Tax Compliance and Tax Morale
An Agent-Based Model Approach

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Abstract

Throughout the usage of an agent-based model (ABM), this research aims to investigate how tax morale may explain a substantial part of tax compliance even in the extreme case scenario where the individuals' subjective probability of being audited is perceived to be null (zero). This work is a modification of the original tax evasion model by Yitzhaki, Allingham (1972) and Sandmo (1974) by proposing a power utility function and a dynamic framework with stochastic parameters. The proposed model accounts for analytical and computational features; particularly, it presents a closed-form solution when the subjective probability is perceived to be either zero or one. Hence, conditions are derived on the tax rate and levels of individual tax morale and risk aversion under which agents will disclose some of their income or be fully compliant even though the audit probability is known to be null. The results show how larger audit probabilities and fine rates have positive effects on the fraction of income declared, while larger tax rates may impose a negative impact. Fine rates, however, become less efficient for lower audit probabilities and for higher tax rates.

Keywords: Tax Evasion, Agent-Based Models, Heterogeneous Agents.
JEL classification: H26, C63.

1 Introduction

Ever since the seminal paper on the economics of crime published by Becker (1968) and the tax evasion model designed by Allingham and Sandmo (1972) there has been an increasing attention to the tax enforcement policies that attempt to tackle the problem of tax evasion and under-reporting. The idea of individuals updating their subjective probability of being audited based on their past experience is quite intuitive. Notwithstanding, as discussed by Bernhofer (2016), the posterior effect on the temporal updating following an audit may be quite ambiguous. As an example, the availability bias presented by Tversky and Kahneman (1973) suggests

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that agents that have just being audited would increase their posterior subjective probability; however, Mittone (2006) found counterexamples of this behavior in a Bomb-Crater Effect in which individuals do not consider consecutive audits as a likely event.

Alm, Bloomquist and McKee (2017) suggest that the behavior of individuals is directly affected by the behaviors of their 'neighbors' (defined as all the other agents from whom they receive information). Such linkages may increase or decrease the individual's tax compliance depending on the signal that is transmitted and perceived. The usage of peer-effects in agent-based models of social interaction is not a novelty anymore.

Tonin (2011) generalized the tax evasion model of Allingham and Sandmo by means of the inclusion of a minimum wage, or a rise increase of such threshold if already existent. Tonin's mathematical model suggested that for economies with a low enforcement level, the introduction (or increment) of a minimum wage would boost the revenues of the tax agency. Moreover, he implies that more research should be targeted in the calibration of an optimal audit policy. An Agent-Based Model was hereby implemented in order to extend and further analyze Tonin's minimum wage model and to distinguish an emergence of behavior in the tax evasion activity (from individual to aggregate level).

We provide a novel adaptation of the tax evasion model in an Agent-Based Model that considers an artificial society with the introduction of minimum wages. Although this is not the first time minimum wages are taken into account inside the environment of tax evasion rates, to the best of the authors' knowledge, it is the first time an Agent-Based Model explores the relationship between the two former variables. An absence of matchable incomes for the individuals allows for frequent opportunities of underreporting and income disguising. Agents update their subjective probability of being audited according to a weighted average of their own experience and the experiences of their "neighbors". Robustness of the model is tested against experimental data that was collected by Alm, McClelland and Schulze (1992). Moreover, we statistically analyze different values of individual parameters used by agents to estimate their effects in the decision-making process of income declaration and tax evasion.

Our model accounts for massive local interactions where agents, at the aggregate level, attempt to discover the true audit probability. We resort to simulation to analyze tax behavior in the presence of heterogeneous agents and see how compliance depends on individual values of tax morale and risk aversion as well as interactions with the perceived audit probability and structural parameters such as tax or fine rates.

Three assumptions are contemplated inside the model: individuals' objective functions belong to the power utility family, the agents have bounded rationality and society is arranged in an random-graph network allowing for information spreading. In particular, if utility is a power of (after tax and fines) personal income, the optimal fraction of declared income can be numerically characterized in terms of the agents' *willingness to pay*, namely a tax-morale measure corrected for risk-aversion. Moreover, according to Andrei et al. (2014) the random-graph or Erdos-Renyi network is the most compelling structure to facilitate the propagation of information among agents in a manner that taxpaying behavior may be significantly influenced by individual interactions.

2 Solving the model

The micro-founded optimization model that individual agents are supposed to solve as presented by Allingham and Sandmo (1972) and Yitzhaki (1974) may be redefined to be expressed as fractions of income instead of absolute values of money as:

$$EU = (1-p)\cdot U[I-\tau(d\cdot I)] + p\cdot U[I-\tau(d\cdot I)-\theta\tau(I-d\cdot I)],$$

where EU represents the Expected Utility, U is the utility function, I is the earned income, d is the optimum fraction of declared income, p is the probability of being audited and τ and θ stand for the tax and fine rates, respectively.

Solving for optimality conditions, the rational taxpayer will declare less than his actual income if the expected tax payment on undeclared income is less than the regular rate, that is, whenever $p\cdot\theta < 1$. There is a widely known substantial drawback for this model in the sense that it highly overestimates the tax evasion rate. Further, in the seminal simplistic version of this expected utility approach, no agent will be tax compliant whenever the audit probability is null.

What causes agents, if any, to comply even when their perceived probability of being audited is zero? Agents solve, under bounded rationality, an analogous maximization problem:

$$EU(d_{i_t}) = p_{i,t}U(X_{i,t}) + (1-p_{i,t})U(Y_{i,t})$$

where $X_{i,t}$ can be understood as the new income in the case where no audit takes places, while $Y_{i,t}$ is the respective income if an audit occurs, and the power utility function follows:

$$U(d_{i,t}) = (1+d_{i,t})^{\kappa_{i,t}} W_{i,t}^{(1-\rho_{i,t})},$$

where the dynamic individual variables for agent i and time t are denoted as: net income $I_{i,t}$, tax-morale $\kappa_{i,t}\in[0,1]$, risk-aversion $\rho_{i,t}\in[0,1]$, period-wealth $W_{i,t}=\{X_{i,t},Y_{i,t}\}$ and the fraction of declared income $d_{i,t}\in[0,1]$. In this sense, a higher the tax morale yields a larger utility of complying; while a higher risk-aversion would yield a lower utility on wealth. Furthermore, the utility function is convex with respect to both $(1+d_{i,t})$ and $W_{i,t}$.

The idea of individuals updating their subjective probability of being audited based on their past experience is quite intuitive. Moreover, the behavior of individuals is likewise affected by the behaviors of their 'neighbors'. Hereafter, The subjective audit probability perceived by agent i at time t can be defined as the weighted average of the agent's previous experience (temporal updating) at and the mean perceived probability of the neighboring individuals time $t-1$ (geographical updating).

