Transition Dynamics, Uneven Development and Business Cycles in Thirlwall's Framework

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Abstract

The theory developed by Anthony Thirlwall (Anthony Philip Thirlwall, 1979) goes far beyond the famous *Thirlwall's law* (Mccombie, 1989). Thirlwall's theory deals not only with countries' external constraints but also with a framework that tackles the question of uneven development between rich and poor countries. The main objective of this paper is to further analyze the canonic Thirlwall model by changing the assumptions that lead to the *Thirlwall's law* - constant terms of trade and zero growth of net financial flows in the long run – observing the adjustment dynamics and its impacts in the relationship between two countries: a technologic developed north and a laggard south. The starting point of this research is the Dutt (2002) model, which explicitly develops the transition between short- and long-run in the Thirlwallian system. After explicitly discussing the properties and the algebra of the model, we (1) add a productivity dynamics to the model by including a *Kaldor-Verdoorn* effect, relating economic activity to labor productivity. Then we (2) model the labor market of the south economy by including a *Phillips Curve* to discuss the relationship between employment and economic activity à la Goodwin (1967). The inclusion of these elements changes the structure of the Dutt model, resulting in a 4-dymensional expanded dynamic system that is able to generate some complex patterns in the trajectory between short- and long-run. This new specification let us investigate further the properties of Thirlwall's framework. The results point to the emergence of a cyclical behavior that endogenously creates fluctuations in laggard economies. The evolution of this trajectory explains (1) why we empirically observe such a divergence between the effective growth rates and the ones calculated using the Thirlwall's law (Ansari, Hashemzadeh, & Xi, 2000), (2) a cause of divergence between rich and poor countries, and (3) Why poor countries show much higher endogenous volatility than rich countries.

Introduction

Thirlwall's framework is one of the most relevant contributions of the Post-Keynesian school of thought to economic theory (Davidson, 1990; Dutt, 2002). His theory finds that the growth rate of an economic system must be compatible with the constraints imposed by the balance of payments. Assuming that terms of trade and financial flows are stable in the long run, Thirlwall derives a rule in which the growth rate of an economy depends directly on the income elasticity ratio between exports and imports, which became known in the literature as the Thirlwall's Law (Mccombie, 1989).

There are many empirical works dealing with the Thirwall's Law by measuring the income elasticity of imports and exports for different countries (Alonso & Garcimartín, 1998). These measures show that the Thirlwall's law is a very good proxy to explain growth rates, especially in developed countries. In developing economies, though, the effects of terms of trade fluctuation and financial flows volatility systematically deviates the actual growth rates from the one predicted in the Thirlwall's law (Thirlwall & Hussain, 1982) . Despite the big relevance of Thirwall's law in the literature, the strict focus on it usually seem in the literature neglects some important aspects of the Thirlwall framework such as uneven development and the short- to long-run dynamics. This article has as its main goal to tackle these aspects that are ignored in the broad debate.

In the short-run, the Thirlwall's framework considers the effects of many variables such as the price elasticities of imports and exports, terms of trade (price level between north and south), and financial flows. In order to take into account the impact of the short-term behavior of these variables in the growth trajectory towards the long run this article derives the Thirlwall framework from explicit equations for imports and

exports. It follows Dutt (2002), who offers a possible solution to the transition between short- to long-run. His model endogenizes the evolution of the terms of trade towards the long-run adjustment. Dutt models a North-South dynamics in which the North follows a monopolistic Keynesian-Kaleckian framework while the south is modelled in perfect competition with a Marx-Lewis (Lewis, 1954) behavior (better explained in the following sections) with a very high elasticity of labor supply.

The use of a North-South model allows us to bring the discussion of uneven development. Since the times of the classic Structuralist theory, with authors such as Raul Prebisch, Paul Singer, among others (Prebisch, 1950; Singer, 1950), there have been challenges to the mainstream economics argument of convergence between north and south. The main argument of the classic Latin American structuralism (Prebish-Singer Hypothesis) focuses on the long-run decline on the terms of trade between north and south, resulting in the emergence of a divergent development process. The north, specialized in high-technology intensive goods, with higher income-elasticity of demand, tend to relatively increase the price of its goods related to the ones produced in the south, which lags behind. This divergent trajectory trend challenges the mainstream argument of a tendency to convergence seen in the Solow-type neoclassical growth models by including a tendency of decline in terms of trade between south and north, which reinforces uneven development.

