Exchange rate dynamics, balance sheet effects, and capital flows. A Minskyan model of emerging market boom-bust-cycles

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Abstract
Business cycles in emerging market economies (EMEs) are characterised by capital inflows, widening current account deficits, and currency appreciation during the boom, and capital outflows, current account reversals, and depreciation during the bust. Moreover, private sectors are often vulnerable to external shocks due to foreign-currency denominated liabilities, which can give rise to balance sheet effects. While there is a sizeable literature on both phenomena, there is little theoretical work on how exchange rate dynamics and balance sheet effects interact. This paper argues that procyclical exchange rate dynamics and balance sheet effects can give rise to boom-bust-cycles. It presents a simple Minskyan model in which exchange rate dynamics are driven by capital and trade flows. A currency appreciation makes balances sheets with foreign currency debt appear more solid. Throughout the resulting boom phase, the exchange rate continues to appreciate and the current account position worsens. Pressures on the domestic exchange rate mount until the currency eventually depreciates. Contractionary balance sheet effects set in as domestic firms face a drop in their nominal net worth. The exchange rate thus exerts a pro-cyclical effect that interacts with foreign-currency debt to generate boom-bust-cycles. Under conservative foreign investors that prevent historically given leverage from rising, the cycles are shock-dependent damped oscillations. Under idiosyncratic investors that target a specific leverage ratio, the fluctuations can assume the form of shock-independent limit cycles. The higher the target leverage ratio, the more financial account regulation is needed to prevent limit cycle dynamics.

Keywords: Business cycles; emerging market economies; balance sheet effects; Minsky

JEL Codes: E11, E12, F36, F41

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1 Introduction

Empirical research on business cycles has repeatedly found that macroeconomic fluctuations in output, exchange rates, and the current account are significantly stronger in emerging market economies (EMEs) compared to industrial economies (Agénor et al., 2000; Lane, 2003; Calderón and Fuentes, 2014). The severity of business cycles in EMEs has led some authors to speak of boom-bust-cycles (Williamson, 2005; Reinhart and Reinhart 2009; Herr, 2013). A common pattern that emerges from these studies is the coincidence of strong gross and net capital inflows, exchange rate appreciation, a widening current account deficit, as well as growing external indebtedness during the boom period. The bust phase, on the other hand, is characterised by capital outflows, currency depreciation, current account reversals, and deleveraging. Thus, unlike in most industrial economies, exchange rates of EMEs behave pro-cyclical (Cordella and Gupta, 2015).

A second characteristic that is highlighted in recent empirical research on EMEs is the significance of foreign-currency borrowing. The literature on 'original sin', which signifies the 'inability of a country to borrow abroad in its own currency' (Eichengreen et al., 2007, p. 122), has shown that foreign-currency denominated borrowing is pervasive in developing countries. Original sin can lead to currency mismatch[1] which exposes economic units to exchange rate risk. A sudden depreciation of the domestic currency raises the nominal value of foreign currency debt reducing the net worth of these units. This, in turn, can lead to a drying up of financial sources or even outright bankruptcy. Indeed, such balance sheet effects played a significant role in the East Asian crisis in the late 1990s (Allen et al., 2002). A rich but mostly mainstream literature on balance sheet effects has developed ever since, in which the implications of foreign currency debt for monetary policy and the optimal exchange rate regime is examined (Krugman, 1999; Aghion et al., 2000; Céspedes et al., 2004; Delli Gatti et al., 2007; Charpe et al., 2011, chap. 2).

Econometric studies confirm that devaluations are more likely to have a negative effect on output and growth in countries with high external debt burdens (Galindo et al., 2003; Bebczuk et al., 2007; Blecker and Razmi, 2007; Janot et al., 2008; Kearns and Patel, 2016). The fact that the share of foreign-currency denominated liabilities on the balance sheets of non-financial corporations has increased sharply in many middle-income countries since the Great Financial Crisis of 2008 (IMF, 2015, chap. 3; Chui et al., 2016), indicates that balance sheet effects will remain an important feature of EMEs.

[1] The denomination of assets and liabilities in different currencies.

While both phenomena, pro-cyclical exchange rates and balance sheet effects, have been studied in isolation, there is much less research on how they interact. The theoretical literature has not fully acknowledged the fact that foreign currency debt not only has contractionary effects during depreciations, but also expansionary effects when the currency appreciates. The focus of models with balance sheet effects is typically on currency crises, not business cycles. In this paper, we examine the role of pro-cyclical exchange rate dynamics in business cycle dynamics in EMEs. We argue that a Minskyan framework is ideal for analysing boom-bust-cycles in EMEs due to its focus on aggregate demand, the role of financial factors in investment decisions, and balance sheet interrelations. Although the recent literature has placed a strong focus on the existence of a global financial cycle that stems from risk appetite and monetary policy in the financial centres (La Marca, 2012; Nier et al., 2014; Rey, 2015), we believe that it is important not to lose sight of domestic economic conditions. From a policy perspective, it is crucial to identify determinants of boom-bust-cycles that are within the reach of domestic policy-makers. If exchange rate dynamics have the effects hypothesised in this paper, a fully flexible exchange rate regime and an open financial account may not be the optimal solution. Managing exchange rates and capital flows then becomes an important policy to curb boom-bust-cycle dynamics.

