Are Kalecki-Minsky models consistent with the data? Evidence from the US

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Abstract: Post-Keynesians have incorporated one or more insights from Minsky's work in their macroeconomic models at an exponential rate during the past 10 years. The time to produce rigorous empirical work in order to discriminate the relevance of different families of models seems ripe. This paper contributes to filling this gap in the literature by estimating various forms of Kalecki-Minsky models for the US whose reduced-form representation are nested by linear and non-linear vector autoreggressions containing leverage and accumulation. The results imply that all models considered in the literature, except for the Charles model, are broadly inconsistent with the data. Many of the non-linear restrictions implied by the Charles model are borne by the data: There is a low leverage and a high leverage equilibrium and in both equilibria accumulation is debt burdened. However, we do not find evidence of increased financial instability in the high leverage equilibria, as proposed by the Charles model, nor do we find any evidence of the Financial Instability Hypothesis or the Paradox of Debt in any of the equilibria. This implies that much more work needs to be done to understand the dynamic effects of leverage on accumulation, instead of focusing on models which emphasize the univariate dynamics of leverage.

Key words: Kalecki-Minsky Models, Leverage, Vector Autoreggression, Non-Linear Time Series
JEL codes: C32, E02, E12

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1 Introduction

Hyman Minsky's (1975, 1982, 1986, 1992) prolific work regarding the financial dynamics of mature capitalists economies has attracted renewed attention both from the mainstream press and heterodox authors since the sub-prime financial crisis. Accordingly, there has been an explosion in the formalization of many Minskian insights into otherwise standard Kaleckian, Harrodian, Goodwinian and Kaldorian macroeconomic models, a literature which has been recently surveyed by Nikolaidi and Stockhammer (2017).[1]

Unfortunately, econometric work aimed at distinguishing the empirical plausibility of these different models is rather scarce, not to say non-existent. This paper aims to fill this empirical gap by investigating the empirical implications of Kalecki-Minsky models. We exploit the fact that most models produced in the literature can be written either as a two dimensional dynamical system on leverage and output or as one dimensional dynamical systems on leverage. This implies that linear and non-linear vector autorregressions (VAR) nest these models as special cases. Indeed, a linear univariate autoregressive model of leverage is implied by the model of Hein (2007), a non-linear univariate autoregressive model is implied by the models described in Taylor (2009), Lavoie (1995;2014), which is examined in depth in Ryoo (2013a), and a non-linear VAR on accumulation and leverage is implied by the more sophisticated Kaleckian model of Charles (2008a).[2]

The few existing empirical investigations regarding Minsky's ideas are not explicitly derived from a heterodox macroeconomic model, and thus, are poorly suited to try to discriminate between competing frameworks in which to understand Minsky's work. There are two exceptions to this rule. The first exception concerns the study by Nishi (2012), where the author claims that he develops a discrete-time version of Charles (2008a) Kalecki-Minsky model, and develops some identifying restrictions for a three-dimensional Vector Autoreggresion (VAR) on accumulation, leverage, and the profit share. The author's main finding is that the Japanese economy exhibits a debt-burdened and profit-led accumulation rate, while the leverage rate is essentially insensitive to accumulation and distribution. While this paper is an important contribution to the literature, it suffers from two flaws: As we discuss later, the paper does not implement the Charles model in the correct fashion, since it uses a linear specification for a non-linear model. Secondly, the paper does not discuss how his findings help to either accept or reject Kalecki-Minsky models or their broader implications for the theoretical literature which seeks to model Minsky.

The second exception to this rule is the small literature which has discussed Minsky's financial instability hypothesis (FIH), which has been mostly interpreted as the idea that business cycles should exhibit pro cyclical leverage. The FIH is typically a building block in Minskian models, since it specifies the dynamic response of leverage to output or accumulation. The first and most influential attempt at testing this question is conducted in Lavoie and Seccareccia (2001), where the authors analyse the correlation between GDP and the aggregate debt to equity ratio for six

[1] Lavoie (2009) provides a comprehensive review on the earlier literature, although in the context of the evolution of Cambridge Post-Keynesians and American Fundamentalists Keynesians.

[2] Of course, as underlined by Nikolaidi and Stockhammer (2017), the modelling of leverage and accumulation is not the only way to formalize Minsky's views. Other Kaleckian formalizations stress the role of interest rate dynamics (Lima and Meirelles, 2007; Fazzari, Ferri and Greenberg, 2008) or the dynamics of the retention ratio set by firms (Charles, 2008b). Also, a smaller but growing literature analyses the roles of asset prices and their interaction with goods market to produce instability and Minsky cycles; see, for example, Ryoo (2013b). However, it seems fair to say that most Kaleckian models have emphasized the leverage - output link as an accurate interpretation of many major Minskian insights, so we postpone an empirical investigation of these families of models for the future.

OECD countries at yearly frequencies. They essentially conclude that the leverage rate is a-cyclical. Furthermore, they estimate a regression of leverage on GDP growth, noting that economic activity is not significant once appropriate controls are included, with the point estimates always displaying a negative coefficient. This would imply that leverage and GDP are counter-cyclical, a phenomena professor Lavoie would latter name 'The paradox of debt': Firms attempt to cut debt during recessions would further depress economic activity, which would conduce to higher leverage ratios during recessions and lower leverage ratios during expansions. As mentioned above, this finding is essentially corroborated by Nishi for Japan.

However, there has also been some evidence contradicting the Paradox of Debt and supporting the FIH. A few representative examples are Charles (2015) and Gonzalez and Perez-Caldentey (2017): While the first paper shows that are low frequency comovements between GDP and aggregate leverage[3] over the 20th century in the US economy, the second paper shows that for a panel of firms in 12 Latin American countries the paradox of debt is rejected in favour of the FIH. These authors also show that there is an important asymmetry regarding the FIH: the correlation between debt and investment is higher during expansions than during recessions.[4]

The main results of our empirical investigation are summarized as follows: First, linear Kalecki-Minsky models, such as the one proposed by Hein (2007), are broadly inconsistent with the data. A linear specification reveals there is no feedback effect whatsoever from leverage to accumulation and from accumulation to leverage. Furthermore, there is substantial evidence of non-linearities in the leverage series. Second, models which are solved in terms of a univariate specification of leverage are also broadly inconsistent with the data. They do not predict the correct number of equilibria empirically nor do they characterize the qualitative behaviour of these equilibria in the correct fashion. Thirdly, the Charles (2008a) model seems to be the most promising candidate to explain the features of our dataset: It correctly predicts the existence of two equilibria - a low leverage equilibria and a high leverage equilibria, and also correctly predicts that accumulation is debt-burdened in this two equilibria. However, it incorrectly characterizes the low leverage equilibria as stable and the high leverage equilibria as unstable. The equilibria estimated in the data present exactly the opposite features: The low-leverage equilibria is stationary while the high-leverage equilibria shows unit-root like behaviour. Furthermore, leverage does not react to lagged accumulation in either of the equilibria, implying that neither the FIH or the Paradox of debt hold in any regime. All in all, while the Charles model fails to explain some important patterns of the data, many of its interesting properties are borne out in our dataset. Thus, further work in the Kaleckian tradition needs to be directed at analysing the dynamic feedbacks from leverage to the goods market, and less time needs to be devoted on models whose emphasis is solely on the univariate dynamics of leverage.