The individuals live in a randomly generated network with (local) social interactions and each period they exchange information with their neighbors, but they never get to know the entire panorama of the society in which they live. Then, agents update their own perceived audit probability by means of a weighted average of three possible channels: their own memory of past audits, their subjective audit rate in the previous period and the signals they received from their neighbors.

$$\hat{p}_{i,t} = (\lambda_1)p'_{i,t-1} + (\lambda_2)\frac{1}{S}\sum_{s=1}^{S} A_{i,t-s} + (1-\lambda_1-\lambda_2)\frac{m_{i,t-1}+A_{i,t-1}}{n_{i,t-1}},$$

where λ_1 and λ_2 are convex averaging weights, $A_{i,t-1}$ is valued one if the agent was audited in the previous period and zero otherwise, S is the memory or number of audit periods that agents can recall in the past, $m_{i,t-1}$ is the number of neighbors that have been audited in the previous period, and n_i is the number of neighbors for the i-th agent (including itself)

Solving the Power Utility model, the condition on fine rates $\theta > 1$ is necessary and sufficient for agents to fully comply whenever audits happen with certainty. Moreover, the enforcement constraint in the original YAS model can be relaxed from $p\theta > 1$ to $\theta > 1$; therefore there is a possibility that a fraction of agents will comply for any value of p, including an absence of audits. Equalizing the partial derivative to zero and solving for $d_{i,t}$ we can derive the optimal declared income in the following expressions:

$$[d^*_{i,t}|_{\hat{p}_{i,t}=1}] = 1 \quad \forall\{i,t\}$$

$$[d^*_{i,t}|_{\hat{p}_{i,t}=0}] = \begin{cases} 0 & \text{if } \gamma_{i,t} \leq \tau \\ (0,1) & \text{if } \tau < \gamma_{i,t} < \frac{2\tau}{1-\tau} \\ 1 & \text{if } \gamma_{i,t} \geq \frac{2\tau}{1-\tau} \end{cases}$$

Where $\gamma_{i,t} = \frac{\kappa_{i,t}}{1-\rho_{i,t}}$ is the *willingness to pay taxes*; understood as the tax morale corrected for risk-aversion. Consequently, if the willingness to pay is high enough with respect to the tax rate, then agents may be full-compliers even in the absence of audits. The term on the right is increasing with respect to τ; meaning that if taxes increase, less individuals would be fully-compliant whenever the perceived audit probability is zero. Notably, there exist agents with a high enough willingness to pay such that they will fully comply even if their perceived probability of being audited is zero.

2.1 Behavior of the Utility Function

It is very helpful to analyze the first and second derivatives of the utility function with respect to wealth $W_{i,t}$:

$$\frac{\partial U(d_{i,t})}{\partial W_{i,t}} > 0, \frac{\partial^2 U(d_{i,t})}{\partial {W_{i,t}}^2} < 0$$

Therefore, considering the optimization problem for the case where no audit takes place; and given the fact that the utility function is concave with respect to wealth, we have:

$$\arg\max_{\{d_{i,t}\}} U(Y_{i,t}) = \arg\max_{\{d_{i,t}\}} Y_{i,t}$$

where $\frac{\partial Y_{i,t}}{\partial d_{i,t}} = -\tau I_i$ and so the negative constant slope confirms that the optimal declared value is $d^*_{i,t} = 0$.

Analogously, solely for the case where audits take place with certainty, we have

$$\arg\max_{\{d_{i,t}\}} U(X_{i,t}) = \arg\max_{\{d_{i,t}\}} X_{i,t}$$

where $\frac{\partial X_{i,t}}{\partial d_{i,t}} = \tau I_{i,t}[\theta - 1]$. Thus, for a fine rate θ lower than 1, the slope is negative. Therefore, in order for agents to not fully-evade systematically, the fine rate must be larger than 1. For the particular case in which the fine rate equals 1, then any declared income is an optimal selection considering only $X_{i,t}$. Thus, the condition on fine rates $\theta > 1$ is necessary and sufficient for agents to fully comply whenever audits happen with certainty.

2.2 Whenever the perceived probability is zero

The previous subsection proved that the enforcement constraint in the original YAS model can be relaxed from $p\theta > 1$ to $\theta > 1$; therefore there is a possibility that a fraction of agents will comply for any value of p, including zero (i.e. an absence of audits).

Let us now assume that the perceived audit probability tends to zero such that the expected utility form can be expressed as:

$$\hat{p}_{i,t} = 0 \text{ such that } EU(d_{i,t}) = U(Y_{i,t})$$

$$\Rightarrow \arg\max_{\{d_{i,t}\}} EU(d_{i,t}) = \arg\max_{\{d_{i,t}\}} (1 + d_{i,t})^{\kappa_{i,t}} [I_{i,t}(1 - \tau d_{i,t})]^{1-\rho_{i,t}}$$

$$\frac{\partial EU(d_{i,t})}{\partial d_{i,t}} = (1 + d_{i,t})^{\kappa_{i,t}} (1 - \rho_{i,t}) [I_{i,t}(1 - \tau d_{i,t})]^{-\rho_{i,t}} (-\tau I_{i,t}) + \kappa_{i,t} (1 + d_{i,t})^{\kappa_{i,t}-1} [I_{i,t}(1 - \tau d_{i,t})]^{1-\rho_{i,t}}$$

Equalizing the partial derivative to zero and dividing both sides by $(1 + d_{i,t})^{\kappa_{i,t}-1}[I_i(1 - \tau d_{i,t})]^{1-\rho_{i,t}}$ we get:

$$(1 + d_{i,t})(1 - \rho_{i,t})(-\tau I_{i,t}) + \kappa_{i,t}[I_{i,t}(1 - \tau d_{i,t})] = 0$$

$$\Rightarrow (1 + d_{i,t})(1 - \rho_{i,t})\tau = \kappa_{i,t}(1 - \tau d_{i,t})$$

Solving for $d_{i,t}$ we can derive the optimal declared income whenever the perceived probability of being audited is assumed to be null as a function of τ, $\rho_{i,t}$, and $\kappa_{i,t}$ as follows:

$$[d^*_{i,t}|_{\hat{p}_{i,t}=0}] = \frac{1 - \frac{(1-\rho_{i,t})\tau}{\kappa_{i,t}}}{\tau + \frac{(1-\rho_{i,t})\tau}{\kappa_{i,t}}},$$

where $d^*_{i,t} \in [0, 1]$.

Plenty of interesting conclusions can be derived from the last equation. Noticing that the denominator is strictly positive then the boundary conditions of full-evasion and full-compliance are computed from the numerator term.

An agent will be a full-evader whenever the numerator is smaller or equal than zero.

$$\frac{1 - \rho_{i,t}}{\kappa_{i,t}} \geq \frac{1}{\tau} \Rightarrow \frac{\kappa_{i,t}}{1 - \rho_{i,t}} \leq \tau$$

Following, a new parameter $\gamma_{i,t}$ is defined as the *willingness to pay* of agent i and may be understood as its tax-morale corrected by its own risk-aversion.

$$\gamma_{i,t} \equiv \frac{\kappa_{i,t}}{1-\rho_{i,t}}$$

The willingness to pay is increasing with respect to both tax morale and risk aversion.In continuation, the last equation can be restated as:

$$\gamma_{i,t} \leq \tau$$

Which implies that whenever the agents' willingness to pay is smaller or equal to the tax rate, and the perceived audit probability is zero, full-evasion will take place.

$$\therefore \text{ if } \gamma_{i,t} \leq \tau \Rightarrow [d^*_{i,t}|_{\hat{p}_{i,t}=0}] = 0.$$

Notwithstanding, it is also interesting to explore the cases, if any, when agents are fully-compliant even in scenarios where the perceived audit probability is zero, i.e., whenever:

$$\frac{1-\frac{1-\rho_{i,t}}{\kappa_{i,t}}\tau}{\tau+\frac{1-\rho_{i,t}}{\kappa_{i,t}}\tau} \geq 1$$

$$\Rightarrow \frac{1-\frac{\tau}{\gamma_{i,t}}}{\tau+\frac{\tau}{\gamma_{i,t}}} \geq 1$$

Applying basic algebra it is straightforward to derive the condition:

$$\gamma_{i,t} \geq \frac{2\tau}{1-\tau}$$

Consequently, if the willingness to pay is high enough with respect to the tax rate, then agents may be full-compliers even in the absence of audits. The term on the right is increasing with respect to τ; meaning that if taxes increase, less individuals would be fully-compliant whenever the perceived audit probability is zero.