The contribution of this article is three-fold. First, it provides a simple dynamic Minskyan model of EM boom-bust-cycles. Thereby, it formalises some aspects highlighted in the non-formal structuralist and Minskyan literature on boom-bust-cycles in EMEs and adds new features to existing Minskyan closed economy models. Second, it proposes a causal mechanism that explains not only the observed pro-cyclicality of exchange rates in EMEs, but also business cycle dynamics. Currency appreciations which improve the balances sheets of foreign currency indebted firms induce an investment boom. Throughout the boom phase, the exchange rate continues to appreciate and the current account position worsens. Pressures on the domestic exchange rate mount until it eventually depreciates. Contractionary balance sheet effects then set in as domestic firms face a drop in their nominal net worth. Although we place a strong emphasis on the interaction of exchange rate dynamics and balance sheet effects, we do no claim that this is the only channel that drives business cycles in EMEs. However, we believe that it is one of the key mechanisms. Third, the article examines how the nature of business cycle dynamics depends on foreign investor behaviour. We distinguish the special case of conservative foreign investors, and the more general case of idiosyncratic foreign investors. Conservative investors possess perfect knowledge of domestic fundamentals and impose a ceiling on the supply of credit to prevent existing debt ratios from rising. Under their dominance, macroeconomic fluctuations are shock-dependent. Idiosyncratic investors target a specific debt ratio under ignorance of domestic fundamentals. If they prevail, business cycle dynamics can assume the form of shock-independent limit cycles around a locally unstable equilibrium. The higher the target debt ratio, the higher the degree of regulation of the financial account that is needed to avoid limit cycle dynamics.

The article is structured as follows. The second section briefly introduces Minskyan and structuralist theoretical approaches and links them to some stylized facts on the cyclical behaviour of exchange rates and real activity in EMEs. The third section then proceeds to develops a simple Minskyan model that explains these stylized facts. It presents a core model, whose dynamic properties are then examined under two different assumptions about foreign investor behaviour. The last section concludes.

2 Minsky cycles in emerging markets

The relevance of financial factors for business cycle dynamics has been highlighted in post-Keynesian approaches for a long time. Drawing on Hyman P. Minsky's *Financial Instability Hypothesis* (Minsky, 2008 [1975], 2016 [1982]), a wealth of models has been developed in which interactions between real and financial variables drive business cycle and debt dynamics (see Nikolaidi and Stockhammer 2017 for a recent survey). A key aspect of the *Financial Instability Hypothesis* is the claim that financial fragility increases during economic booms to the point where it spills over to the real economy and turns them into busts. Every boom thus prepares its own bust, and it is finance that plays a decisive role in driving these cycles. Structuralist economists, furthermore, highlight the role of the balance-of-payments in open developing economies in turning external financial shocks into business cycles (Ocampo et al., 2009, chap. 5 and 7; Ocampo, 2016). A crucial factor are pro-cyclical capital flows that often induce private spending booms, which in turn attracts further inflows up to the point where risk concerns take over and capital pulls out.

There is also a branch of the Minskyan literature that argues that recessions and debt crises in EMEs follow a pattern that is in line with Minsky's theory (Taylor, 1998; Kregel, 1998; Arestis and Glickman, 2002; Wolfson, 2002; Cruz et al., 2006; Frenkel and Rapetti, 2009; Harvey, 2010; Agosin and Huaita, 2011). However, as pointed out by Ryoo (2010 p. 184) 'few attempts have been made to formalize the ideas and to propose precise mechanisms behind them'. Among the few exceptions are the models by Foley (2003), Taylor (2004a, chap. 10), and Gallardo et al. (2006). Foley (2003) shows how conservative monetary policy in developing countries can lead to financial fragility that may generate boom-bust-cycles. Taylor (2004a, chap. 10) presents a model in which risk premia on interest rates are sensitive to the stock of foreign reserves. Boom periods lead to a loss of foreign reserves which pushes up interest rates to the point where the economy contracts and the current account reveres. Both models, however, completely abstract from exchange rate issues and focus solely on interest rates. Gallardo et al. (2006) introduce exchange rate dynamics, but focus on the interaction between exchange rates and foreign reserve dynamics to assess under which conditions a fixed exchange rate regime is stable.[2] However, given that the majority of EMEs currently follows some form of floating (see Figure 1 in Ghosh et al., 2015), models with fixed exchange rates do not capture the reality of exchange rate dynamics in most EMEs.

The importance of flexible exchange rates has been pointed out in the recent policy literature, which suggests that capital flow-driven exchange rate dynamics and balance sheet effects may increase macroeconomic fluctuations:

> 'In economies that have net external liabilities denominated in foreign currencies, exchange rate fluctuations also have the real balance effects […]. These effects are pro-cyclical: real appreciation during the boom generates capital gains, whereas depreciation during crises generates capital losses. These balance sheet effects associated with currency fluctuations increase the amplitude of economic fluctuations. The exchange rate fluctuations are themselves a result of some of the same forces that give rise to the economic fluctuations: capital inflows can fuel real exchange rate appreciation, at the same time that they lead to a private spending boom, while depreciation may have the opposite effects. In broader terms, in open developing economies the real exchange rate is an essential element in the dynamics of the business cycles' (Stiglitz et al., 2006, p. 117).