As a third element beyond the (1) transitional dynamics and (2) uneven development, (3) volatility is added to the divergence and external constrains discussion. It is possible to observe that regular patterns of oscillation emerge in the transitory dynamics between short-run and long-run. When considering the Thirlwall's complete framework, the sources of this volatility comes from the behavior of terms of trade and financial flows, which is disregarded in Thirlwall's law, but affect the growth rate compatible with stability in the balance of payments. The concept of volatility in the Thirlwall model is directly related to the concept of fragility[1].

The income and price elasticities are considered exogenous in the Thirlwall's law. These elasticities are linked in the literature to the economic structures. They have a dynamics on their own that is more stable than the short-run effects of ToT and FF. If we consider that short-run changes are actually able to change the behavior of these elasticities (through structural changes), then new dynamics may emerge.

The objective of this research is to observe the transition dynamics in the Thirlwall's model by endogeneizing parameters that are related to the productive structure (productivity) and labor market. We focus on the productivity, on the income and price elasticities of the southern economy, and how they interact with changes in the terms of trade. We follow the baseline model defined by Dutt (2002). In this sense we define a framework that observes how the transitional dynamics operate in terms of volatility and uneven development between north and south.

1. Literature review

1.1. Thirlwall's Law and the Thirlwall Model

The Thirlwall model (Thirlwall, 1979) discusses the growth rate compatible with the constrains imposed by the balance of payments. The system is derived from the behavior of the external sector. The Thirlwall model can be explained by the following specification, starting with the definition of an import and an export function:

[1] Fragility related to resilience to external shocks

$$M = \theta_M (1/P)^{-\mu} Y^{\varepsilon} \quad (1)$$

$$X = \theta_X (P)^{-\nu} Y_f^{\delta} \quad (2)$$

Where M and X represent total import and total export respectively. θ_M and θ_X are constants. Y is the domestic income/output and Y_f the foreign income/output. The relative price P represents the price ratio between domestic prices (P_d) and foreign prices (P_f), in domestic currency (E). $P = P_d/EP_f$. μ and ν are the price elasticities of imports and exports, respectively. Finally, ε and δ are the income elasticity of imports and exports, respectively. So we see that imports increase with higher domestic production while exports grow with higher foreign production. Import falls with increases in the relative price while exports increase. Price elasticities define the sensitivity of growth to price changes and income elasticities the sensitivity of growth to output/income changes.

Considering the existence of financial flows (F), the equilibrium of the balance-of-payments can be described when net exports and net financial flows are balanced, as described by Dutt (2002):

$$PX + F = M \quad (3)$$

Writing eq.(3) in terms of growth rates:

$$[1 - (F/M)][p + x] + (F/M)f = m \quad (4)$$

In which the lowercase letters implies growth rates. When we replace the growth rates of M, X and P by eq. (1) and (2) we end up with the Thirlwall growth equation for the **short run**, which is given by:

$$y = (1/\varepsilon)\{(1 - \mu - \nu)p + [1 - (F/M)]\delta y_f + (F/X_N)[f - (1 - \nu)p]\} \quad (5)$$

In the long-run, Thirlwall considers that there is no change in the Terms of Trade ($p = 0$) and Capital Flows ($f = 0$). This gives us **Thirlwall's Law**. Net capital flows are stable in the long run, and the growth rate will depend on the ration between income elasticities of exports and imports multiplied by the growth rate of the foreign sector.

$$y = (1/\varepsilon)\ \delta y_f \quad (6)$$

or

$$y = (1/\varepsilon)\ x \quad (7)$$

Growth rate then is a function of the elasticities, which in this first specification is considered exogenous. The topic of endogenizing the income elasticities is a relevant discussion recently debated in the structuralist theory (Cimoli & Porcile, 2014; Porcile & Spinola, 2018).

This specification has the problem of ignoring the transitional dynamics. It does now allow observing the trajectory between the short- and long-run, which neglects some possible effects in the adjustment process that may affect the final outcome (steady state). This problem can be clearly seen by the fact that Thirlwall's law does not empirically sustain itself for many developing regions (effective rate diverging from the income elasticity ratio) (Thirlwall & Hussain, 1982).