The previous findings can be summarized in the following expressions:

$$[d^*_{i,t}|_{\hat{p}_{i,t}=1}] = 1 \quad \forall\{i,t\}$$

$$[d^*_{i,t}|_{\hat{p}_{i,t}=0}] = \begin{cases} 0 & \text{if } \gamma_{i,t} \leq \tau \\ (0,1) & \text{if } \tau < \gamma_{i,t} < \frac{2\tau}{1-\tau} \\ 1 & \text{if } \gamma_{i,t} \geq \frac{2\tau}{1-\tau} \end{cases}$$

Where $\gamma_{i,t} = \frac{\kappa_{i,t}}{1-\rho_{i,t}}$ is the *willingness to pay taxes*; understood as the tax morale corrected for risk-aversion.

Perhaps it would be of some interest to study the distribution of the willingness to pay taxes. The notion that a majority of people is not very keen to pay taxes, whereas a small population is highly law-abiding is quite intuitive. For a fixed society-level value of tax morale (κ) and a uniformly distributed risk-aversion level between zero and one (ρ) a generic distribution of the willingness to pay taxes (γ) can be portrayed in the following figure. Such graph displays how the willingness to pay taxes (WTP), for a society-level tax morale of 0.20, follows a power law distribution, where the frequency of people decreases as a function of the WTP taxes.

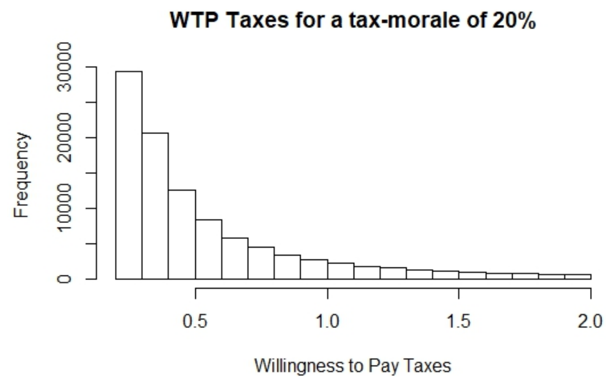

Figure 1: Willingness-to-pay-taxes distribution

2.2.1 Tax-Compliance Area

Kirchler et al. (2008) presented the slippery slope framework, which suggests that tax compliance can be achieved by means of power (enforcement) or trust (voluntary) in authorities. Our power utility model incorporates these two parameters as the subjective audit probability ($\hat{p}$) and the tax morale (κ), respectively, inside an expected utility model.

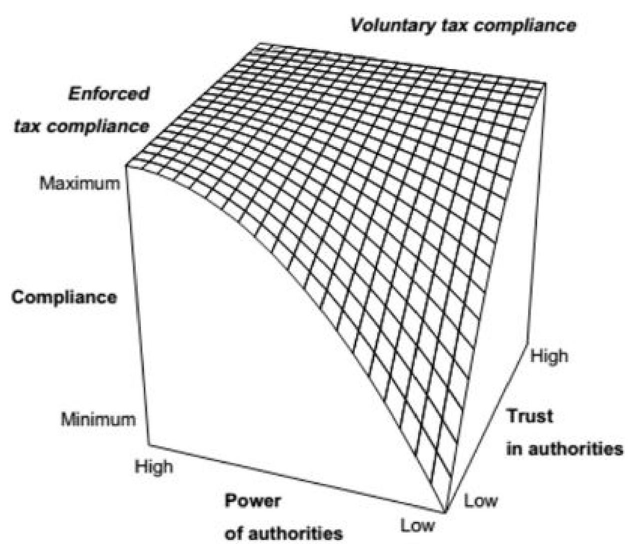

Figure 2: Source: Kirchler et al. (2008)

Alm, McClelland and Schulze (1992) explored why do people pay taxes even when they have an incentive to evade by employing experimental data. Among the interesting results presented by the authors, they find that people in the experiment complied to pay taxes even in the absence of audits. Moreover, several individuals fully evaded taxes under high audit probabilities;

even when the expected utility from the 'evasion gamble' was negative. Therefore, the results predicted by the expected utility theory where not supported. Given that there were public goods involved in the experiment and that most declarations where either full-compliance or full-evasion, Alm et al. suggest that agents might use a cut-off rule; but not the expected utility theory.

The last equation in Subsection 2.2, whenever equalized, gives a linear function which can be conveniently represented in a Cartesian plane. For an arbitrarily fixed time t, denote

$$\gamma_i \equiv \frac{\kappa_i}{1-\rho_i} = \frac{2\tau}{1-\tau} \equiv m$$
$$\Rightarrow \kappa_i = m - \rho_i m,$$

where m is the indifference curve for full-compliance of tax payments. Such curve would be the slope of the Tax-Compliance Area for each agent and across all time periods. Denote the size of the Tax-Compliance Area as Γ, which is constant for all agents i and all time periods t.

$$\Gamma \propto^{-1} m, \text{ and } m \propto \tau$$

$$\Rightarrow \Gamma \propto^{-1} \tau.$$

Therefore, whenever the perceived audit probability is null, the agents' tax-compliance area is inversely proportionate to the tax rate. Such proposed 'Tax-compliance Area' might suggest a potential explanatory cut-off rule that agents follow under the experiment by Alm et al. (1992) to compute their optimal declared income in the absence of audits.

The way to interpret such a Tax-Compliance Area, as seen in the next figure, is to think that given an agent's values for tax morale (κ) and risk-aversion (ρ), such an agent would fully-comply to pay taxes if such pair of coordinates could be mapped inside the dark gray area of Figure 3. If the individual's coordinates would happen to be inside the light-gray area, such an agent would be either a partial or a full-evader. It is important to recall from the last equation that the slope of the Tax-compliance area is dependent, and inversely proportionate, to the tax rate that the individual faces at the moment of the decision-making process.