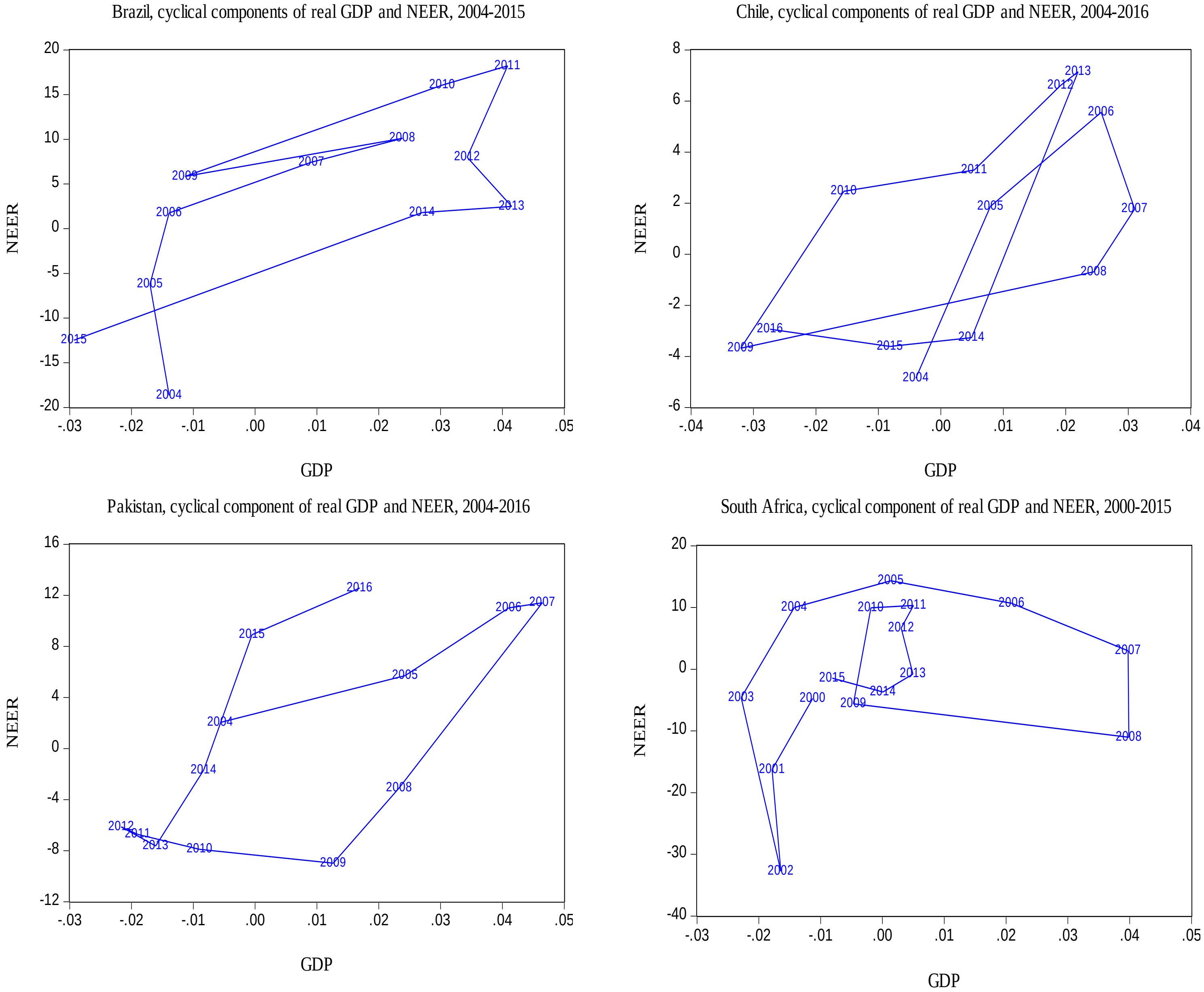
The link between exchange rate dynamics and balance sheets has also been termed 'the risk-taking' or 'financial channel' of exchange rates. If the exchange rate of a global funding currency (typically the US-dollar) depreciates against the domestic currency, balance sheets of economic units with currency mismatch improve. This stimulates domestic investment and increases the demand for further external credit (Shin, 2015; 2016; Kearns and Patel, 2016).

2 Their model aims at explaining Mexico's collapsing fixed exchange rate regime in 1994.

> 'The financial channel can thus lead to a cycle through which appreciation against global funding currencies increases the supply, and reduces the cost, of foreign lending. This will boost interest-sensitive domestic spending. Conversely, a contraction in financial conditions and economic activity will follow a local currency depreciation' (Kearns and Patel, 2016, p. 98).

The implied procyclicality of exchange rates in EMEs has also been confirmed by recent empirical studies (Calderón and Fuentes, 2014; Cordella and Gupta, 2015). We investigate the nature of output and exchange rate cycles further by examining phase diagrams in which the cyclical component of the nominal effective exchange rate is plotted against the cyclical component of log real GDP. This allows us to draw a conclusion about the direction of the cycles. Figure 1 depicts a set of these phase plots for selected EMEs between 2000 and 2016.

Figure 1: Phase diagrams for Brazil, Chile, Pakistan, and South Africa



Data sources: IFS (IMF), World Bank.

Notes: NEER: Nominal effective exchange rate (index, 2010 = 100). GDP: Natural logarithm of real gross domestic product. The cyclical components have been extracted using the Hodrick-Prescott filter with a smoothing parameter of 100. Note that an increase in the NEER indicates an appreciation of the domestic currency.

In Brazil, a sustained appreciation of the exchange rate began in 2004 and lasted until 2011. It was accompanied by an expansion of output, which was only interrupted by the 2009 global recession. The currency then began to depreciate from 2011 onwards, followed by a contraction in output between 2013 and 2015. Chile underwent two output-exchange-rate cycles between 2004 and 2016. An expansionary appreciation phase set in from 2004 onwards. The currency began to depreciate from 2006 on, which was followed by a contraction in output between 2007 and 2009. A second cycle began in 2009 with an expansionary appreciation lasting until 2013. A contractionary depreciation phase followed from 2013 to about 2015. A new cycle might be in the beginning since 2016. In Pakistan, a full cycle can be observed between 2004 and 2014/2015, starting off with an expansionary appreciation phase from 2004 to 2007, which then turned into a contractionary depreciation until about 2010. Although the currency began to appreciate again between 2010 and 2012, this was interrupted by a slight depreciation between 2011 and 2013. From then on, an expansionary appreciation phase set in which completed the cycle that began in 2004. The last two years suggest the beginning of a new cycle. Lastly, South Africa exhibits two output exchange rate cycles between 2000 and 2015. A short cycle from about 2009 to 2014/2015 was embedded in a longer cycle lasting from 2000 to 2015. The long cycle began with a contractionary depreciation between 2000 and 2002, which was then followed by a marked appreciation from 2003 to 2005 that was accompanied by output expansion. From then on, the currency began to depreciate slightly while the expansion continued up until 2008. The strong contraction of 2009 set the start for a short clockwise cycle between 2009 and 2014. By 2015, the long cycle that began in 2000 has been completed.