This paper is organized as follows: Section 2 briefly reviews a series of Kalecki-Minsky models, paying special attention to the dynamical systems that describe the evolution of the economy over time. Section 3 presents our data sources, with special care of our definition of Aggregate Debt, and some stylized facts of our univariate series. Section 4 starts by tracing the empirical implications of the various vintages of Kaleckian models presented earlier. It then proceeds to show how vector auto regressions can be used to test these implications. Section 5 presents our main results, where the number and qualitative properties of the equilibria present in the data are analysed. Section 6

[3]The aggregate leverage ratio is measured as the debt to GDP ratio

[4]Evidently, other empirical work exists, such as Isenberg (1988) or more recently, Davis, de Souza and Hernandez (2017). This article claims that this rich empirical work either doesn't nest or doesn't test an explicit macroeconomic model.

presents some last remarks on the Minsky research agenda and concludes.

2 Minsky's insights in a Kaleckian framework.

As noted by Nikolaidi and Stockhammer (2017), there has been an explosion of formal presentations of Minsky's insights during the last decade. Many differences stand out across these models: Among them, the choice of endogenous variables, the focus on cycles or explosive instability, and their use of different heterodox authors to complement Minsky's work.

However, it seems fair to say that extensions of the benchmark Neo-Kaleckian model, presented in texbook form in Lavoie (2014) and Hein (2014), for example, are one of the most popular ways to formalize Minsky's work. This section starts presenting the Kalecki-Minsky model of Charles (2008a), and then shows how successive simplifying assumptions yield different models popular in the literature.

2.1 Model 1: Sluggish adjustment in the goods market

Professor Charles assumes a closed economy without government in continuous time[5], which abstains from technological progress and depreciation. There are only two sectors: The household sector and the firm sector. Banks are not explicitly modelled. The equations that fully characterize the model are:

$$i = i^* + \phi l \tag{1}$$

$$g^s = s_f(r - il) + s_h[(1 - s_f)(r - il) + il] \tag{2}$$

$$g^i = \gamma_0 + \gamma_1 s_f(r - id) \tag{3}$$

$$\dot{l} = g(1 - l) - \frac{s_f}{(s_f + s_c(1 - s_f))(g - s_c id)} \tag{4}$$

$$\dot{g} = \delta(g^d - g) \tag{5}$$

The endogenous variables of the systems are the profit rate (r), the growth rate of capital (g) and leverage (l). The model does not contain any exogenous variables.[6] They key structural parameters are: the saving propensity of capitalists (s_c), the retention ratio of the corporate sector (s_f), the parameters of the investment function (γ_0, γ_1) and the reaction of the interest rate to changes in corporate sector leverage, ϕ.

Equation 1 describes the behaviour of interest rates: the author assumes that there is an interest rate from a risky asset (i) which is assumed to be equal to the rate of interest of a risk-free asset (i^*)

[5]For notational simplicity, we drop the dependence on time for all variables. Thus, $x(t)$ is written as x. A dot over a variable $\dot{x}$ denotes its instantaneous change, and growth rates are written as $\frac{\dot{x}}{x}$.

[6]Formally, the profit share and the capital-output ratio are exogenous variables, but they do not impact the relevant dynamical system which is of interest.

plus a risk premium associated with the level of leverage of the corporate sector.[7] The rationale for this equation is Kalecki's (1937) principle of increasing risk.

Equations 2, 3 and 5 characterize the behavior of the goods market: The growth rate of savings (2) is decomposed in savings from the corporate sector, which retain a constant fraction (s_f) of their retained earnings ($r - il$), and the household sector, in which only capitalists save a constant fraction of their dividend income, and their rentier income. The growth rate of investment is a function solely of retained earnings and a constant. Finally, equation 5 assumes that actual accumulation adjusts slowly to the desired by capitalists.[8]

Equation 4 is obtained by assuming that investment can only be financed by means of retained earnings or by issuing debt[9]; thus, equity finance plays no role in this model. As mentioned before, since internal finance is simply a fraction of retained earnings, debt grows to close the gap between desired investment and internal finance, bearing all of the adjustment in the firm's budget constraint.

The model is closed by assuming that savings (2) must be equal to actual accumulation, and plugging desired investment (3) into the adjustment equation (5). Replacing the interest rate equation into the investment function gives a two-dimensional dynamical system in leverage and accumulation which is quadratic:

$$\dot{g} = a_1 + a_2 g - a_3 l - a_4 l^2 \tag{6}$$

$$\dot{l} = b_1 g - b_2 g l + b_3 l + b_4 l^2 \tag{7}$$

Where the reduced-form vector of coefficients $\{\mathbf{a}, \mathbf{b}\}$ are combinations of the structural parameters of the model. [10] The model can be readily analysed by plotting the nullclines of the system in the (g, l) space, as shown below:

The equilibrium relations of this model have several features which are common across Kaleckian specifications. First, there are two equilibria: A 'good' equilibria with low leverage and high accumulation, and a 'bad' equilibria with high leverage and low accumulation. The first equilibria is locally stable, while the second one is locally unstable. In the 'bad' equilibria, the leverage ratio exhibits a counter-cyclical property: Whenever the economy is shocked to the right, its leverage ratio increases indefinitely while the accumulation rate tends to 0. Thus, the economy exhibits a 'Paradox of debt': Firms attempt to cut debt ratios will cause aggregate profits to fall faster than investment, which then leads to increasing debt ratios.

An additional feature of this model which is uncommon across Kalecki-Minsky model is that the accumulation rate is always debt-burdened; that is, a negative function of contemporaneous debt. This arises because the parameter configuration of the model always makes the negative effects on investment bigger than the increase in consumption which arises from the re-distribution of income

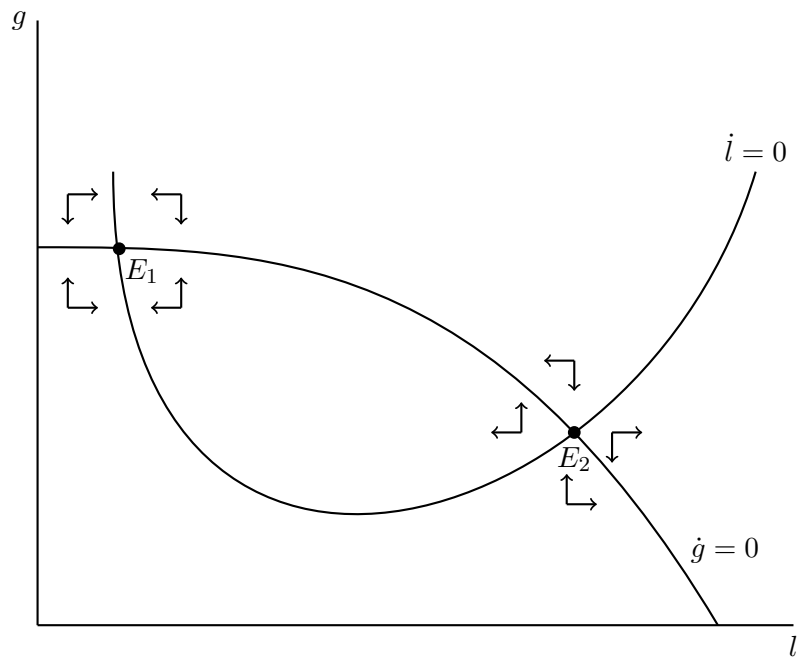
[7] Strangely, only the income flows from the risky asset appear in the savings function. The income coming from the risk-free asset is nowhere to be found. This renders the model stock-flow inconsistent.

[8] Although the author cites lags between the formation and realizations of expectations as the result of this adjustment, it's also possible to justify this adjustment equation by appealing to adjustment costs.

[9] Since banks are not present in the model, it seems that only the household sector acts as a lender.

[10] The reader can refer to the original paper to map these reduced form coefficients into the structural coefficients.

Figure 1: Phase Diagram of the Charles model



from firms to rentiers whenever the leverage rate increases. The models we review below are more flexible in this regard, since they allow accumulation to be debt-led.