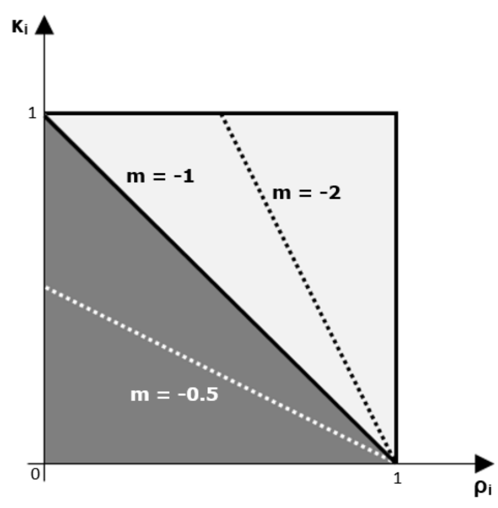

Figure 3: Tax-Compliance Area (light gray)

The previous Cartesian plane depicts the tax-compliance area (light gray) and the non-compliance area (dark gray) for a slope of $m = -1$, which is the corresponding value for $\tau = 33.33\%$; its respective indifference curve is represented by a solid black diagonal line. Analogously, a curve for $m = -2$ or $[\tau = 50\%]$ and $m = -0.5$ or $[\tau = 20\%]$ are illustrated as dotted black and white lines, respectively. When $\tau = 0\%$ both the slope and intercept become zero; if $\tau \to 100\%$ then $m \to +\infty$. The intuition behind this is quite straightforward, given the notion that whenever the tax rate is zero, then all agents are full-compliers by definition. Meanwhile, when the tax rate is one hundred percent (and there is no redistribution effect of wealth), no agent would have incentive to enter a labor market where it would be obliged to give away its entire wage and remain empty-handed.

The next figures show the results found by the authors (left) under a public investment tax frame for a multiplicative factor of 2 , a tax rate of 40%, a fine rate of $\theta = 15$ and three different audit probabilities; whereas on the right the collective utility comes from a tax morale utility parametrized by a tax morale of $\kappa = 0.20$ and a true audit rate of zero (right). Comparing both scenarios where the audit probability is set to be null there are some clear similarities between the experimental and the simulated declarations, nevertheless there is a seeming over-compliance in the latter with respect to the former.

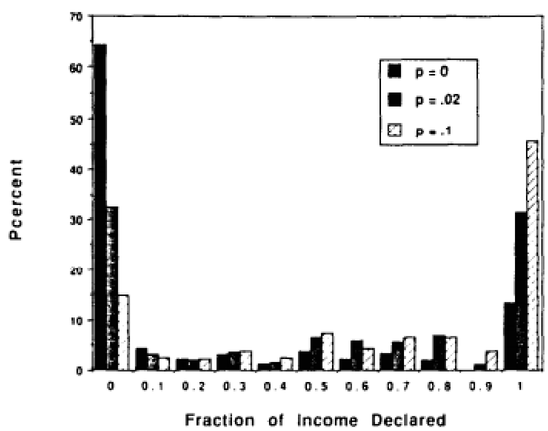

Figure 4: Source: Alm et al. (1992)

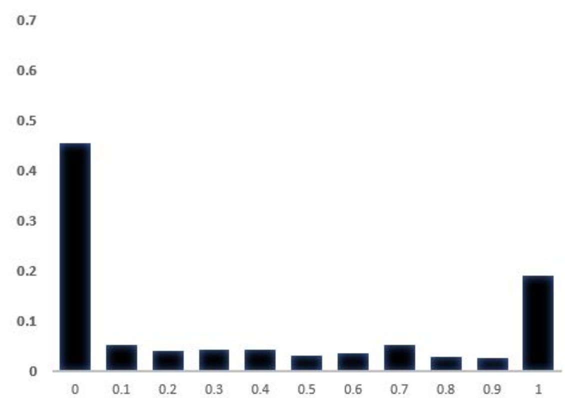

Figure 5: Declared d^* given $\hat{p}_{i,t}$=0, $\kappa = 0.20$

There are two very important similarities in the histograms depicted in Figure 4 and 5. First, the optimal declared incomes are practically dichotomous, having most of the agents either fully-complying or fully-evading, whereas just a rather smaller fraction actually decides an intermediate value of declared income. Second, there is a fraction of agents that fully-comply even when the true audit rate is zero whenever there is a public investment game (Figure 4, empirical data) or there is an extra utility to be gained by complying (Figure 5, simulated data). In this way, the artificially generated data in our power utility Agent-Based Model potentially provides a rather accurate representation of the empirical data collected by Alm et al.

2.3 Whenever the perceived probability is one

In a similar fashion to the previous subsection, the assumption that agents may have a perceived audit probability of one is studied and properties are derived in order to assess which agents, if any, would evade even when facing a certain audit.

$$\hat{p}_{i,t} = 1 \text{ such that } EU(d_{i,t}) = U(X_{i,t})$$

$$\Rightarrow \arg\max_{\{d_{i,t}\}} EU(d_{i,t}) = \arg\max_{\{d_{i,t}\}} (1+d_{i,t})^{\kappa_{i,t}} [I_{i,t}(1-\tau d_{i,t} - \theta\tau(1-d_{i,t}))]^{1-\rho_{i,t}}$$

$$\frac{\partial EU(d_{i,t})}{\partial d_{i,t}} = (1+d_{i,t})^{\kappa_{i,t}}(1-\rho_{i,t})[I_{i,t}(1-\tau d_{i,t}-\theta\tau(1-d_{i,t}))]^{-\rho_{i,t}}[I_{i,t}(-\tau+\theta\tau)] + \kappa_{i,t}(1+ d_{i_t})^{\kappa_{i,t}-1}[I_{i,t}(1-\tau d_{i,t}-\theta\tau(1-d_{i,t}))]^{1-\rho_{i,t}} = 0$$

After equalizing the partial derivative to zero and dividing both sides by $(1+d_{i_t})^{\kappa_{i,t}-1}[I_{i,t}(1-\tau d_{i,t} - \theta\tau(1-d_{i,t}))]^{-\rho_{i,t}}$ we can simplify to:

$$(1+d_{i,t})(1-\rho_{i,t})(-\tau+\theta\tau) + \kappa_{i,t}(1-\tau d_{i,t} - \theta\tau(1-d_{i,t})) = 0$$

$$\Rightarrow (1+d_{i,t})(1-\rho_{i,t})(\tau)(\theta-1) = -\kappa_{i,t}(1-\tau d_{i,t} - \theta\tau(1-d_{i,t}))$$

$$\Rightarrow (1+d_{i,t})\frac{(1-\rho_{i,t})}{\kappa_{i,t}}(\theta-1) = d_{i,t}(1-\theta) + \theta - \frac{1}{\tau}$$

Applying basic algebra the optimal value of declared income may be computed in an analogous way to the case where the perceived probability was null. Moreover, it is derived as:

$$[d^*_{i,t}|_{\hat{p}_{i,t}=1}] = \frac{\frac{\theta\tau-1}{\theta-1} - \frac{(1-\rho_{i,t})\tau}{\kappa_{i,t}}}{\tau + \frac{(1-\rho_{i,t})\tau}{\kappa_{i,t}}},$$

which is very similar to the case where $\hat{p}_{i,t} = 0$; however the first numerator term became $\phi \equiv \frac{\theta\tau-1}{\theta-1}$, which is a function of both θ and τ and is constant for all agents and all periods.