1.2.Uneven Development and the Transition Dynamics

The questions posed in this research concern the dynamics between the "short-run" version of the Thirlwall model (Thirlwall's complete framework) and the "long-run" version (Thirlwall's Law). Dutt (2002) proposes a transition dynamics in an open north-south framework. In this model he focus on uneven development, an important but usually neglected matter raised by Thirwall (2012).

As already mentioned, the debates around the Thirlwall model mainly focus on discussing and measuring the parameters of the Thirlwall's law. Nevertheless, uneven development is a central aspect to study the behavior of developing economies. This discussion contraposes itself to the one raised by Solow-type neoclassical growth models and approaches itself to the old Structuralist ideas of Prebisch and Singer.

In the old structuralism, the position of an economy in the international division of labor defines its role in the system and its development possibilities. A country specialized in the production and export of raw materials tends to progressively lag behind than those that produce and export manufactured goods. Products are heterogeneous in terms of their price and income elasticity of demand. Products with higher income elasticity of demands (manufactured goods) see a rise in their demand as international economy grows, resulting in higher relative prices than raw materials. This ends up resulting in the emergence of an uneven development in which the core countries of the system advances in their productive structure while the periphery remains trapped in the exports. The solution in this system is increase government intervention towards the creation and development of modern manufacture sectors – through a process of import substitution industrialization (ISI).

The Thirlwall framework does not focus directly on the productive heterogeneity, but on the role of changes in terms of trade (prices of exports divided by prices of imports) and financial flows to define the growth rate considering the structural aspects of the economy (captured by price and income elasticity of imports and exports). There is no inherent trend of decline of the terms of trade as in the old structuralist ideas, but price behavior play a role in stablishing the growth possibilities. It is important to say that in Thirlwall's model, autonomous demand[2] is endogenous to the behavior of the external sector, so investments are then endogenous to the balance of payments possibilities.

In order to expand the Thirlwall framework, by discussing uneven development, we use a north-south model (Dutt, 2002) in which two economies with different dynamics interact defining the dynamics od terms of trade and the evolution of the specialization pattern. Section 2 discusses this model, explaining its internal coherence, its assumptions and the mathematics of the model. Section 3 expands the model by adding a Kaldorian productivity dynamics and a Phillips curve à la Goodwin to the labor market. This expansion completely changes the dynamic properties of the model. The original Dutt model resulted with one dynamic equation explaining the evolution the terms of trade. The expansion results in a 4-dimensional dynamic system capable of generating other economic outcomes, such as cycles. This allows us to capture the volatility in developing countries (South) that comes from the external sector and affect the whole adjustment trajectory of these economies.

2. The baseline model

In the Dutt Model the author explicitly define sthe dynamic properties between short-run and long-run in the Thirlwall system. In this section we briefly define these properties and how he builds his model of uneven development.

[2] Consumption, Investment and Government Spending.

a) Thirlwall Model of Uneven Development

A basic North-South model based on the **Thirlwall's law** states that the relationship between growth rates in North and South depends on the ration between income elasticity of imports in the North and South.

$$y_S/y_N = \varepsilon_N/\varepsilon_S \tag{8}$$

In order to have a more broad approach to consider the transitional dynamics of the Thirlwall system in this article it is relevant to explicitly define the macroeconomic equations related to the economies developed in this system. In order to do so Dutt (2002) models two economies that interact through their external sector: (i) a south economy, marked by perfect competition and by a Marx-Lewis structure (Lewis, 1954) with fixed real wage and unemployment labor, and (ii) a north economy that has imperfect competition, in which firms practice mark-up pricing and excess capacity, with a Kalecki-Keynes structure (Bhaduri & Marglin, 1990).

The monopolistic north has its price level defined by a markup function over costs, with price-making firms are related to the supply structure:

$$P_N = (1 + z)W_N b_N \tag{9}$$

In which P_N is the price level in the north; z consists in the mark-up ($z \geq 1$); W_N is the wage level in the north and b_N is the fixed unit labor requirement for the northern good (also understood as the inverse of labor productivity). An increase in markup and/or on costs (unitary wages) raises price levels, as well as a reduction in labor productivity (increases in productivity have a negative impact on prices).