2.2 Model 2: Instantaneous adjustment in the goods market and equity finance

Next, we explore the reduced-form implications of the models developed by Taylor (2009), Lavoie (2014) and extensivly analyzed in Ryoo (2013a). The main differences with the previous model are the following: First, the goods market adjusts instantaneously, and thus, accumulation has no law of motion. Second, commercial Banks are explicitly modelled, and it is assumed that the interest rate is exogenous and does not react to the level of leverage held by firms. Third, the model allows firms to finance part of its investment through equity, which introduces wealth in the saving and investment function, in the form of Tobin's q. We maintain the assumptions of continuous time, no government or external sector. Depreciation is assumed away, as is technological progress. The model is composed of the following systems of equations:

$$i^D = i^B = \bar{i} \tag{8}$$

$$B = D \tag{9}$$

$$g^s = s_f(r - \bar{i}l) + s_h[u(v - r) + (1 - s_f)(r - \bar{i}l) + \bar{i}l] - c_q q \tag{10}$$

$$g^i = \gamma_0 + \gamma_u u - \gamma_l \bar{i} l + \gamma_q q \tag{11}$$

$$\dot{l} = g(1 - x) + (s_f \bar{i} - g)l - s_f r \tag{12}$$

Compared with the previous model, the utilization rate is introduced as an endogenous variable which adjusts to clear the goods market, which is a defining characteristic of Kaleckian models. The model now has three exogenous variables: the capital to potential output ratio (v), the profit share (π) and the interest rate (i). They key structural parameters are: the saving propensity of the household sector (s_h), the retention ratio of the corporate sector (s_f), the percentage of investment financed by equities, (x), the parameters of the investment function ($\gamma_0, \gamma_u, \gamma_l, \gamma_q$), and the propensities to consume out of wealth, c_q.

Equations (8) and (9) describe the behaviour of banks: The interest rate on deposits is set equal to the interest rate on loans, which is exogenous. Additionally, since it is assumed that the public does not hold money and that banks don't hold reserves, bank loans (B) are equal to bank deposits (D).

Equation (10) now assumes that workers do save as a class, and thus we analyse the savings of the household sector as a whole via the parameter s_c. It is also know assumed that there are some wealth effects from consumption, contained into Tobin's q. Equations (11) and (12) describe the behaviour of the firm: Investment now additionally depends on the utilization rate and Tobin's q. Retained earnings now do not appear as a determinant of investment; only interest payments out of debt.[11] The differential equation for leverage is derived from the finance frontier of the firm, which states that the growth of investment expenditures net of wage payments must be financed either by issuing equities, bank loans or retained earnings. As mentioned before, equation (12) is extended to include the fact that equity finance is a fixed percentage of investment, while internal funds enter the finance frontier in the same way.

Long-run equilibrium requires that the goods market clears in every period (which implies (10) = (11)), and that investment expenditures are consistent with the budget constraint of the firm (which implies (11) = (12)). It is assumed that the utilization rate adjusts to clear the goods market, while leverage adjusts to bring in planned investment in line with the budget constraint of the firm. Since only leverage exhibits dynamic behaviour, however, the whole economy can be characterized by solving the dynamics of leverage: we can obtain the trajectories of the growth rate of capital, the profit rate and the utilization rate all as functions of leverage.

Thus, if we denote $g(l)$ as the equilibrium growth rate of capital as a function of leverage, which is derived from the equilibrium on the goods market, equation (12) becomes:

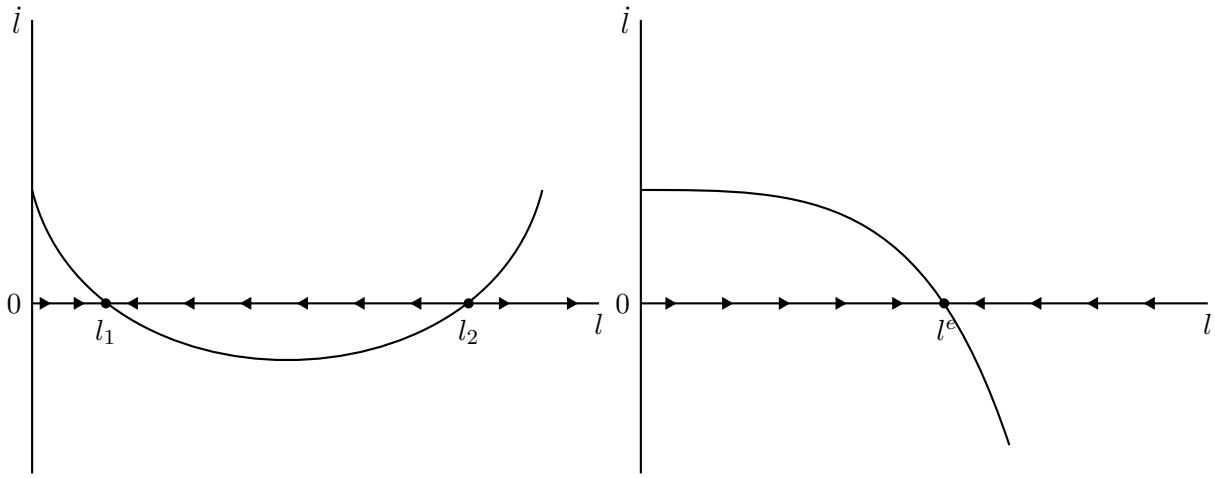
$$\dot{l} = g(l)(1-x) + (s_f\bar{i} - g(l))l - s_f r \tag{13}$$

It can be shown that $g(l)$ is a linear function of leverage, which implies that (7) will be a quadratic function of leverage. The reaction of the goods market to leverage will be crucial to determining equilibrium: if the goods market is 'debt-led'($g'(l) > 0$), the quadratic equation will have only one equilibrium with positive leverage, which will be globally stable. If the goods market is 'debt-burdened', ($g'(l) < 0$), the above equation will have two equilibiria: One with low leverage and high growth, which will be locally stable, and one with high leverage and low growth, which will be locally unstable.[12] Figure 2 plots the phase line for both cases.

[11] Thus, in this variant of the Kalecki-Minsky models, it's not clear what explains the impact of interest payments of investment, since if a cash-constrained story would hold, we would expect the profit rate to enter the investment function with the same parameter value as interest payment, as in the Charles model.

[12] See Ryoo (2013), pages 6-7 for a more detailed discussion of these results.

Figure 2: Dynamics of Model 2 with $g'(l) < 0$ (left) and $g'(l) > 0$ (right)



In the 'debt-burdened' case, this model displays the same dynamics that the model with sluggish adjustment in the goods market: There is a bad equilibrium with low growth and high leverage, and a good equilibria where the converse is true. The stability properties of the model are also identical. The main difference is that *all the information which pertains to the reaction of the accumulation rate to the leverage rate* is contained in the leverage equation. As thus, one equation disappears from the previous dynamical system.

In the 'debt-led' case, the model predicts that leverage will behave in a pro-cyclical way, and thus, contains a Kaleckian formalization of the 'Financial Instability Hypothesis'. Essentially, the model requires that the negative effects of debt on investment are low, and the difference between the propensity to save out of the household sector and the retention rate is large: Whenever debt increases, the redistribute effects increase aggregate demand by more than the negative effects from investment.

A crucial difference between this model and the previous model is that the interest rate is assumed to be exogenous. Interestingly, the effects of interest rates on the dynamics of accumulation are ambiguous on many variants of the model, such as the one presented here: If interest rate rises, investment will contract, but a redistribution of income from firms to rentiers will take place, which will boost consumption. Thus, there will be 'puzzling' scenarios where an increase in the loan rate will increase accumulation.