We conclude that all four countries exhibit clockwise cycles between output and the nominal effective exchange rate since the millennium. Converted into the more conventional definition of the exchange rate as domestic currency units per foreign unit, where an increase in the exchange rate indicates a depreciation, we would thus expect counter-clockwise cycles in economic activity and the exchange rate. The counter-clockwise direction of the cycles would indicate that peaks and troughs in economic activity precede peaks and troughs in the exchange rate. This pattern points to predator-prey dynamics where economic activity takes on the role of the prey that is being squeezed by a rising exchange rate (i.e. depreciation), while the currency behaves like a predator that grows together with the prey.

3 A Minskyan open economy model

We argue that the observed cyclical dynamics between the exchange rate and economic activity can be explained within a Minskyan framework. From a Minskyan perspective, financial fragility plays a key role in generating business cycle dynamics. We suggest that in EMEs balance sheet effects due to foreign-currency denominated private debt are a key propagation mechanism that translates exchange rate dynamics into episodes of financial stability and instability giving rise to business cycles. Moreover, it will be demonstrated that different types of investor behaviour and their consequences for the dynamics of firm leverage generate different kinds of business cycle dynamics. We distinguish conservative investors that impose a ceiling on the credit supply to prevent given leverage ratios from rising based on perfect knowledge about the domestic rate of capital accumulation, and idiosyncratic investors that pursue a self-defined target leverage ratio depending on liquidity and risk appetite in financial centres and that do not possess perfect knowledge of the investment rate or simply do not take into account domestic fundamentals.

The dominance of conservative investors gives rise to a nonlinear two-dimensional continuous time model that, under certain conditions, generates shock-dependent damped oscillations in the exchange rate and the investment rate towards a locally stable equilibrium. An external shock leads to an appreciation of the currency. Balance sheets improve and firms increase investment. A worsening current account throughout the boom puts increasing pressure on the domestic exchange rate. The currency eventually depreciates, which induces contractionary balance sheet effects. The reduction in income will re-balance the current account until the pressure on the exchange rate is removed, so that the cycle may repeat itself when the next shock occurs.

Under idiosyncratic investors, the described dynamics persist but may turn into shock-independent endogenous limit cycles that repeat itself even without the occurrence of external shocks.[3] Leverage ratios now fluctuate jointly with the investment and exchange rate. The higher the target debt ratio of foreign investors, the more regulation of the financial account is needed to avoid such limit cycle dynamics.

[3] See Gabisch and Lorenz (1989, chap. 2 and 4) on the distinction between shock-dependent and shock-independent business cycle models.

3.1 The core model

The goods market of the model is kept simple and resembles other Minsky models, such as Foley (2003), and Charles (2008a,b). The economy consists of one sector that produces a homogenous good using capital and labour, which can be used for consumption and investment. For simplicity, there is no depreciation of the capital stock and no overhead labour. The technical coefficients of labour and capital are assumed to be constant, so there is no substitution between capital and labour, and no technical progress. There is Keynesian quantity adjustment to changes in demand. For the sake of simplicity, there is no government sector, and no inflation. We normalise the domestic and foreign price level to unity. Furthermore, there is no substitution between the imported good and the domestic good.

The economy is small and open, so that all foreign variables are exogenously given. The exchange rate is flexible. The domestic economy only consists of workers and firms. Workers consume their entire income which exclusively consists of wages. Their budget constraint is always satisfied. Firms can finance their investment expenditures (I) through profits net of interest payments (R^{Net}) and via floating foreign currency-denominated bonds abroad $\left(s\dot{D^f}\right)$ – a practice that has taken place on a large scale in emerging market economies in the last decade (IMF, 2015, chap. 3; Chui et al., 2016). The aggregate firm budget constraint reads:[4]

$$(1) \qquad I = R^{Net} + s\dot{D^f} = R - si^f D^f + s\dot{D^f},$$

where s is the spot exchange rate defined as units of domestic currency per unit of foreign currency[5] and i^f is the interest rate on foreign-currency denominated bonds (D^f). The superscript f denotes foreign variables.

In an open economy without government, national income (Y) consists of consumption (C), investment (I) and net exports $(X - sM)$ net of factor payments abroad $(si^f D^f)$. The national income identity $Y \equiv C + I + X - sM - si^f D^f$ is always satisfied due to the inclusion of unplanned inventory accumulation in the investment term. The economy consists of workers who earn wages (W) and firms who earn profits net of interest payments. Thus, another way of writing the national income identity is $Y \equiv W + R - si^f D^f$.

The balance-of-payments (BoP) as an accounting identity is given by

[4] A list of a symbol definitions can be found in appendix A1.

[5] Hence, an increase in s corresponds to a currency depreciation. Notice the difference to the empirically widely used nominal effective exchange rate in Figure 1, where an increase indicates an appreciation.

(2) $(X - sM - si^f D^f) + \left(s\dot{D^f}\right) \equiv s\dot{Z}$,

where the first term in brackets represents the current account, i.e. the trade surplus minus interest payments abroad, and the second term is the financial account, i.e. net capital inflows. A surplus in the current (financial) account that is not fully matched by a deficit in the financial (current) account leads to an accumulation of foreign reserves $\left(s\dot{Z}\right)$.