The south follows a perfect competition specification, so the mark-up in the south is equal to 1. The GDP in the south (Y_S) operates at full capacity, so it can be written as the relationship between capital stock in the south (K_S) and the fixed capital-output ratio in the south (a_S) – the inverse of capital productivity.

$$Y_S = K_S/a_S \tag{10}$$

Real wages in the south (V_S) is defined as the ratio between nominal wages (W_S) and price index in the south (P_S):

$$W_S/P_S = V_S \tag{11}$$

Consumers in the north consume all their income, while capitalists save a fraction (s_N) of their income. The north spends a fraction α of their consumption expenditure on southern goods (and the rest on the northern goods). This fraction is equal to:

$$\alpha = \alpha_0 Y_N^{\varepsilon_N - 1} P^{1-\mu_N} \tag{12}$$

In which α_0 is the autonomous part of the northern expenditure in southern goods; Y_N is the GDP in the north. The terms of trade (P) is given by the ratio between prices in the south (P_S) and prices in the north (P_N):

$$P = P_S/P_N \tag{13}$$

In the south, workers spend all their income on southern goods and capitalists save a fraction (s_S) and consume the rest – being a part of this total consumption (β) spent on the northern good. Analogous to α; β can be described as:

$$\beta = \beta_0(\sigma_S Y_S)^{\varepsilon_S - 1} P^{1-\mu_S} \tag{14}$$

In which β_0 is the autonomous part of the south expenditure in northern goods; and σ_S is the profit share of income. This profit share is the residual from the wage share on total output ($b_S V_S$), which can be specified as the part of total income that does not go to wages:

$$\sigma_S = (1 - b_S V_S) \tag{15}$$

The investment function follows a Kaleckian (Bhaduri & Marglin, 1990) specification, in which capacity utilization affects the capitalist perception of economic activity. When capacity utilization increases, capitalists perceive it as an increase in effective demand, which stimulates them to immobilize capital to sustain the increases in the demand (and the opposite in periods of reduction in demand). Investments in the north are given by:

$$I_N / K_N = \gamma_0 + \gamma_1(u) \tag{16}$$

In which I_N is total investment in the north, γ_0 and γ_1 are positive constants. u consists on the rate of capacity utilization which is given by $u = Y_N / K_N$. The next step is to find explicit equations for northern and southern exports. Considering the equations for P_S and X_S, the total value of southern exports is given by:

$$P_S X_S = \alpha\{[1 + (1 + s_N)z]/(1 + z)\} P_N Y_N \tag{17}$$

Using eq.(12) on eq.(17), we end up with the equation for southern exports, which can be given in its reduced form as:

$$X_S = \theta_S P^{-\mu_N} Y_N^{\varepsilon_N} \tag{18}$$

In which $\theta_S = \alpha_0[1 + (1 - s_N)z]/(1 + z)$.

For northern exports, its value can be given as a part of southern value of exports:

$$P_N X_N = \beta \sigma_S P_S X_S \tag{19}$$

Using the eq.(14) on eq.(19), the equation for northern exports is given by:

$$X_N = \theta_N (1/P)^{-\mu_S} Y_S^{\varepsilon_S} \tag{20}$$

In which $\theta_N = \beta_0 \sigma_S^{\varepsilon_S}$.

With this simple model we highlight the static properties of the north-south interaction. Southern and Northern exports are explicitly addressed and in order to find themselves in equilibrium in current account, the values of exports in north and south must be balanced.

In the next section we define how growth and external sector evolve dynamically in this model.

b) Dynamics properties of the Thirlwall model

The dynamic properties of the Dutt (2002) model are derived from the excess demand (ED) functions in the north and south. In the south, excess demand (ED_S) it is given by:

$$ED_S = C_{SS} + I_{SS} + X_S - Y_S \tag{21}$$

While analogously, excess demand in the north (ED_N) is given by:

$$ED_N = C_{NN} + I_N + X_N - Y_N \tag{22}$$

$$ED_N = I_N - S_N + X_N - PX_S \tag{23}$$

Following a market clearing equilibrium, we can state that the equilibrium condition is that there is no excess demand in the long-run. This results in:

$$ED_i = 0 \tag{24}$$

The equilibrium condition can be used in eq. (21) and (23). When substituting all variables and applying the equilibrium in eq. (24), the results give us the following static equations for terms of trade and capacity utilization:

$$P = [(\theta_S/\theta_N)(uK_N)^{\varepsilon_S}(K_S/a_S)^{\varepsilon_N}]^{1/(\mu_N+\mu_S-1)} \tag{25}$$