2.3 Model 3: Instantaneous adjustment in the goods market without equity finance

Our final - and most simple models - are based on the first vintage of Kalecki-Minsky models, developed by Lavoie (1995) and Hein (2007). Professor's Lavoie model assumes that the evolution of leverage is determined solely by the rentiers behaviour. Denoting the propensity to save of rentiers as s_r, the growth of firm debt is given by:

$$\dot{l} = [s_r r - g(l)]l \tag{14}$$

As before, two equilibria exists with the same characteristics as the previous model. Thus, we defer to commenting it further. Professor's Hein model is the simplest model of the ones considered here: There is istantaneous adjustment in the goods market, workers do not save, dividends are not distributed ($s_f = 1$), there are no equities in the model, and a class of pure rentiers exists which saves in the form of deposits. The growth rate of leverage is thus a simplified version of (12) which equals:

$$\dot{l} = g - (1 - l)r \tag{15}$$

The linearity of the leverage function now guarantees the existence of only one steady state whether the accumulation rate is debt-led or debt-burdened. However, the debt-led regime will be globally stable, and the debt-burdened regime will be globally unstable.

To summarize our discussion, various Kalecki-Minsky models imply a dynamical system which exhibits multiple equilibria. In general, the high leverage equilibria will be unstable, while the low leverage equilibria is stable. These multiple equilibria can arise either from specific assumptions regarding how firm's finance investment - specifically, assuming that equity finance is a fixed proportion of investment, or by assuming that the risk premium is a function of leverage. We will discuss how to test these and other implications below; however, we first turn to discussions related to data sources.

3 Data Sources and Stylized Facts

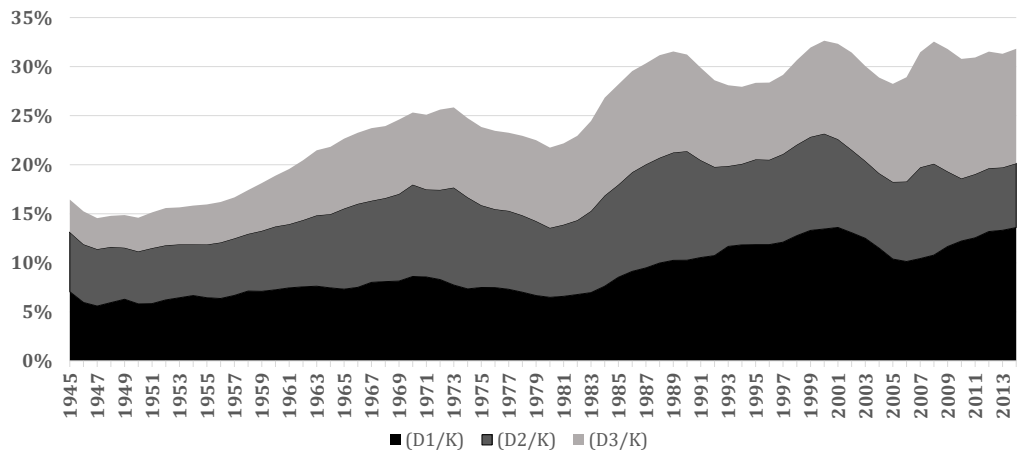
In order to appropriately estimate the Kaleckian-Minsky model outlined above, we need two relevant time series. The first one concerns the accumulation rate, while the second one concerns the measure of leverage employed in the model. Unfortunately, previous empirical work has not been consistent when measuring leverage. Lavoie and Secareccia (2001) use a measure of debt securities over equity, while Charles (2008) uses a measure of aggregate debt over GDP. However, the Kaleckian model employs aggregate bank loans over tangible capital as its relevant measure of leverage. Thus, in order to stay as closely as possible to our theoretical model, we attempt to construct a time series consistent with this notion.

We first obtain our series of annual capital stock as the current-cost gross stock of private fixed assets constructed by the Bureau of Economic Analysis. This constitutes the denominator of our series. The numerator of our series should be a measure of aggregate debt. If we were to take our model literally, this measure should only contain aggregate bank loans of the corporate and non-corporate financial sector, which is readily available from the flow of funds accounts. However, one could also argue that one should not take this simplification literally and should add other debt securities used by the corporate sector, such as corporate bonds, mortgages, and so on. We choose to employ this latter measure of aggregate debt. The capital accumulation series is simply constructed by taking total investment in fixed assets by the private sector and dividing them by our measure of the capital stock. All series are measured in real terms.[13]

[13] Links to all data sources.

Figure 3 shows the long-run movements of the leverage ratio as we defined it and the different components of debt it entails. D1 includes all Debt Securities of the Non-Financial Corporate Sector, D2 includes all Loans from Banks and other financial institutions to the Non-Financial Corporate Sector, while D3 stands includes the same debt components as D2 but directed to Non-Financial Non-Corporate Business. The trends of the different components of business debt show very interesting behavior: While Debt securities remained essentially stable at 5% between 1945 and 1980, they started growing rapidly until 1999 and then stabilized around 10% for the last fifteen years. The most notorious increase in debt has come from bank loans to the non-corporate sector. The leverage ratio for this particular measure of debt evolved from roughly 3% to a staggering 15%.

Figure 3: Aggregate Leverage and its components. USA, 1945 - 2014



Next, Figure 4 plots the contemporaneous relation between aggregate leverage and capital accumulation. If the Charles model was an accurate representation of the data, we should see the data cluster around two equilibria, one with low leverage and high accumulation, and the other one with high leverage and low accumulation, with possible shifts between decades accounting for changes in loan interest rates. If one ignores the two outliers of at the bottom left of the panel, there seems to be one regime before the leverage rate crosses 30%, with an accumulation rate around 7-8%, and one regime after leverage crosses 30%. The volatility of the accumulation rate around these leverage rates has certainly increased, but one could even argue that, without taking into account the global financial crisis, which displays itself around the bottom-right of the figure, the accumulation rate had been slightly *increasing* around this newer steady-state leverage rates.

Finally, before proceeding to the econometric analysis, Table 1 shows some important characteristics of our aggregate time series: Their persistence, volatility, and co-movement. g denotes the accumulation rate, while l denotes our measure of aggregate leverage. Persistence is measured as the first-order autocorrelation of each series, comovements as the correlation between our two series, and volatility as the ratio between the standard deviation and the mean of each series.

As it can be seen, the leverage series is very persistent, showing almost unit-root like behaviour,

Figure 4: Scatterplot between the accumulation and the leverage rate. USA, 1945 - 2014

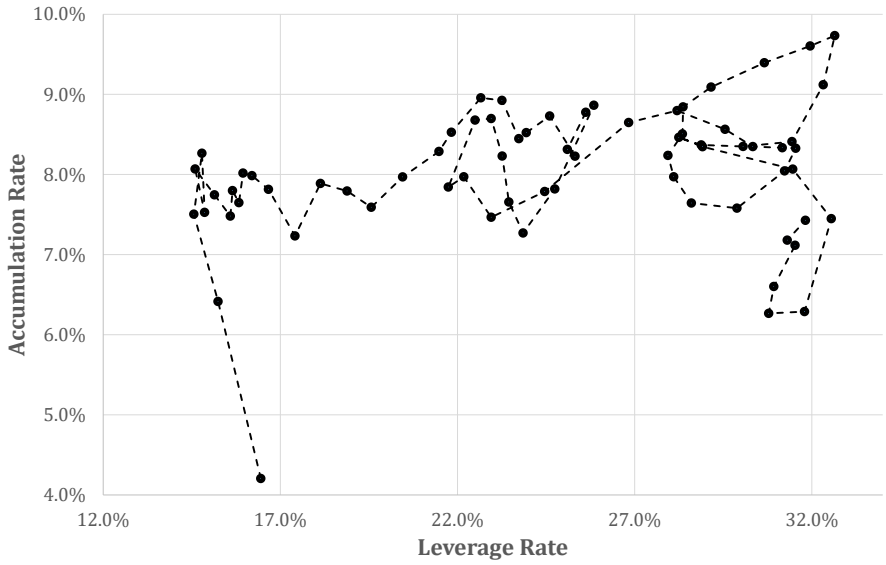


Table 1: Moments of Time Series

Moment	Persistence	Volatility	Comovement
g	0.66	0.11	1.00
l	0.96	0.24	0.27

while the accumulation rate is much less persistent. The volatility of leverage is higher than the volatility of the accumulation rate. As documented first by Lavoie and Seccareccia (2001), the leverage rate seems to be essentially a-cyclical; while it shows a small positive correlation with the accumulation rate, the wage rate, which is widely regarded to be a-cyclical by the profession, shows a correlation with output of 0.35 (see King and Rebelo (1990) and Stock and Watson (1999) for a discussion). With these stylized facts in mind, we move on to present our econometric strategy.