Consequently, agents will be full-compliers whenever the latter value is equal or larger to one. Substituting for ϕ and $\gamma_{i,t}$ we derive:

$$\text{If } \frac{\phi\frac{\tau}{\gamma_{i,t}}}{\tau\frac{\tau}{\gamma_{i,t}}} \geq 1 \Rightarrow [d^*_{i,t}|_{\hat{p}_{i,t}=1}] = 1$$

Solving the left hand inequality the necessary condition is expressed as:

$$\text{If } \gamma_{i,t} \geq \frac{2\tau}{\phi-\tau} \Rightarrow [d^*_{i,t}|_{\hat{p}_{i,t}=1}] = 1$$

The denominator is non-positive whenever $\phi \leq \tau$

$$\phi \leq \tau \Leftrightarrow \frac{\theta\tau-1}{\theta-1} \leq \tau \Leftrightarrow \theta\tau - 1 \leq \theta\tau - \tau \Leftrightarrow \tau \leq 1$$

Noticing that $\gamma_{i,t}$ and τ are non-negative and $\phi-\tau$ is non-positive, then the former condition is void. In other words,

$$[d^*_{i,t}|_{\hat{p}_{i,t}=1}] = 1, \forall\{i,t\}.$$

3 Simulation and Robustness Analysis

Korobow, Johnson and Axtell (2007) approached the tax evasion problem through the usage of social networks (Moore neighborhood) in an ABM setup. Agents live in a network with limited knowledge about the true enforcement parameters and can merely perceive the audit probability. Following, agents can take three actions: fully evade, fully comply, or underreport. We, however, have also given a mathematical proof for the conditions in which each of the three states prevails with respect to the willingness to pay taxes of the individuals. The results by Korobow et al. on how limited knowledge about neighbors' payoffs produces a higher tax compliance can be confirmed by the overshooting during the first periods of our simulations, where agents have not yet acquired information from their peers. Additionally, several of their findings were also present in our simulation: the chaotic behavior of individual agents, a non-linear convergence of the society level tax compliance or how fine rates losing their effect to deter tax evasion as they increase. Furthermore, in Mittone and Pattelli (2000) three types of agents are assumed to exist: always honest, imitative and perfect free-riders. We mathematically proved the existence for all of them in any given society under our framework.

Each agent has a set of endogenous variables that are stochastically dynamic: a uniformly $U(0,1)$ distributed tax morale and a uniform $U(0,1)$ risk aversion, a Gaussian distributed gross income (strictly larger than zero), and age. No agent knows the complete picture and it is not necessary for them to know it in order to make choices or take action. The optimal fraction of income to be declared is a non-linear function of the willingness to pay. Simulations where ran for values $\tau = 20\%$, $\theta = 2$ and $p = 2\%$; individuals start with a randomly assigned subjective probability of being audited and modify it as time runs.

There is a bottom-up emergence of patterns, both in the fraction of declared income as a society and for the mean perceived audit probability. Even if the agents act independently and do not know the exact declared income of others, they converge to an aggregate level of tax compliance.

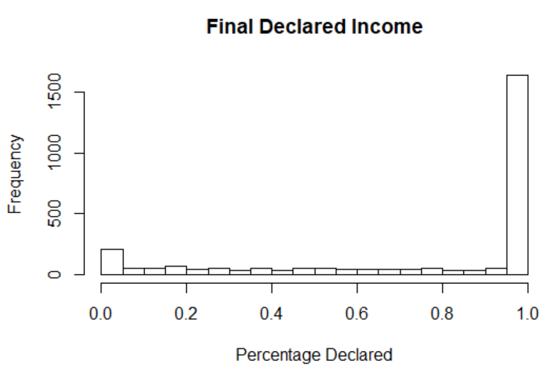

Figure 6: Individual $d_{i,t}$

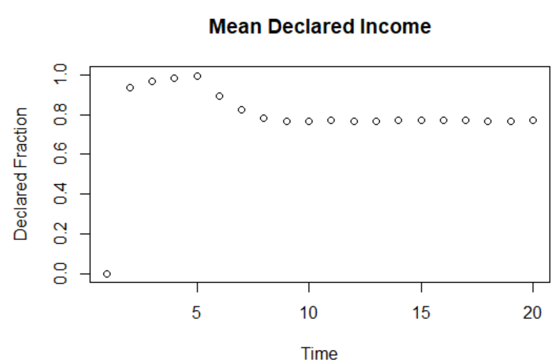

Figure 7: Collective $d_{i,t}$

The left figure depicts a high level of full tax compliance even, relatively low full evasion and a continuous distribution of agents that partially complied. Meanwhile, the overall fraction of declared income as a society, initialized in zero, quickly overshoots the steady level and converges after a few steps (the actual speed of convergence is inversely proportional to the memory span of the agents).

Concerning the perceived audit probability, on average four out of five agents underestimate and may even consider it to be practically zero, however a few overestimate it significantly. The explanations can be recalled form the Bomb-Crater effect and Availability Bias.

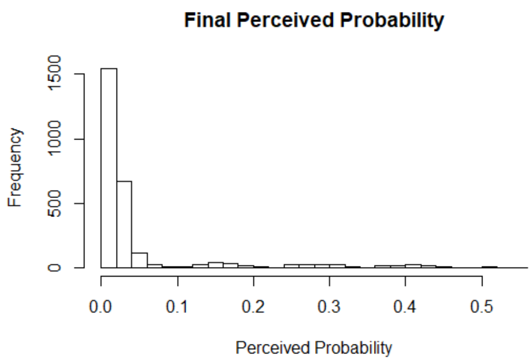

Figure 8: Individual $\hat{p}_{i,t}$

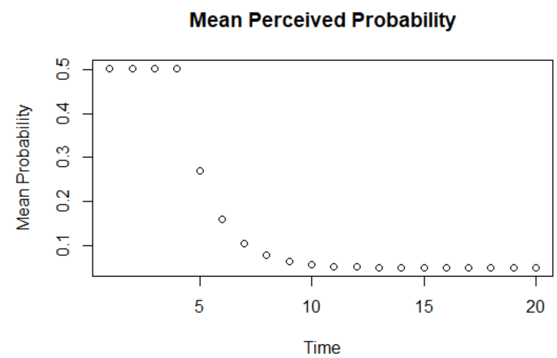

Figure 9: Collective $\hat{p}_{i,t}$

Whenever in the static scenario, there is a bottom-up emergence of patterns; both in the fraction of declared income as a society and for the mean perceived audit probability. Although no agent ever discovers the true audit rate, at the aggregate level, they discover the true p level. Even if the agents act independently and do not know the exact declared income of others, they converge to an aggregate level of tax compliance. Moreover, they find the true audit probability at the aggregate level. Consistently the t-test $H_o : \sum_{i=1}^{n} \frac{\hat{p}_{i,t}}{n} = p$ had a p-value above 0.70, meaning that the null hypothesis, the collective perceived audit probability is equal to the true audit rate, cannot be rejected.

The optimal fraction of income declared may be defined as a function of the willingness to pay once we filter by subjective audit probability. In the next figure, the pattern is similarly characterized irrespectively of whether the income is uniform or variant among agents. IT must stated that the right-hand side of the graph was cut-off given that all agents whose willingness to pay was larger than a certain threshold (as defined in Section 2) fully complied.