We now turn to the equilibrium conditions of the model. Equilibrium in the goods market requires that national income (Y) equals aggregate demand (Y^D). Aggregate demand in an open economy that is a net debtor is composed of planned consumption (C^P), planned investment (I^P), and planned net exports $(X^P - sM^P)$ minus interest payments abroad:

(3) $Y = Y^D = C^P + I^P + X^P - sM^P - si^f D^f$,

Equilibrium in the balance of payments, on the other hand, is given when reserve changes are zero:

(4) $(X - sM - si^f D^f) + \left(s\dot{D^f}\right) = 0$

Only workers consume, and for simplicity we assume they consume all their wage income. Considering that national income consists of wages and net profits, we have the following consumption function:

(5) $C^P = W = Y - (R - si^f D^f)$.

Inserting the consumption function into the goods market equilibrium condition, we have:

(6) $R - I^P = X^P - sM^P$

$\Leftrightarrow I^P - R = sM^P - X^P$

Thus, whenever investment exceeds profits, the economy runs a trade deficit. Substitution into the equilibrium condition for the BoP (4) yields:

(7) $\quad (R - I^P - si^f D^f) + (s\dot{D^f}) = 0.$

When the goods market is in equilibrium, we have $I^P = I$. In this case, equation (7) is indeed equivalent with the budget constraint of the firm sector (1). Thus, when the goods market and the BoP is in equilibrium, the firm budget constraint is satisfied. In temporary disequilibrium, the firm sector may not always satisfy its budget constraint, which can result in temporary payment arrears. In this case, changes in foreign reserves have to fill the payment gap. We will examine the forces that re-establish equilibrium in the BoP.

We now turn to the determination of the remaining components of aggregate demand. We scale all variables by the capital stock (K) and use lower case letters henceforth.[6] Based on the assumption about consumption already made, we have the following consumption function:

(8) $\quad c \equiv \frac{C}{K} = u - (r - si^f\lambda), \qquad u \equiv \frac{Y}{K};\ r \equiv \frac{R}{K};\ \lambda \equiv \frac{D^f}{K}$

We assume a highly simplified investment function in which the rate of, and the exchange rate as the only two arguments. The use of the profit rate as a measure aggregate demand[7] and internal funds in the investment function is conventional in post-Keynesian model (Foley, 2003; Oreiro, 2005). However, inclusion of the exchange is not. The economic rationale for this is the presence of foreign-currency denominated debt on the balance sheet of emerging market firms. Changes in value of the domestic currency thus exert a strong impact on the net worth of firms, which in turn can affect investment demand. From a Minskyan perspective, the link between net worth and investment is due to 'borrower's risk', which is the subjective risk of illiquidity and bankruptcy of the entrepreneur due to the possibility of lower than expected cash flows despite fixed payment obligations (Keynes, 2013 [1936], pp. 144–145; Minsky, 2008 [1975], pp. 104–110). Here, we follow the approach of Charpe et al. (2011, chap. 2) by including the exchange rate in the desired investment (g^d) function but instead use a simpler,

[6] We drop the superscript P for planned magnitudes as we will be only concerned with planned expenditures from here on.

[7] The profit rate is, by definition, composed of the rate of capacity utilisation $\left(\frac{Y}{Y^P}\right)$, the profit share $\left(\frac{R}{Y}\right)$ and the output-capital ratio $\left(\frac{Y^P}{K}\right)$: $r = \frac{R}{K} = \frac{Y}{Y^P}\frac{R}{Y}\frac{Y^P}{K}$. Since we assume the profit share and the capital-output ratio to be constant, the profit rate is monotonically related to the rate of capacity utilisation, which in turn measures aggregate demand. We abstract from interest payments in the investment function to keep the model simple and to focus on stocks (i.e. balance sheet) effects. For a Minskyan model that examines the role of interest payments in EMEs, see Foley (2003).

linear function.[8] Note that price competitiveness effects of the exchange rate affect desired investment expenditures only via their impact on the profit rate which, in equilibrium, is positively related to the exchange rate. Hence, the coefficient g_s solely captures balance sheet effects.

(9) $$g^d = g_0 + g_r r - g_s s, \qquad g_r \in (0,1);\ g_s > 0.$$

Lastly, we use a net exports function that relates the net exports ratio (b) to the foreign profit rate (r^f), the domestic profit rate, and the exchange rate. This is similar to standard Kaleckian open economy models (Blecker, 2011), where, however, the rate of capacity utilisation rather than the rate of profit is used as a measure for aggregate demand. However, this does not lead to major differences.

(10) $$b \equiv \frac{X-sM}{K} = b_{rf} r^f - b_r r + b_s s, \qquad b_{rf} > 0;\ b_r \in (0,1).$$

The foreign rate of profit is assumed to improve the trade balance as it translates into export demand for the home country, so the parameter b_{rf} is positive. The inclusion of the rate of profit, however, requires that the domestic and foreign capital stock grow at the same rate. Second, the domestic rate of profit has a negative effect on the trade balance, as an increase in domestic demand will increase import demand. Third, whether the effect of an increase in the real exchange rate on the trade balance is positive depends on whether the Marshall-Lerner condition (MLC) holds, which is captured by the sign of parameter b_s.[9] The empirical evidence on the MLC is mixed.[10] We will assume that b_2 has a low absolute magnitude and can assume positive or negative values .

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[8] Using only the exchange rate rather than the debt-to-capital ratio in domestic currency in the investment function has the advantage of making the model analytically more tractable.

