the numerical order of magnitude of the effects that we obtained. The theoretical discussion would profit from their detailed criticism and alternative approaches to determine desired utilization by an explicit profit maximization argument.

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