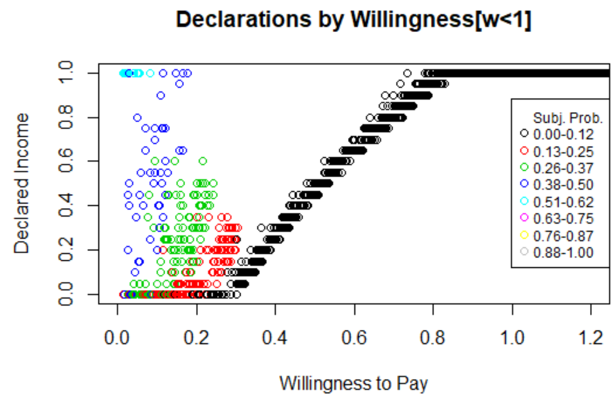

Figure 10: Optimally Declared fractions of income

Employing a dynamic framework the willingness to pay taxes fluctuates and the outcomes are of particular interest in two different ways. The aggregate fraction of declared income by the society as a whole reaches the steady state in an oscillatory way but evidently more sinuous than in the static case.

Whenever we allow for tax morale and risk aversion to fluctuate stochastically as the agents turn older, the willingness to pay taxes fluctuates as well. In order to keep the results comparable with the previous section, the life-average tax morale and risk aversion of agents are computed in the same way the values for $\kappa_{i,t}$ and $\rho_{i,t}$ were obtained previously. Ensuing this mechanism, the values for $\gamma_{i,t}$ increase stochastically as individuals age from twenty until sixty years old while constraining $\kappa_{i,t}$ and $\rho_{i,t}$ on the previously specified conditions.

The outcomes of the dynamic case are of particular interest in two different ways. The aggregate fraction of declared income by the society as a whole reaches the steady state in an oscillatory way but evidently more sinuous than in the static case. Furthermore, a second result shows that the society altogether not only is incapable of finding the true audit rate p, but also their averaged perceived audit probability systematically overestimates the actual value.

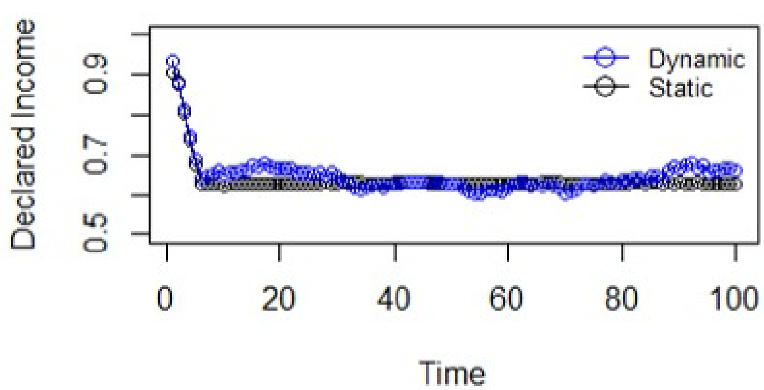

Figure 11: Average declared incomes $d^*_{i,t}$

At the aggregate level, society converges to a steady state of declared income both under static and dynamic frameworks; where the second is oscillatory given the stochastic parameters and the introduction (exit) of new (old) agents.

3.1 Internal and External Validation

The Laffer Curve is the non-linear representation of tax revenues as a function of the tax rate. Governments cannot over-raise the tax rate as it would incentive agents to evade taxes, reducing the governmental revenue. The following graphs depict the tax evasion as a function of the tax rate (left) and the corresponding Laffer Curve (right) for the simulated society. Given that incomes may differ among the agents, tax evasion is computed as one minus the ratio of declared earnings over the true income.

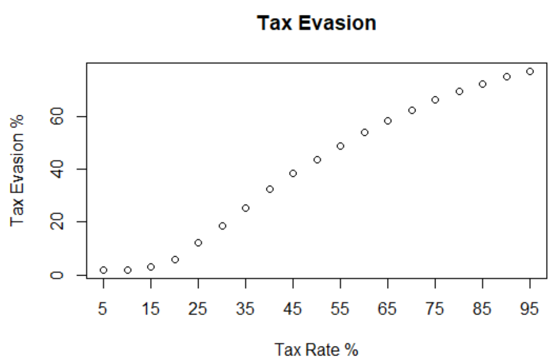

Figure 12: Tax evasion as a function of τ

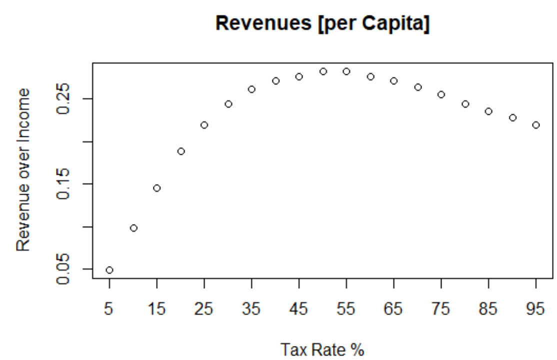

Figure 13: Laffer Curve

Furthermore, the Behavioral Space tool from Netlogo was exploited to empirically test the effects of an increasing fine rate on tax compliance. Assuming nonnegative incomes, such that the government cannot take more than the entire gross earnings, fine rates are tested from $\theta \in [1; 5]$, tax rates from $\tau \in [0; 1]$ and the audit probability from $p \in [0.01; 0.10]$. Increasing the fine rate does have a positive effect in the fraction of declared income, however, this effect diminishes for higher levels of tax rate. Moreover, for tax rates above 50%, which are not very common to begin with, the fine rates seem to lose their power to modify the declarations. The possible reason may be that, for such high values of tax rates, any fine rate larger than 2 will implicate that full-evaders need to hand in their entire income, which is the maximum payment possible.

Another interesting result is that the audit probability also plays a role in the effectiveness of fine rates; the effects of fine rates diminish as the tax rate increases. The smaller the audit probability becomes, the less importance agents give to the fine rate parameter, ergo, for lower values of p, the fine rate loses most of its effect on compliance. Two interesting cases arise whenever the tax rate is either 0% or 100%. For such cases the fine rate has absolutely no effect, given that in the first scenario all agents are full-compliers and in the second one they are all full-evaders by definition. Therefore, the fine rates are more useful as an enforcement enhancer whenever tax rates are low, when the audit probability is high, or both things simultaneously.

Alm , Jackson and McKee (1992) study the response of individual tax compliance behavior with respect to different parameters. They find that income, audit rates and the existence of public goods have a positive elasticity with respect to the declared income, whereas tax rates have a negative elasticity. Moreover, fine rates have a weak, positive effect on declarations. Additionally, rising the audit rate does not have a significant effect on future declared income. All of these results have been shown as well by our minimal Agent-Based Model simulations; with exception of the income effect for random audits. Whenever the audit probability is exogenous, i.e., the audit rate is completely randomizes, then the income of agents plays no role. However, whenever the audit rate is endogenous, i.e., the larger your income the higher your probability of being audited, then there is a non-linear effect of income that channels through an augmented subjective probability of being audited. Therefore, for endogenous probabilities, there is an income effect in which richer individuals tend to report a larger fraction of their income compared to low-income agents.

3.2 Statistical Analysis of parameters

In order to test the statistical significance and effects of each parameter on the fraction of income declared, data was simulated in 25 different scenarios for 100 runs in each one, varying the tax rate from 10% to 50%, fine rates from 1 to 15 times the evaded income, tax morale ranging from 0.1 to 0.5, five different income levels and a true audit rate oscillating from 0% to 10%. The results were analyzed by means of a censored regression where the dependent variables lives in the range of zero to one, inclusive. The audit mechanism was the one of randomized audits.