[9] It has to be noted that the linear specification that is being used in standard Kaleckian open economy models is in fact a special case. It assumes away a non-linearity that stems from the valuation of the imports by the exchange rate (see Gandolfo, 2016, pp. 102–106 on this aspect of the MLC). It can be shown that the implicit assumption behind equation (10) is an exchange rate elasticity of import demand of -1.

[10] In a survey of empirical studies over the past 50 years, Bahmani et al. (2013) show that empirical estimates of the MLC have often been either contradictory or changed over time. They conduct a meta-analysis of existing studies and find that the MLC is only statistically significantly satisfied in just under 30 percent of 92 estimated elasticities. Moreover, the authors conduct their own empirical analysis for a set of 29 countries over the period 1971-2009 and find the MLC to be met in only three countries.

The goods market equilibrium condition (3), scaled by the capital stock ($u = c + g + b - si^f\lambda$), together with the consumption (8) and net export function (10) yield the following goods market equilibrium profit rate:

$$(11) \quad r^* = \frac{g + b_{r^f} r^f + b_s s}{1 + b_r} = \theta\left(g + b_{r^f} r^f + b_s s\right), \qquad \text{where } \theta(b_r) \equiv \frac{1}{1 + b_r} \in (0,1).$$

Following Charles (2008a,b), we introduce finite adjustment of the actual rate of capital accumulation to the desired one:

$$(12) \quad \dot{g} = \gamma(g^d - g).$$

Substituting the equilibrium rate of profit (11) into the investment demand function (9), we obtain:

$$(13) \quad g^d = g\theta g_r + s(\theta b_s g_r - g_s) + g_0 + \theta b_{r^f} r^f g_r.$$

Plugging (13) into the law of motion of the investment rate yields:

$$(14) \quad \dot{g} = \gamma\left[g(\theta g_r - 1) + s(\theta b_s g_r - g_s) + g_0 + \theta b_{r^f} r^f g_r\right].$$

Now we address the question how equilibrium in the BoP is established. The normalised form of the BoP equilibrium condition is:

$$(15) \quad r - g - si^f\lambda + s\left(\frac{\dot{D^f}}{K}\right) = 0,$$

$$\Leftrightarrow b - si^f\lambda + s\left(\frac{\dot{D^f}}{K}\right) = 0.$$

From the time derivate of the external debt-to-capital ratio $\frac{d\lambda}{dt} \equiv \dot{\lambda} \equiv \frac{\dot{D^f}}{K} - g\lambda$, we can derive $\frac{\dot{D^f}}{K} \equiv \dot{\lambda} + g\lambda$. Substituting this expression into equation (15) yields:

$$(16) \quad r - g(1 - s\lambda) - si^f\lambda + s\dot{\lambda} = 0$$

According to Gandolfo (2016, pp. 346-349) the BoP can be interpreted as a market-clearing condition for the foreign exchange market, since it contains all the sources of supply and demand for foreign currency. Whenever equality (16) is violated, there is excess demand or supply in the foreign exchange market. Excess demand for foreign currency may arise from a domestic trade deficit and the necessity to pay foreign interest, while excess supply of foreign currency can stem from an increase in the supply of foreign credit.

Substituting the equilibrium rate of profit (11) into (16), we have:

$$(17) \quad \left(\theta b_{r^f} r^f\right) + g(s\lambda - \theta b_r) + s\left(\dot{\lambda} + \theta b_s - i^f \lambda\right) = 0.$$

Equilibrium in the foreign exchange market is established by an endogenously adjusting exchange rate, however, this is accomplished only with finite speed. Following Bhaduri (2003), the speed of adjustment μ can be interpreted as the degree of deregulation of the financial account.

$$(18) \quad \dot{s} = \mu\left[g(\theta b_r - s\lambda) + s\left(i^f \lambda - \dot{\lambda} - \theta b_s\right) - \theta b_{r^f} r^f\right]$$

Exchange rate dynamics are thus determined by the rate of capital accumulation, which in turn affects the trade balance, as well as by the exchange rate itself, which influences net exports and the value of interest payments, as well as capital flows. Moreover, an exogenous increase in foreign profitability affects the exchange rate through its impact on the import demand for domestic goods.

Before we can examine the stability and trajectory of the dynamic system in (14) and (18), we need to specify how financial flows are determined. We will distinguish two cases: first, the prevalence of conservative foreign investors with perfect information who seek to maintain the current leverage ratio of the firm; and second, the dominance of idiosyncratic investors with imperfect information who form their own target leverage ratio. Although both types of investors ultimately pursue a fixed target external debt ratio, only conservative investors are successful in keeping the actual debt ratio constant. We regard the scenario with conservative investors as a special case, and the scenario with idiosyncratic investors as a generalisation.

3.2 Stability and dynamic behaviour under conservative foreign investors

Let us focus on the special case first, and suppose that foreign investors are conservative. Rather than pursuing their own target debt ratio, conservative foreign investors seek to prevent the actual, historically given external debt-to-capital ratio from rising, e.g. to prevent financial fragility. They impose a ceiling on the supply of credit that ensure that that the debt-to-capital ratio does not increase over time. From the law of motion of the debt-to-capital ratio, this requires:

(19) $\frac{\dot{D^f}}{K} \leq \lambda g.$

Thus, the share of investment that is financed by foreign debt must be lower or equal to the actual debt-to-capital ratio to prevent this ratio from growing over time. We shall assume that the demand of domestic firms for foreign credit always hits that ceiling, so that $\frac{\dot{D^f}}{K} = \lambda g$. Under this assumption, we have $\dot{\lambda} = 0$, so the law of motion of the exchange rate reduces to:

(20) $\dot{s} = \mu\left[g(\theta b_r - s\lambda) + s(i^f\lambda - \theta b_s) - \theta b_{r^f} r^f\right]$