4 Econometric Strategy

What kind of econometric tools do we have at our disposal to test the models outlined above? Two roads can be taken: The first is to estimate the structural equations of Kalecki-Minsky models. However, this route poses several difficulties, since the models outlined above is heavily under identified. The second route is to estimate the reduced-form of the model, and to ask what kind reduced-form predictions the Kalecki-Minsky model implies. This paper purses the later route.

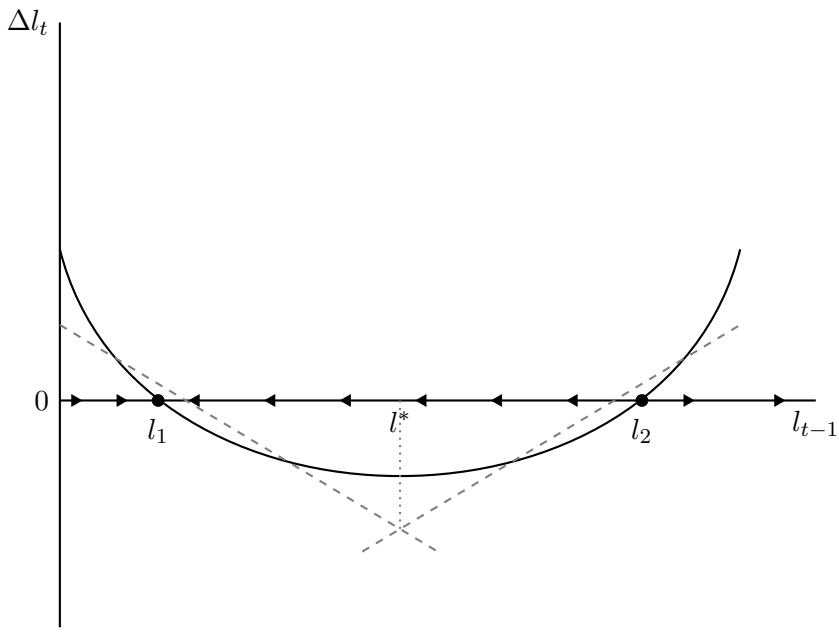
A natural framework in which to specify a statistical model which can nest both the DGP and the Kalecki-Minsky model is with VAR models. As it has been discussed elsewhere at length (Juselious, 2006; Canova chap. 4, 2007), VAR models, in their unrestricted lag form, are merely a reformulation of the covariance structure present among variables, and thus it can be argued that a properly specified VAR model and 'the data' are synonyms. It then becomes a natural to ask what kind of restrictions does the Kalecki-Minsky model place on the VAR model to be estimated.

4.1 Testable Implications

Fortunately, variants of Kalecki-Minsky models are rich with implications. One common feature of all the models explored above, with the sole exception of Hein (2007) is that these models feature multiple equilibria: there is typically an bad equilibria with low growth and high leverage, and a good equilibria with the opposite characteristics. To fix ideas as to how this multiple-equilibria might be tested in this setting, let us reproduce graphically the dynamics of leverage from model 2 in the debt-burdened case, but in discrete time.

Figure 5 shows us an interesting feature: Suppose an econometrician where to estimate by least squares a regression of the form $\Delta l_t = \alpha + \beta l_{t-1} + u_t$. This linear approximation would do a poor job of fitting the data if the model was the DGP, due to the quadratic nature of our equation. However, if the econometrician knew where point l^* is located, he could estimate the above regression for two regimes: whenever $l < l^*$, he would estimate one regression, and whenever $l > l^*$, he would estimate another regression. The dashed grey lines show the presumed estimated slopes in each regime by the econometrician.

Figure 5: Multiple Equilibria and Threshold Autorregresive Models



This kind of model has been widely studied in macroeconometrics, since the seminal contribution by Tong (1990): It is called the threshold autoregressive (TAR) model. Its name derives from the fact that the DGP exhibits a point on any observable variable (the threshold) which acts as a regime shifter for the data. In the model studied above, the threshold variable is leverage itself. TAR models exhibit a number of attractive features: they are relatively easy to estimate, they can be derived directly from economic theory (as in our example), and they can capture a rich number of non-linear features of the data.

In the case shown by figure 6, where the goods market is debt-burdened, a number of additional predictions emerge. Besides the existence of a threshold effect on leverage, the model predicts at

most two distinct regimes, with each regime having distinct time series properties. The regime with low leverage was shown to be locally stable, which implies that leverage should be stationary in this regime, while the regime with high leverage is locally unstable, which means that leverage should be non-stationary in this regime. Furthermore, the (sum of) coefficient(s) on the autorregresive terms should change signs with each regime. Finally, we should expect to see a debt-burdened goods market: capital accumulation should react negatively to leverage in order to observe such a non-linearity.

It can be shown that all of these results carry on to the Charles model.[14] The key additional prediction of the Charles model is that the debt-burdened demand effects on accumulation have dynamic implications; that is, past values of leverage should also enter the accumulation rate. The sluggish goods-market assumption guarantees that this holds, and this generates a two-dimensional dynamical system which in turns implies a Threshold Vector Autorregression (TVAR) with the rate of growth of capital and leverage as endogenous variables. As before, the threshold variable should be leverage, not accumulation; there should be at most 2 regimes, one regime with low leverage and high growth, and one regime with high leverage and low growth. The first regime should be stationary, while the second should be non-stationary, the autoregressive coefficient on the leverage equation should change signs when we change from one regime to another and finally, as before, these qualitative features of the data should be accompanied by a debt-burdened accumulation rate on each equilibrium.

4.2 Vector Autorregressions

In light of the above discussion, it seems fair to say that a one way to test the implications of the Kalecki-Minsky model would imply the following steps: First, estimate the following linear VAR(p) model:

$$y_t = c + A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \tag{16}$$

Where $y_t = (l_t, g_t)$ is our vector time series containing leverage and accumulation, c is the vector of constants, A_i is the matrix accompanying the i lagged vector of dependent variables and u_t is a vector of white noise shocks on each equation. Provided that our VAR passes the relevant diagnostic misspecification tests, we could claim that we can pit the best linear approximation of the data against a global linear alternative. If we reject the assumption of linearity, we would expect the non-linearity to take a threshold form and the threshold variable to be leverage or lagged values of leverage, but not the accumulation rate. In this case, the above model would take the form:

$$y_t = \begin{cases} c_1 + A_1^1 y_{t-1} + ... + A_p^1 y_{t-p} + u_{t1} \text{ if } l_{t-d} \leq l^* \\ c_2 + A_1^2 y_{t-1} + ... + A_p^2 y_{t-p} + u_{t2} \text{ if } l_{t-d} > l^* \end{cases}$$

Where l^* is the threshold level of leverage, which we must estimate. With this formulation estimated, we can investigate whether the predictions outlined in the section above are fulfilled by our TVAR model. This requires a few distinct steps; first, we need to test the existence of a threshold effect; second, we need to develop a method to distinguish which is the threshold variable, and

[14] One might suspect that the threshold variable in the Charles model is the accumulation rate; however, given the logical bounds which must be imposed on a number of parameters, the non-linearity of the accumulation nullcline will be generally much less pronounced that the one from the leverage nullcline. This makes us suspect that leverage will also be the threshold variable in the Charles model.

to estimate its location; finally, we need to estimate the VAR model within each regime. Details regarding all of these issues as well as a review of influential applications of TAR and TVAR models are surveyed in Hansen (2010) and Hubrich and Teräsvirta (2013).