Dependent variable: Fraction of income declared [d^*]

Parameter	**Flat Taxes**	**Stepped Taxes**
(Intercept)	0.4750***	0.0832***
tax_morale	1.7516***	2.0966***
tax_morale^2	-1.4411***	-1.8326***
audit_prob	1.1562***	1.1654***
audit_prob^2	-1.4743	-0.5288
fine_rate	0.0337***	0.0540***
fine_rate^2	-0.0019***	-0.0035***
income	0.0003	-0.0032
income^2	-0.0001	-0.0022
tax_rate	-1.6818***	
tax_rate^2	0.8753***	

Note: *** stands for a significance level of 0.001

The regression analysis sheds light on the individual effects that each parameter imposes in the optimal fraction of income declared. It is clear to see that both tax morale and fine rates have positive effects on tax compliance with a decreasing reaction whenever the tax rate reaches high levels. However, it is important to notice that the effect of fine rates in tax compliance is not of a large magnitude; this could imply that increments in the form of harsher fine rates may not be the optimal policy to counterattack tax evasion. The audit probability seems to have a linear effect in the incrementation of income declaration. Income however, under random audits, is not statistically significant in the determination of tax compliance. Lastly, the tax rates have a negative impact in the tax payment process; for higher tax rates people are less willing to comply with their due payments. This result follows in line with the *willingness to pay taxes* condition that was derived from the power utility model in the previous section.

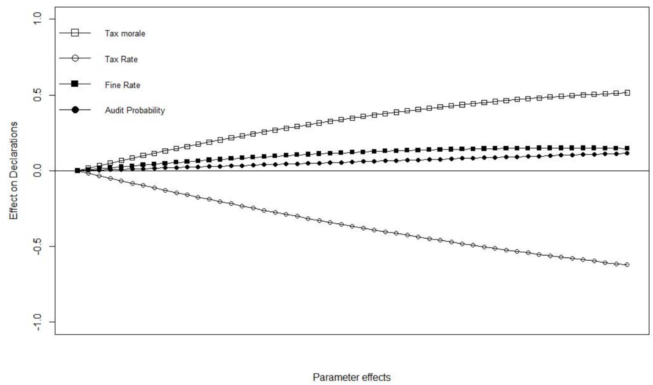

Figure 14: Parameter effects on Income Declaration

Figure 14 shows an approximate representation of the behavior of the optimal fraction of declared income [d^*] as a function of each parameter individually *ceteris paribus*. The magnitudes should not be taken literally as each parameter lives in a different range of values, however the shape and convexity/concavity of the functions yield an important intuition on the first and second derivatives of the fraction of income declared with respect to each parameter individually.

Further exploration regarding the income effect on tax compliance was performed, modifying the Tax Agency's audit strategy from random to endogenous. Therefore, for each agent, the probability of being audited was set to be endogenous in accordance to its income level with respect to society. Following, the true audit rate for agent i is the society-level average audit probability $\bar{p}$ multiplied by the weight of the agent's income over the total income of the population.

$$p_{i,t} = \frac{I_{i,t}}{\sum_{j=1}^{N} I_{j,t}} \cdot \bar{p}$$

Modifying the probability from exogenous to endogenous generates interesting results. The population was divided into three general income-based categories: low-class, middle-class and high-class agents. Under random audits, there is no statistical significance among the fractions declared across all groups. On the other hand, under endogenous audits, a t-test showed that there is a statistically significant difference between the fraction of income declared by poor and rich agents. In such statistical test it is shown that rich individuals tend to declare more than their low-budget counterparts. There is a non-linear effect of income on tax compliance in the following channel: a larger income accounts for a higher true audit rate, which reflects in an increased subjective audit probability, which in return incentives the agent to raise its fraction of income declared and reduce its tax evasion.

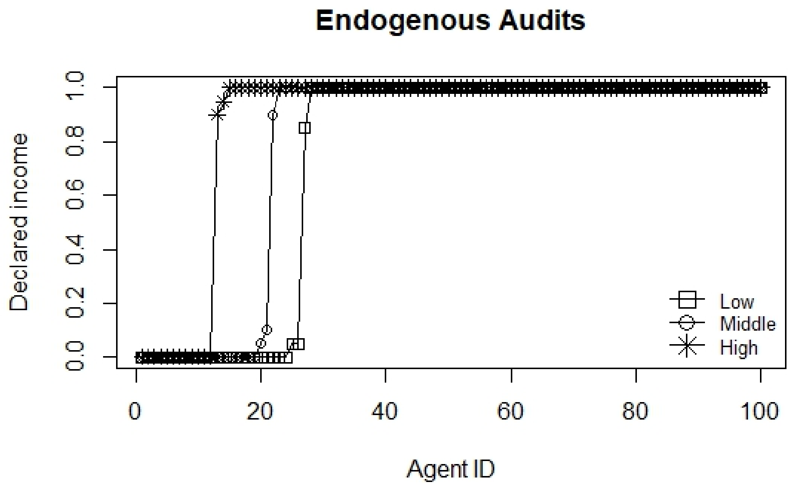

Figure 15: Income effects on Tax Compliance

It may be observed in Figure 15 how the distribution of declared income fraction is larger for high-income agents and lower for poorer ones. The graph shows the fraction of income declared by income-level increasingly by tax compliance inside each category. The next tables depict the p-value results from the t-tests performed to compare the distributions of income declared across the three wealth categories.

p-values for Randomized audits

Income Level	Low	Middle	High
High	0.5946	0.1853	-
Middle	0.4269	-	-
Low	-	-	-

p-values for Endogenous audits

Income Level	Low	Middle	High
High	0.0118**	0.0928*	-
Middle	0.3946	-	-
Low	-	-	-

Lastly, one could interpret from the previous p-values that under randomized audits, there is no income-level effect on tax compliance. Nevertheless, there is a difference between high-income and middle-income agents and also between high-income and low-income individuals with significance levels of 0.10 and 0.05 respectively.

4 Application of a Minimum Wage

Perhaps one of the first papers to study the implications of minimum wages in tax evasion was the one by Tonin (2011). Consequently, the distribution of reported income becomes a discontinuous and two-spike distribution. Where the optimal reported incomes d^* cluster both around

$d = 0$ and $d = w$, and follows a smooth and continuous distribution for $d > w$. In other words, the introduction of a minimum wage divides the individuals in three main categories: those who declare nothing (low productivity), those who state the minimum wage (medium productivity) and those who stay unaffected since their income was already superior to the minimum wage (high productivity). Workers whose income is below the newly set minimum wage may to opt for increasing compliance and report the new threshold or to withdraw from the labor market and report zero as their income.

Afterwards, by establishing a fixed minimum wage w, the space of possible reported incomes X is diminished from $D \in [0, y]$ to $D \in [w, y]$, where y is the actual earned income. Notwithstanding, it is important to note that the case of reporting an income of zero, i.e., abandoning the labor market, is also feasible. Therefore, the reported income space becomes $D \in \{0\} \cup [w, y]$.