Indeed, this supply function for external debt implies that foreign investors have perfect information about the rate of capital accumulation. On the demand side, it assumes that firms readily pick up the exact amount of credit that is supplied by foreign creditors. As a result, the actual debt ratio, λ, is constant. We will regard this as a special case whose consideration allows us to focus our attention on the dynamics of capital accumulation and the exchange rate. The laws of motion of capital accumulation (14) and the exchange rate (20) then constitute a two-dimensional dynamic system. Its Jacobian matrix is:

(21) $\boldsymbol{J}(g, s) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s) \\ \mu(\theta b_r - s\lambda) & \mu[(i^f - g)\lambda - \theta b_s] \end{bmatrix}.$

The dynamic behaviour of the system depends on the signs of the elements of the Jacobian. The element J_{11} constitutes a version of the Keynesian stability condition. For the goods market to be stable, the marginal effect of an increase in the rate of profit on domestic saving and net exports must exceed the static accelerator effect on investment (g_r). Kaleckian Minsky models assume that the stability condition is satisfied.[11] We follow this approach here. The sign of J_{12} appears to be ambiguous. The first term expresses the positive effect of a depreciation on the trade balance which is mediated by the price elasticity of net exports (b_s) and the sensitivity of investment with respect to profitability (g_r). The second term captures the contractionary (expansionary) balance sheet effect of a currency depreciation (appreciation) on investment (g_s). Based on our previous discussion of the empirical evidence, we suppose that the second effect outweighs the first effect: balance sheet effects are typically strong in EMEs, while price elasticities are low. We thus assume that $J_{12} < 0$. The sign of the element J_{21} is positive only if $\theta b_r > s\lambda$. The first term captures the fact that an increase in the rate of investment leads to a trade deficit, which in turn creates excess demand on the foreign exchange market. This effect is attenuated by the growth in the capital stock which raises the supply of foreign credit. Lastly, J_{22} determines the stability of the BoP. A destabilising element is interest payments on foreign currency debt. Capital accumulation and the exchange rate sensitivity of net exports are stabilising factors.

Given the assumptions made so far, we have the following sign structure of the Jacobian matrix:

$$sgn[\boldsymbol{J}(g,s)] = \begin{bmatrix} - & - \\ ? & ? \end{bmatrix}.$$

We want to know under what conditions the system exhibits oscillatory behaviour as observed in Figure 1. A sufficient condition is $(J_{11} - J_{22})^2 < |4J_{21}J_{12}|$ (see appendix A2). This requires, first, that $J_{12}J_{21} < 0$, i.e. the off-diagonal elements of the Jacobian matrix must have opposite signs. Such a configuration is given in our system if $J_{21} > 0 \Leftrightarrow \theta b_s > s\lambda$, i.e. if the negative effect of an increase in capital accumulation on the trade balance exceeds the positive effect on the supply of foreign credit. Economically, this requires a relatively large propensity to import, which is typically the case in developing countries due to a lack of domestically produced manufactured goods. Together with the negative effect of a depreciation on investment ($J_{12} < 0$), this makes for the required interaction mechanism. Second, it requires that the off-diagonal elements are not only signed oppositely, but that their coefficients are large enough (in absolute

[11] On the distinction between Kaleckian Minsky models that assume a stable goods market and Kaldorian Minsky models in which the product market is unstable, see Ryoo (2013). Nikolaidi and Stockhammer (2017) provide a systematic review of Minsky models along this distinction.

terms) to dominate the dynamic behaviour of both state variables. Formally, this can be expressed as:

$$\{\gamma(\theta g_r - 1) - \mu[(i^f - g)\lambda - \theta b_s]\}^2 + 4\mu(\theta b_r - s\lambda)\gamma(\theta b_s g_r - g_s) < 0.$$

In our system, this is given when the trade balance responds strongly to domestic demand ($b_r \gg 0$), which we just argued to be likely, and investment is very sensitive towards the exchange rate ($g_s \gg 0$), which is also typical for developing countries where balance sheet effects have been shown to be strong. Thus, let us suppose that the sufficient condition for complex eigenvalues is satisfied. Under this condition, an exogenous shock that pushes the economy off equilibrium leads to a cyclical adjustment path back to equilibrium – provided the equilibrium is stable.

Stability of a two-dimensional system requires a negative trace and a positive determinant (Chiang and Wainwright, 2005, p. 627). The trace is given by:

$$Tr(J) = \gamma(\theta g_r - 1) + \mu[(i^f - g)\lambda - \theta b_s],$$

where $Tr(J) < 0 \iff \gamma\theta g_r + \mu i^f \lambda < \gamma + \mu(\theta b_s + g\lambda)$.

Thus, the trace is negative if the accelerator effect as well as interest payments on foreign debt are relatively low, while the exchange rate sensitivity of net exports and investment are sufficiently high.

Next, consider the determinant:

$$Det(J) = \gamma\mu\{(\theta g_r - 1)[(i^f - g)\lambda - \theta b_s] - (\theta b_s g_r - g_s)(\theta b_r - s\lambda)\},$$

where $Det(J) > 0 \iff (\theta g_r - 1)[(i^f - g)\lambda - \theta b_s] > (\theta b_s g_r - g_2)(\theta b_r - s\lambda)$.

Given our previous assumptions, the right-hand side of the above inequality is negative. A positive determinant, which guarantees that the fixed point is not a saddle, requires the left-hand side to be either positive or smaller in absolute value: $|J_{12}J_{21}| > |J_{11}J_{22}|$. Economically, this requires that there is a strong interaction between the dynamics of capital accumulation and the exchange rate, while these variables do not exhibit a strong effect on themselves. This is in line with our already made assumptions that net exports respond strongly to domestic profits ($b_r \gg 0$) and investment is very sensitive towards the exchange ($g_s \gg 0$). If, on top of that, interest payments are not too large relative to the investment rate, this condition is easily met.