5 Results

5.1 A linear VAR benchmark

The first step towards specifying a linear VAR model is to select the maximum lag order, k. In order to do so, we compute different information criteria for up two 5 lags. The results are presented in Table 2. The results of this procedure are encouraging since all lag selection criteria suggest a VAR(2) model.[15]

Table 2: Lag-Length selection criteria

Lag	AIC	BIC	HQ
1	-15.01	-14.81	-14.93
2	**-15.67**	**-15.33**	**-15.54**
3	-15.66	-15.19	-15.47
4	-15.57	-14.96	-15.33
5	-15.46	-14.73	-15.17

The coefficient estimates by least squares are shown in table 2. The most striking feature of the point estimates is the inexistence of any kind of dynamic feedback between the accumulation rate and the leverage rate. Indeed, a linear Granger causality test on both equations using the Wald statistic cannot reject the null that leverage does not Granger cause accumulation and vice-versa at any conventional significance level. The results also reveal that the accumulation rate is a fairly persistent stationary series, while the leverage rate is a very persistent series, exhibiting unit-root like behaviour. These results serve as an important benchmark point to compare the threshold VAR specification that is implied by Model 1.

Table 3: Estimation Results: Linear VAR(2) model

Equation	Leverage (l_t)			Accumulation (g_t)		
	Coef.	Std. Er.	p-val	Coef.	Std. Er.	p-val
c	0.01	(0.009)	0.22	0.02	(0.006)	0.01
l_{t-1}	1.65	(0.096)	0.01	-0.03	(0.063)	0.65
l_{t-2}	-0.66	(0.095)	0.01	0.03	(0.062)	0.66
g_{t-1}	0.12	(0.178)	0.47	1.06	(0.115)	0.01
g_{t-2}	-0.20	(0.145)	0.17	-0.29	(0.097)	0.01
R^2	0.98	-	-	0.68	-	-

[15]Evidently, if we hadn't had the luck to obtain the same lag length selection from this criteria, we would have used the BIC criteria, since it tends to penalize over-fitting less than the HQ criteria (and consequently, minimizing the chance of omitted variable bias), and also because its a consistent model selection method along with the HQ criteria.

In order to interpret the point estimates and p-values of our linear VAR correctly, a number of assumptions need to be satisfied which can be readily checked using misspecification tests. The first and foremost assumption that has to be satisfied is that the residuals of the VAR behave jointly as white noise, that is, lagged values of the residuals series u_t^l and u_t^g cannot help *jointly* to predict leverage or accumulation. The first column of table 3 shows the result of a simple Lagrange Multiplier test which achieves this goal: a VAR(0) of the residual needs to always outperform a VAR(p) of the residuals in log-likelihood terms. As it can be seen, our model behaves excellent: All lagged values of the residuals considered do not help to predict any of the series at any conventional statistical level. A second assumption regards the behaviour of second moments: implicitly, the VAR model assume that any lagged value of the variance does not help to predict the current variance; that is, the variance also behaves as white noise. The consequences of ignoring ARCH-like effects when they exists can be disastrous for inference of the parameters contained in A_i.[16] This test is carried out in the second and third columns of table 3. These results also display excellent results, since ARCH-like effects are also rejected at any conventional statistical level. Finally, we test that the residuals follow a bivariate normal distribution using the Doornik-Hansen (1994) test statistic. The residuals reject the null of normality at any conventional significance level. This result is driven by the fact that the distribution of the accumulation rate residuals are skewed to the left, while the residuals of the leverege rate are skewed to the right. There is no excess curtosis in any of the series.

Table 4: Misspecification tests on the VAR(2) residuals

Lag	Autocorrelation	ARCH(l_t)	ARCH(g_t)
1	0.10	0.30	0.71
2	0.34	0.19	0.94
3	0.88	0.34	0.52
4	0.76	0.51	0.76
5	0.91	0.65	0.49

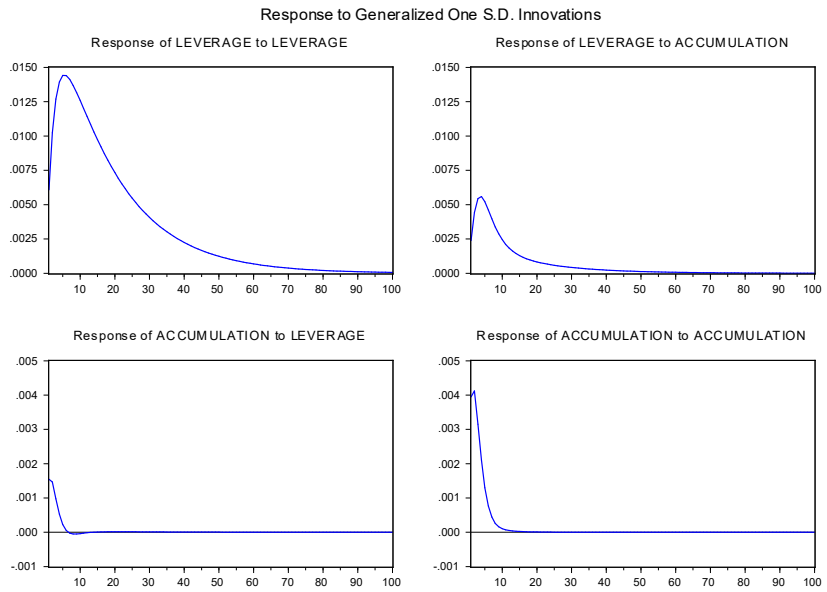
Our negative normality result has two implications: First, it implies that the distribution of our tests on individual and joint parameters cannot be based on exact distributions and must rely on asymptotic approximations. If the sample size is small relative to parameters, this could led to some distorted inference on the other tests we have performed. However, since our VAR model is very parsimonious compared to our sample size, this is not an important issue. Thus, our crucial result - the rejection of any systematic feedback effects from leverage to accumulation - can be viewed with confidence. The second implication is that our impulse-response function will have their standard errors calculated via bootstrap methods, since asymptotic approximations rely on the normality assumption. We now turn to computing our impulse response functions.

Since we are only interested in the reduced-form implications of the models outlined above, we will use Pesaran and Shin (1996) generalized impulse-response functions. These impulse response functions avoid identifying assumptions regarding the contemporaneous relationships across the variables used above, and thus, each shock to each variable is merely a linear combination of some underlying structural shocks into the data - which could be shocks to mark-ups, the bargaining power of workers or shareholders vis-a-vis management, animal spirits in the investment function, and so on. As mentioned above, the standard errors are computed using bootstrapped standard

[16] See Hamilton (2008) for a discussion.

errors which are drawn from 1000 repetitions.