Consequently, the distribution of reported income $g(d)$ becomes a discontinuous and two-spike distribution. Where the reported incomes cluster both around $d = 0$ and $d = w$, and follows a smooth and continuous distribution for $d > w$. In other words, the introduction of a minimum wage divides the individuals in three main categories: those who declare nothing (low productivity), those who state the minimum wage (medium productivity) and those who stay unaffected since their income was already superior to the minimum wage (high productivity). Workers whose income is below the newly set minimum wage may to opt for increasing compliance and report the new threshold or to withdraw from the labor market and report zero as their income.

$$d^* = \begin{cases} 0 & \text{, low productivity} \\ w & \text{, medium productivity} \\ \left(1 - \dfrac{1}{p\theta}\right) y & \text{, high productivity.} \end{cases}$$

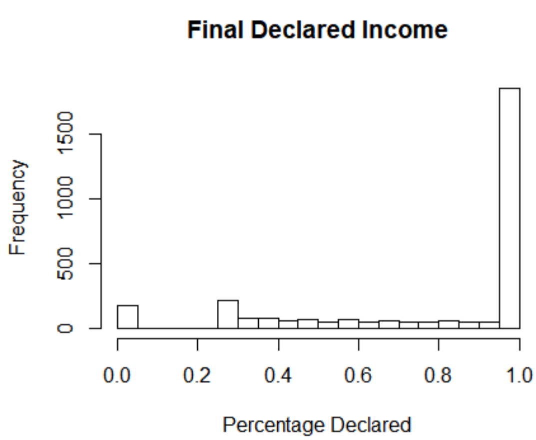

Figure 16: Minimum contribution

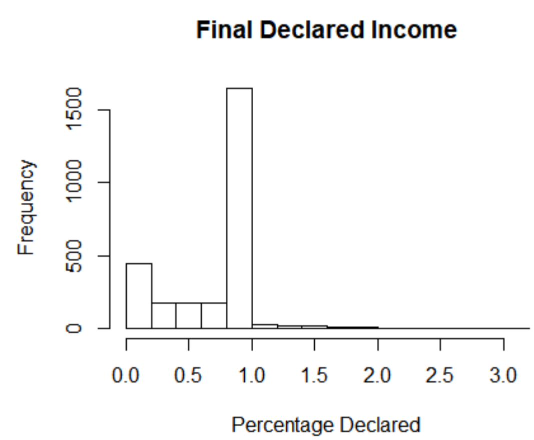

Figure 17: Minimum wage

Moreover, it can be seen that after the application of a common minimum wage, many individuals felt oblige to overpay their taxes in order for them to not leave the labor market. Few agents had to declare even shy above twice their income in order to be part of the fully compliant citizens. Therefore, from the fiscal point of view, setting a minimum wage threshold too high is not optimal, as many agents will leave the labor market.

Effects on tax compliance and full evasion were tested for different minimum wage levels and degrees of inequality with respect to the gross income. Individuals can comply at any level above the threshold or abandon the labor market and declare zero earnings. The next figures represent the effects of a minimum wage application of 30%, 60% and 90% of the mean gross income, respectively. The dynamic case with a minimum wage yielded two compelling preliminary results to our study. First, the tax evasion rate of the society decreased in a statistically significant manner for low level tax rates, while it increased of high level tax rates. Secondly, the number of full-evaders rose noticeably. Both preliminary results reflect how the incorporation of a minimum wage in a society with low tax enforcement would help decrease the fiscal evasion with the cost of allowing an expanse on the number of individuals working in the shadow economy as repercussion.

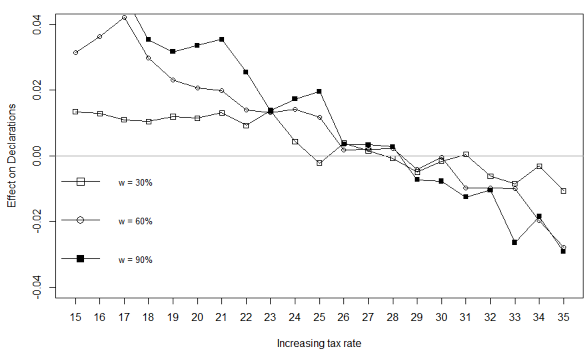

Figure 18: Effect in Declarations

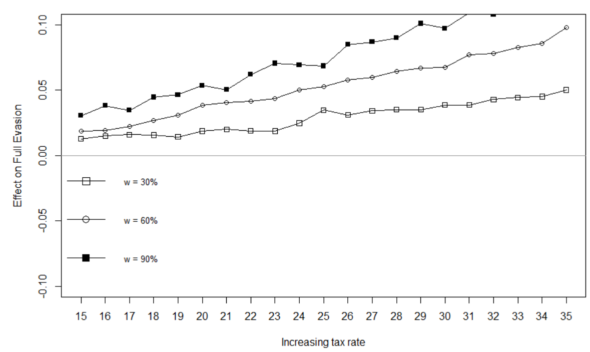

Figure 19: Effect on Evasion

Let us denote two regions for simplicity: South and North. North has a larger mean gross income and a lower tax evasion than South; moreover, given a step-wise or regressive tax system, the tax pressure is larger in North than in South. If the same minimum wage is established for both regions, it would represent a larger fraction of regional average income for South than for North. In other words, it would be considered to be 'high' in South, yet 'low' in North. In this sense, a minimum wage would increase the declared income, while the effect will be larger for the Southern region and lower for the North. There could be, however, a large drawback: the larger the minimum wage, the larger the effect on full-evasion. Therefore, the poor regions may have a larger increase in the black economy. The previous figures plot how for South, the effects will correspond to the filled-squares, while for North the hollow-squares would be applied. Given the notion that for North (South), the minimum wage represents a lower (larger) percentage of the mean, the minimum wage would increase the tax compliance more in the South than in the North; attempting to equalize both tax evasion levels in a new, lower, stationary state.

5 Results and Conclusion

A modification of the YAS Tax-Evasion Model was implemented through an Agent-Based Model approach with the inclusion of tax-morale and risk-aversion of agents. Individuals might fully-comply even in the perceived absence of audits; which would not be possible in the standard YAS model. A main result would be that a risk-aversion corrected tax morale can explain a substantial part of tax compliance for real case scenarios, even when the audit probability is set to zero.

Ours is a potentially interesting new proposal in which probability can be internally computed rather than exogenously given. The optimal fraction of declared income for individuals is a non-linear function of their willingness-to-pay; a tax-morale measure corrected for risk-aversion preferences. Both tax rates and audit probabilities have an effect on the fine rate reinforcement; fine rates become less efficient for lower audit probabilities, higher tax rates or both.

A compelling advantage of our modified model is its ability to capture the quasi-dichotomy of tax declarations which is present in existing empirical data. Moreover, the behavior of our artificial society resembles the experimental results for an environment where the true audit rate is null and agents get an extra utility for complying, whether it is a public investment game or a tax morale.

Running ABM simulations, society as a whole discovers the true audit rate under an static environment, i.e., agents live forever and there is no entering nor exiting allowed; however, society systematically overestimates the audit probability under dynamic frameworks. Moreover, there is a non-linear convergence to a steady state of the aggregate level of declared income in both scenarios, though it is an oscillating equilibrium under the dynamic setup.

Additionally, the installation of a nation-wide minimum wage, or the increment of an existing one, might enhance an homogenization of the tax compliance rates among regions. The poorer regions would increase their tax compliance more than their richer counterparts. Nevertheless, a possible side effect may present in the form of an increased full evasion level in the poorer regions, which could be understood as a boost of the shadow economy. In consequence, further study is suggested in the usage of a minimum wage to deter tax evasion.

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