From the Saving-Investment balance condition in the north we have:

$$u = \gamma_0/[s_N\sigma_N - \gamma_1] \tag{26}$$

In the long-run the **capital stock grows according to the rates of capital accumulation** in the two regions ($g_i = I_i/K_i$). The short-run conditions are always satisfied ($ED_i = 0$). In this sense accumulation in the north is given by:

$$g_N = \gamma_0 + \gamma_0\gamma_1/[s_N\sigma_N - \gamma_1] \tag{27}$$

In south, savings determine investments. And only southern capitalists save, not workers. The savings function is then given by the propensity to save times the profit share, times output:

$$S_S = s_S\sigma_S\, K_S/a_S \tag{28}$$

The investment function is then given by the value of total savings in domestic currency:

$$I_S = P^{\xi} S_S \tag{29}$$

The next step is to define the savings – investment conditions ($S_i = I_i$). Combining eq. (28) and (29) to the south we have the following equation for capital accumulation:

$$g_S = s_S P^{\xi}\, \sigma_S/a_S \tag{30}$$

When deriving eq.(25) we get the dynamic properties of the model. Terms of trade then fluctuate following the relationship between capital accumulation in the north and south:

$$p = [1/(\mu_N + \mu_S - 1)](\varepsilon_N g_N - \varepsilon_S g_S) \tag{31}$$

Which means that terms of trade (P) will fluctuate depending on the gap between investment in north and south weighted by their respective income elasticity of imports.

We just stated the main equations of the canonical model of Dutt (2002). In the next session we explain the expansion of this model by adding a productivity dynamics and by modeling the labor market. These modifications completely change the characteristics of the dynamic model, resulting in the emergence of new patterns.

3. Expansion of the Dutt-Thirlwall Model

The original Dutt model results in one dynamic equation for terms of trade dynamics. This dynamics depends on the gap between capital accumulation in the north and the south. In this expansion we focus on creating a productivity dynamics that is able to define other patterns rather than monotonic convergence and/or divergence.

3.1.Productivity Dynamics

The price level in the north is defined as function of the mark-up over costs $P_N = (1+z)W_N b_N$. From this equation we introduce an initial productivity dynamics and a wage rate dynamics in the north following constant rates of growth:

$$\widehat{b_N} = -\beta_N \tag{32}$$

And

$$\widehat{W_N} = \beta_N \tag{33}$$

In which β_N is a constant. Labour productivity in the north grows exogenously, and wages track productivity, growing at the same rate. $Y_S = K_S/a_S$ remains for the south, considering that there is no idle capacity, and that the capital-output ratio is constant. $W_S/P_s = V_S$ defines the value of real wages. So in this sense productivity and real wages follow the same path, growing according to technological progress. Technological progress in the north is assumed constant and stable, in order to simplify the model[3].

In Dutt (2002), the real wages in the south (V_S) are fixed. The capitalist's income in south is equal to $(1-b_sV_s)P_sY_S$ and the share of that in total income is $\sigma = (1-b_sV_s)$. b_s and V_s are endogenied in order to develop a productivity (labor productivity) dynamics. There is also the need to specify b_sV_s, so they stay within bounds (wage share cannot be smaller than zero or higher than one).

Labor productivity can be defined as the inverse of the unit labor requirement for the production of a good:

$$\lambda_i = \frac{1}{b_i} \tag{36}$$

Using this this definition, productivity gap[4] (G) between north and south can be discussed as:

$$G = \ln\left(\frac{\lambda_N}{\lambda_S}\right) = \ln\left(\frac{b_S}{b_N}\right) \tag{37}$$

[3] We understand, on the other hand, the central aspect of evolutionary major technological change happening in waves. (Schumpeter, 1939)

[4] Cimoli & Porcile (2014)

In the north, productivity grows at a constant rate $\widehat{\lambda_s} = \beta_N$ and labor productivity growth in south can be written as a constant rate (β_S) plus the effect of the productivity gap – a productivity catching-up effect:

$$\widehat{\lambda_S} = \beta_S + \rho G \tag{38}$$

Being $\beta_S < \beta_N$ and considering the definition of productivity gap on eq.(37), we can work out the dynamics o the technology gap ($\hat{G}$) as:

$$\hat{G} = (\beta_N - \beta_S) - \rho G \tag{39}$$

In this sense the catching-up literature can be brought to the context of this research (Verspagen, 1991).