Figure 6: Generalized Impulse Response Function



The impulse response functions show that all shocks lead to an instant positive response on accumulation and leverage, but with different persistence profiles. The accumulation rate experiences a positive reaction to a leverage that lasts for 5 years, while the leverage rate experiences an instant positive increase which reaches its peak roughly 5 yeas after the shock, after that, it slowly dies out, dissipating completely after 60 years. Own-variable shocks also exacerbate these persistence profiles: While Leverage takes 100 years to dissipate a shock to itself, leverage takes only 10 years.

Since we established before that there is no dynamic feedback across leverage and output, why do impulse response functions show results that do not seem consistent with this notion? The answer to this question is that impulse response function are picking up the correlation across the contemporaneous *residuals* of leverage and accumulation; in other words, there is a third omitted variable which is responsible for these correlations among the unobserved components of accumulation and leverage which is driving these results.

As we discussed extensively in the previous section, a linear vector autorregression could be a poor approximation to the data if the true data generating process is model 1: due to the multiple equilibria of this model, the data would be expected to show strong signs of non-linearities of a specific kind, namely, a threshold auto regression non-linearity. However, it seems useful to use as a diagnostic test a global test of non-linearity, that is, a test that can detect any possible form of non-linearity which includes a threshold effect. The rationale for doing this exercise is the following: If we reject linearity as a null and then we find a threhold effect, the global non linearity test operates as a robustness test; if we reject linearity as the null but we don't find evidence of any threshold effects, this implies that a consistent modelling effort should be made in the future to approximate the non linear behaviour of leverage and accumulation by other popular non-linear models, such as Markov-Switching models or stochastic volatility models.

An appealing test for global non-linearity is the Brock-Dehert-Scheinkman-LeBaron (1996) test (BDS, from now own). This test is a global test of non-linearity which has reasonable power against a number of non-linear alternatives popularly used in the literature, such as ARCH models, GARCH models, and TAR. The philosophy behind the test is the following: Since we usually use autocorrelations - which are a form of linear dependence - to detect misspecified linear models, we should be able to construct a measure of non-linear dependency in the residuals in order to check that some form of non-linearity exists. Thus, the null of the test is that the residuals of each equation behave as i.i.d errors, while the alternative is that some form of non-linear dependence occurs.[17]

Table 5: Results of the BDS test for non-linearity

Dimension	l_t	g_t
2	0.18	0.43
3	0.04	0.19
4	0.02	0.08

Note 1: To compute the test, a measure of distance between the points of the residual is needed. As in the original BDS paper, we use 0.5 standard deviations of the distribution as a measure of distance. Since the residuals were found to be non-normal, the empirical distribution is bootstrapped 10.000 times.

These results embedded in table 5 strongly suggest that there are omitted non-linear structures in the leverage equation, while there is marginal evidence of non-linearity for the accumulation equation. Thus, this indicates that model 1 and model 2 get one feature of the reduced-form equations of the data right: The leverage rate is strongly non-linear, while accumulation could be non-linear. We now turn to examine the specific form of non-linear relation that the various Kalecki-Minsky models imply: Threshold non-linearity.

5.2 A threshold VAR

The first step towards estimating a TVAR is to conduct a linearity test on the VAR model. Given the results on the BDS test, we already know that there non-linearities in our data set; the question now is if the threshold VAR, which is implied by the models examined, is a good candidate to explain the non-linearity present in the data.

In order to test the existence of threshold effects in a VAR setting, we follow the approach of Lo and Zivot (2001). Their paper extends the approach surveyed by Hansen (1999) to test linear AR models against TAR models. See the original papers for a discussion. The null hypothesis of this likelihood ratio test is:

$$LR_{ij} = T\left(\ln|\hat{\sum_i}| - \ln|\hat{\sum_j}|\right) \tag{17}$$

[17]See Barnett et. al (2004) for additional Monte-Carlo evidence that the BDS test has very reasonable power properties against deterministic non-linear models and even chaotic models.

Where the sub-indices i denote the number of regimes, and thus, $i-1$ is the number of thresholds. j is taken to be the null model, which will be the linear VAR model estimated above. Σ is the variance-covariance matrix of the residuals of each model. The distribution the test are simulated by bootstrap, with 1000 repetitions used. The results of this test are presented in table 6, where both leverage and accumulation are treated as the threshold variables. We test the null of a linear VAR model against the alternative of a two regime threshold model, and a three regime threshold model.

Table 6: Test for threshold effects in leverage

Number of Thresholds	0 vs 1	0 vs 2	2 vs 3
P-values	0.01	0.01	0.12

Notes: The bootstrap uses 1.000 replications to draw the empirical distribution. In each case, 10% of the sample is trimmed at the left and the right to search for the threshold. Both contemporaneous and up to two lags of the relevant variables to search for threshold effects are contemplated. The variable which minimizes the p-value across specifications is picked, favouring the alternative hypothesis.

The test results lend support to model 1: Leverage acts as a threshold variable, and there is only one threshold point, which implies two regimes. The estimated threshold point is a leverage rate of 30%. It's interesting to note that this implies that the US economy was in a low-leverage regime between 1945 and 1986, after that, the economy has switched between the low and high leverage regime. The high leverage regimes correspond to the periods of 1987-1990, 1998-2003 and 2007 - 2014. This implies that the threshold VAR has not been picking up only a structural break in the US economy; there has been substantial genuine regime switching in the last 30 years. With this in mind, we turn to analyse the coefficient estimates for both regimes.

Table 7: Estimation Results: Threshold VAR(2) model

Equation	Leverage (l_t)			Accumulation (g_t)		
	Coef.	Std. Er.	p-val	Coef.	Std. Er.	p-val
Regime 1: Low Leverage (T=50)						
c	0.01	(0.016)	0.66	0.07	(0.010)	0.01
l_{t-1}	1.51	(0.149)	0.01	0.21	(0.091)	0.01
l_{t-2}	-0.50	(0.140)	0.01	-0.17	(0.086)	0.05
g_{t-1}	-0.06	(0.240)	0.80	0.29	(0.147)	0.05
g_{t-2}	-0.00	(0.169)	0.99	-0.22	(0.103)	0.03
Regime 2: High Leverage (T=18)						
c	0.29	(0.102)	0.01	0.13	(0.063)	0.04
l_{t-1}	0.47	(0.327)	0.15	-0.40	(0.200)	0.05
l_{t-2}	-0.39	(0.196)	0.05	0.03	(0.120)	0.80
g_{t-1}	0.55	(0.377)	0.14	1.14	(0.232)	0.01
g_{t-2}	-0.54	(0.378)	0.15	-0.25	(0.232)	0.28
Threshold value: $l_t = 0.3$						

The results that stem from table 7 are not coherent with model 1 with regards to three predictions. The first wrong prediction concerns the level of the accumulation rate in each regime: In the low

leverage regime, the accumulation rate is 7,5%, while in the high leverage regime, the accumulation rate is 14,6%[18]. Model 1 predicts that a high-leverage equilibrium should have a lower-steady state accumulation rate, not a higher one.

The second wrong prediction relates to the stability properties of each equilibria: While the leverage rate shows unit-root like behaviour in the low leverage regime, with the consequent non stationarity behaviour, it exhibits stationary behaviour in the high leverage regime. The accumulation rate shows the opposite behaviour: it's stationary around the low-leverage equilibrium and shows unit-root like behaviour in the high leverage equilibrium. Model 1 predicts exactly the opposite with regards to the leverage rate: The economy is stable around a low leverage equilibrium, while its unstable around the high leverage equilibrium.[19]

Finally, while model 1 predicts that leverage should react to lagged accumulation, this prediction does not hold true for neither of the two regimes. This impressive result means that neither the FIH or the Paradox of debt hold for either a linear or a non-linear specification.