3.2. **Labor market and Phillips curve**

A second addition to the model consist in developing a labor market dynamics. In order to do so we endogenize real wages. We use a Phillips curve (Barbosa-Filho & Taylor, 2006), which relates wages to employment, apllying it to the south:

$$\widehat{V_S} = -m + n\left(\frac{L_S}{\Lambda_S}\right) = -m + nl_s \tag{40}$$

In which m and n are constants. L_S consists on total employment, and Λ_S the total workforce. In this sense, the employment rate (l_s) can be defined as:

$$l_s = \frac{L_S}{\Lambda_S} \tag{41}$$

This addition change the characteristics of the model. equations (12) - (16) stay the same, but the profit share $\sigma = (1 - b_s V_s)$ is no longer a constant. Defining the wage share in the south as $\phi_S = b_S V_S$, we have that its growth rate is given by:

$$\widehat{\phi_S} = \widehat{b_S} + \widehat{V_S} = -\beta_S - \rho G - m + nl_S \tag{42}$$

Note that $\theta_N = \beta_0 \sigma_S^{\varepsilon_S}$

Equation (27) does not change. But for equations (30) and (31) σ_S becomes a variable, not a parameter. From equation (31) we then have that:

$$p = P = \frac{1}{\mu_N + \mu_S - 1}(\varepsilon_N g_N - \varepsilon_S g_S) \tag{43}$$

If we expand g_N and g_S from equations (27) and (30) respectivelly, we end up with the following equation for the evolution of the Terms of Trade:

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left(\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right) \tag{44}$$

And the growth rate of the profit share in the south (σ_S) follows then the opposite variation of the wage share (ϕ_S), which gives the following equation:

$$\widehat{\sigma_S} = -\widehat{\phi_S} = -\beta_S + \rho G + m - nl_S \tag{45}$$

In order to find a differential equation for the evolution of the employment rate $l_s = \frac{L_S}{\Lambda_S}$ we consider first that population growth is equal to zero, which gives us:

$$\widehat{l_s} = \widehat{L_S} \tag{46}$$

Being total labor growth in the south equal to:

$$L_S = b_S Y_S \tag{47}$$

In growth rates, this gives us:

$$\widehat{L_S} = \widehat{b_S} + \widehat{Y_S} \tag{48}$$

Eq. (10) says that $Y_S = K_S/a_S$. As there is no depreciation, therefore $\widehat{K_S}K_S = I_S = P^{\xi}S_S = P^{\xi}s_S\sigma_S\frac{K_S}{a_S}$ (we used the equations in-between (27) and (30)). This result takes takes us to:

$$\widehat{K_S} = P^{\xi}s_S\sigma_S\frac{1}{a_S} \tag{49}$$

Which gives us finally the dynamic equation for the employment rate:

$$\widehat{L_S} = \widehat{b_S} + \widehat{Y_S} = \widehat{b_S} + \widehat{K_S} = \widehat{b_S} + P^{\xi}s_S\sigma_S\frac{1}{a_S} = -\beta_S - \rho G + P^{\xi}s_S\sigma_S\frac{K_S}{a_S} \tag{50}$$

In summary, we end up with a system of four differential equations:

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left(\varepsilon_N\gamma_0\left(1 + \frac{\gamma_1}{s_N\sigma_N - \gamma_1}\right) - \varepsilon_S\frac{s_S P^{\xi}\sigma_S}{a_S}\right)$$

$$\widehat{\sigma_S} = -\widehat{\phi_S} = \beta_S + \rho G + m - n\frac{L_S}{\Lambda_S}$$

$$\widehat{L_S} = -\beta_S - \rho G + P^{\xi}s_S\sigma_S\frac{1}{a_S}$$

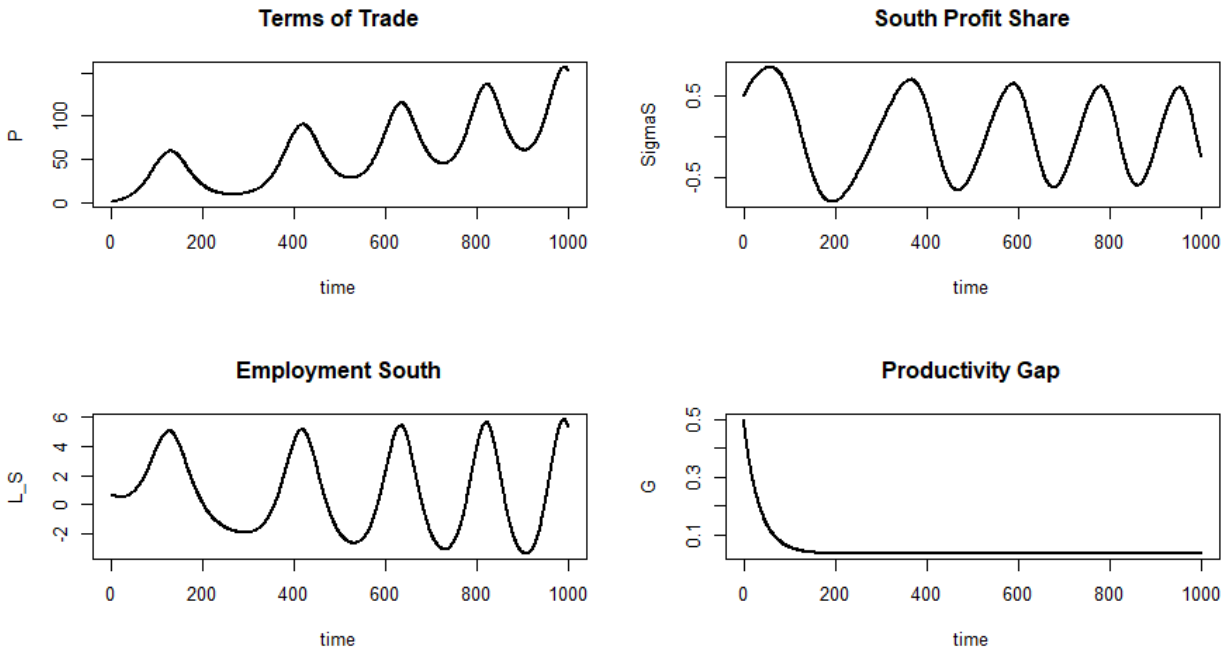
$$\hat{G} = (\beta_N - \beta_S) - \rho G$$

These equations show the north-south dynamics between terms of trade, profit share in the south, employment level in the south and productivity gap between south and north. The trajectory defines the relationship between the short- and the long-run and partially gives a supply-side interpretation to the Thirlwall framework.

The next step is to analyze the dynamic properties of this system.

4. Dynamic properties of the expanded model

In this section we discuss the dynamic properties of the expanded model. The paper is in a working version and will be later developed from here.



Parameters of the Model

```
a_s=4 # Capital-Output Ratio
xi=0.9 #Esponential to the Terms of Trade
u_n=0.2 # Price Elasticity imports North
u_s=0.6 # Price Elasticity imports South
e_s=1.9 # Income Elasticity imports South
e_n=2.4 # Income Elasticity imports North
s_n=0.25 # Propensity to Save in the North
s_s=0.18 # Propensity to Save in the South
b_s=0.7 # Fixed unit labor requirement for southern good
b_N=0.9 # Fixed unit labor requirement for northern good
W_s=15 # Fixed money wage in the South
W_N=30 # Fixed money Wage in the North
y_0=0.4 # Autonomous Investment parameter
y_1=1 # Sensitivity of Investment to Capacity Utilization
m=0.01 # Constant growth of real wages
n=0.02 # Sensitivity Real Wages to Labor in the South
sig_n=0.6 #Profit Share
o_s=0.4 #Total Workforce
bet_s=0.01 # Constant part of productivity growth in the South
bet_n=0.03 #Productivity Constant Growth Rate in the North
ro=0.3 # Sensitivity of Gap growth to Gap Level (Catching-Up)
```

4.1.Steady State

In this session we establish the steady state conditions of the model.