Despite these negative results, there is one key prediction of the Charles model which is borne out by the data: The accumulation rate does show a substantial response to lagged values of the leverage rate in both regimes. On the low-leverage regime, the accumulation rate is almost horizontal on the long-run with regards to lagged leverage. Indeed, the hypothesis that the long-run effects of leverage are 0 on the low leverage regime cannot be rejected at any conventional significance level. Despite this horizontal long-run nullcline, the leverage rate contributes substantially on the short run to fluctuations in accumulation, since both coefficients are statistically significant and with opposite sign. On the high leverage regime the long-run impact of a one point increase in accumulation is -0.37 over leverage, which implies that accumulation is debt burdened. This implies that a 5 percentage point increase in the leverage rate, decreases the accumulation rate by 1.8 percentage points on the long run. This seems a sizeable effect, especially when one considers that a 5 percentage point fluctuation in leverage has been common at business-cycle frequencies in the US experience in recent years. Taken together, this results imply that feedback effects from leverage to accumulation have some potential to explain fluctuations in the accumulation rate at business cycle frequencies in both regimes, while fluctuations in the leverage rate are a good candidate to explain differences in the growth performance of the US economy over the long run in the high leverage regime.

It's important to remember that the Charles model is constrained to have a debt-burdened accumulation rate because it's assumed that the negative effects of leverage on investment outweigh the positive effects on rentiers consumption. This is the only model among all models considered which imposes this reduced-form restriction on its dynamical system. These results are also coherent with Nishi's (2012) results for Japan, which also find a debt-burdened goods market.

[18]The value for the high leverage regime holds if the leverage is assumed to be stationary in this regime; if it is assumed that it follows unit-root behaviour, then the mean is simply 13%.

[19]IMPORTANT NOTE: It seems that the high-leverage equilibria of the Charles model is a saddle; as such, the accumulation rate always has self-stabilizing behaviour and leverage dynamics are responsible for instability. Here, we obtained that the goods market is unstable on the high-leverage equilibria, which is also at odds with the model. This should be checked latter.

5.3 Are the models consistent with the data?

Given our previous discussion, it seems safe to say that model 3 - the variants proposed by Proffesor's Hein and Lavoie - are not consistent at all with the data. As we've seen, a linear specification implies essentially no feedback from the relevant endogenous variables, which implies that a linear model, such as the one developed by Professor Hein, misses most of the important reduced-form features of the data. The problem with Professor Lavoie's model is that his low-leverage equilibrium has a steady-state value of 0 - a prediction also rejected by the data.

Models 2 and 3 can be said to be inconsistent with the data in another very important dimension: Using a one dimension dynamical systems ignores all the relevant feedback from leverage to accumulation. As we've shown, this is the most important empirical fact that arises when we use a non-linear model, since there doesn't seem to exist any dynamic effects from accumulation to leverage whatsoever. In this sense, future models should seek to explore the dynamic effects of leverage on aggregate demand, and not only the contemporaneous effects as the literature has done so far - with the exception of model 1.

One could object to this claims by suggesting that models 2 and 3 predict precisely that accumulation has no dynamic effect on leverage; a result which was borne by the data. As such, it could be argued that these models are a reasonable approximation to understand the dynamics of leverage as an univariate process. In Appendix A1 we investigate this claim and provide a negative answer: Whenever we consider leverage in isolation, our data finds 2 thresholds, not 1, as predicted by model 2; all the qualitative characteristics of this equilibria are inconsistent with the claims of the model. As such, even when we give model 2 and 3 a fair chance, they get the non-linear features of the data wrong.

Two robustness results which do not essentially change the above discussion are conducted in appendix A1 and A2. In appendix A1, we consider the interest rate of the commercial banking system exogenous, as it is also assumed in variants of model 2, and include the loan prime rate as an additional regressor. The results on our bivariate or univariate specification do not change at all, since the loan prime rate is at best marginally significant in all specifications. In appendix A2, we take our dependent variable as output growth instead of accumulation. Since the capital to potential output should be fixed in the long run, this implies that these two series should exhibit the same long-run growth rate. Furthermore, one could argue that Minsky's reflection about the business cycle were done in terms of output growth, not in terms the accumulation rate. The results from this econometric exercise are disappointing for the Kalecki-Minsky models: In all specifications, there are no feedback effects from output growth to leverage in any sense. Furthermore, leverage does not show omitted non-linearities in this specification and output growth acts as the threshold variable. Thus, if we take output growth instead of the accumulation rate as the dependent variable, all of the predictions by Kalecki-Minsky models are rejected.

In light of the above discussion, we think that the verdict of this empirical investigation is that the Charles model is the only Kaleckian specification which shows some promise of capturing relevant features of the data. More work needs to be done on the feedback channels from lagged leverage to current accumulation, and less work on models which have a solution in terms of a one dimension dynamical system containing leverage - be it linear o non-linear, since these simple Kalecki-Minsky models are not relevant to explain the patterns of the data, at least in the US case. However, more empirical work needs to be done taking this family of models to other countries to test their

implications before any definite conclusions are reached. Moreover, the implications from Kaldorian and Goodwinian models with Minskian insights also need to be tested, since many variants of these models also imply multiple equilibria and non-linear behaviour.

6 Conclusion

This paper investigates the empirical relevance of Kaleckian model which are augmented by business debt to produce Minskian insights. In order to do so, it formulates several variants of the Kalecki-Minsky models found in the literature, explores what are their main reduced-form implications on the joint dynamics of accumulation and leverage, and shows how linear and non-linear vector autorregressions can be used to test their implications. Specifically, a common feature of Kalecki-Minsky models is their multiple equilibria nature, where one stationary equilibria featuring low leverage and high accumulation and one non-stationary equilibria featuring high leverage and low growth are implied by the models. This implies that the Data Generating Process should be a threshold vector auto-regression, which is a specific form of non-linearity.

Using 70 years of US data, this paper finds that the two-equilibria description of accumulation and leverage is borne by the data; furthermore, there exists a threshold effect caused by leverage, not accumulation, with at most 2 regimes, as predicted by the Charles model. However, many of the qualitative behaviours expected from the Charles model are not borne by the data: One equilibria has low average leverage and accumulation, while the other has high average leverage and accumulation. Furthermore, there is no feedback from accumulation to the leverage rate, contrary to what is predicted by any of the Kalecki-Minsky variants considered. One key prediction which is characteristic of the Charles model is that the accumulation rate is debt-burdened; this implies that the negative effects of leverage on investment are higher than their effects on rentiers consumption. It is found that the high-leverage regime is substantially more debt-burdened that the low leverage regime, which implies that the financial behaviour of firm's have become more important to explaining business cycles and long-run accumulation in the highly leveraged economy that is the US. Overall, while the Charles model can be used as a baseline to understand some relevant features of the data, many work still remains to be done to explain the wrong predictions of the Charles model.

Two last comments are in order. First, non-linearity or multiple equilibria are *not* exclusive features of the Kalecki-Minsky family, for example, the Goodwin-Minsky model of Keen (1994) is also shown to have two equilibria in Grasselli and Costa Lima (2012). The particular feature of Kalecki-Minsky models is that the source of non-linearity is in the behaviour of leverage, while in the Goodwin-Minsky models the Wage Share behaves non-linearly to generate two distinct equilibria. As such, it is certainly promising to study if the labour market feed backs into the accumulation rate in a way that the goods market does not. Second, we ignored Kalecki-Minsky models which examine the joint behaviour of accumulation and interest rates (Lima and Meirelles, 2007) or of the retention rate and accumulation (Charles, 2008b). Given the results reviewed here, it seems that other big families of minskian models or other variants of Kalecki-Minsky models might be promising avenues of research.

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