$$\hat{P} = \frac{1}{\mu_N + \mu_S - 1}\left(\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right)$$

$$\widehat{\sigma_S} = -\widehat{\phi_S} = \beta_S + \rho G + m - n\frac{L_S}{\Lambda_S}$$

$$\widehat{L_S} = -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S}$$

$$\hat{G} = (\beta_N - \beta_S) - \rho G$$

$$\hat{G} = 0 \rightarrow (\beta_N - \beta_S) - \rho G = 0 \rightarrow G^* = \frac{(\beta_N - \beta_S)}{\rho}$$

$$\widehat{\sigma_S} = 0 \rightarrow \beta_S + \rho G + m - n\frac{L_S}{\Lambda_S} = 0 \rightarrow \beta_S + \rho\frac{(\beta_N - \beta_S)}{\rho} + m - n\frac{L_S}{\Lambda_S} = 0 \rightarrow$$
$$n\frac{L_S}{\Lambda_S} = \beta_S + \rho\frac{(\beta_N - \beta_S)}{\rho} + m \rightarrow L_S = \frac{\Lambda_S}{n}[\beta_S + (\beta_N - \beta_S) + m] \rightarrow L_S = \frac{\Lambda_S}{n}[\beta_N + m]$$

$$\widehat{L_S} = 0 \rightarrow -\beta_S - \rho G + P^{\xi} s_S \sigma_S \frac{1}{a_S} = 0 \rightarrow -\beta_S - \rho\frac{(\beta_N - \beta_S)}{\rho} + P^{\xi} s_S \sigma_S \frac{1}{a_S} = 0 \rightarrow -\beta_N + P^{\xi} s_S \sigma_S \frac{1}{a_S} = 0$$
$$\rightarrow -\beta_N + P^{\xi} s_S \sigma_S \frac{1}{a_S} = 0 \rightarrow P^{\xi} s_S \sigma_S \frac{1}{a_S} = \beta_N \rightarrow \sigma_S = \frac{\beta_N a_S}{P^{\xi} s_S}$$

$$\hat{P} = 0 \rightarrow \frac{1}{\mu_N + \mu_S - 1}\left(\varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}\right) = 0$$
$$\rightarrow \varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) - \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S} = 0 \rightarrow \varepsilon_N \gamma_0 \left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right) = \varepsilon_S \frac{s_S P^{\xi} \sigma_S}{a_S}$$
$$\rightarrow P^{\xi} \sigma_S = \frac{a_S \varepsilon_N \gamma_0}{\varepsilon_S s_S}\left(1 + \frac{\gamma_1}{s_N \sigma_N - \gamma_1}\right)$$

Table 1. Data for Brasil

Annual Growth Rates	2002-2013	2014-2016	Annual Growth Rates	2002-2013	2014-2016
GDP (Effective Rate)	**3.8%**	**-2.3%**	**Natural Rate**	3.1%	-0.9%
Per Capita GDP	3.8%	-2.5%	Labor Productivity	1.6%	-2.1%
			Labor Supply	1.5%	1.2%
Consumption	4.0%	-1.7%	E = Effective - Natural	3.1%	-0.9%
Investment	5.4%	-8.2%			
Government Spending	3.7%	-0.9%	**BoP Compatible rate**	**3.6**	**0.6**
Imports	9.9%	-8.9%	Income Elasticity of Imports	1.9	4.4
Exports	4.9%	2.3%	Income Elasticity of Exports	2.4	1.0
Exchange Rate	3.3%	-17.3%	Income Elasticity Ratio	1.2	0.2
			Growth World GDP	2.9	2.7
Iron Ore Price	20%	-12%			
Crude Oil Price	16%	-16%	Agriculture VA (constant)	3.5%	-0.2%
Soybeans Price	10%	-9%	Manufacturing - Constant VA US$	2.5%	-6.8%
Beef Price	8%	2%	Services VA (constant 2010 US$)	4%	-1%
Minimum Wage	5.2%	0.7%	Animal Spirit (annual growth)	1.5%	-10.0%
Number Employed people	2.1%	-0.2%	Animal Spirit total growth	16%	-27%
Average Employment Rate			NUCI Growth	3%	-7%
P.P. change in unemployment rate	-2.6	5.1	NUCI Average level	81.9	78.5
Interest rate p.p. change		5.7	Current account balance	-3.8	1.7
FDI Inflow	14%	4%	Total Reserves	0.2	0.0
FDI Outflow	18%	-5%			

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