In order to examine the dynamic trajectories, consider the isoclines:

$$(22) \quad s^*_{|\dot{g}=0} = \frac{g(\theta g_r - 1) + g_0 + \theta b_{rf} r^f g_r}{g_s - \theta b_s g_r}; \qquad \frac{\partial s^*_{|\dot{g}=0}}{\partial g} = \frac{\theta g_r - 1}{g_s - \theta b_s g_r} < 0$$

$$(23) \quad s^*_{|\dot{s}=0} = \frac{\theta\left(b_{rf} r^f - g b_r\right)}{(i^f - g)\lambda - \theta b_s}; \qquad \frac{\partial s^*_{|\dot{s}=0}}{\partial g} = \frac{\theta\left[b_r(\theta b_2 - i^f \lambda) + \lambda b_{rf} r^f\right]}{\left[(i^f - g)\lambda - \theta b_s\right]^2} \lesseqgtr 0.$$

The denominator of the $\dot{g} = 0$ isocline is positive given our assumption that the contractionary balance sheet effect of a depreciation on investment outweighs the expansionary import demand effect. Together with the assumption that the Keynesian stability condition holds, this makes for a negative slope of the isocline. The $\dot{s} = 0$ isocline is a rectangular hyperbola with two branches. Its vertical asymptote is given by $\bar{\bar{g}} = \frac{i^f \lambda - \theta b_s}{\lambda}$. The asymptote is linked to the self-stability of the BoP: it constitutes the investment rate, for which the element J_{22} becomes zero. If $g > \bar{\bar{g}}$, the BoP is stable; if $g < \bar{\bar{g}}$, it becomes unstable. The asymptote thus determines the minimum rate of capital accumulation that is required to maintain a stable BoP. The higher the foreign interest rate, the given stock of external debt, and the import propensity, the higher will be the rate of capital accumulation that is required to maintain stability. The price elasticity of net export, in contrast, constitutes a stabilising factor that lowers the required rate of accumulation. Given that the Marshall-Lerner condition has been frequently rejected in empirical work (Bahmani et al., 2013), and that developing countries often have to cope with high debt service burdens (Frenkel, 2008), let us consider the case where the asymptote of the $\dot{s} = 0$ isocline is positive $i^f \lambda > \theta b_s$. Note that this scenario does not require a negative exchange rate sensitivity of net exports (i.e. a violation of the MLC). The sensitivity only has to be low, while interest payments on external debt are sufficiently large. If the steady state rate of capital accumulation is then below that asymptote, we have $J_{22} > 0$, so the BoP is unstable.[12] To maintain overall stability of the system, the trace of the Jacobian matrix must remain negative, so we require that $|J_{11}| > J_{22}$. Economically, this implies that the instability of the

[12] Notice that our qualitative findings do not hinge on this assumption. It can be shown that similar results arise for a stable BoP.

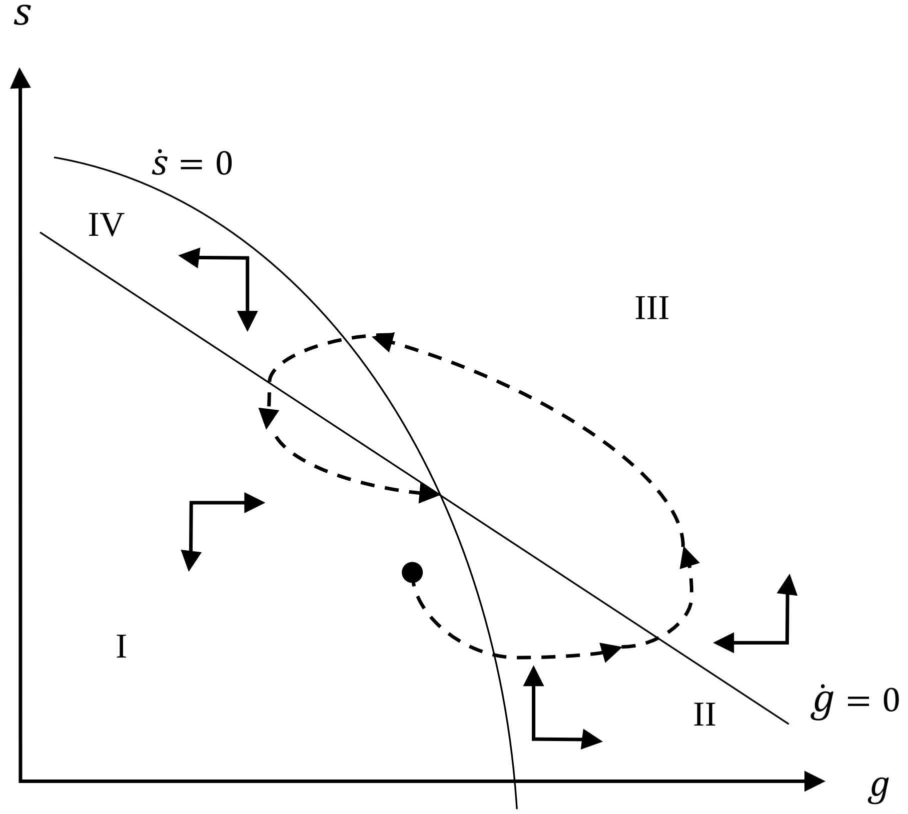
foreign exchange market must not be too strong compared to the self-stability of the goods market. We then have the following sign structure:

$$sgn[\boldsymbol{J}(g,s)] = \begin{bmatrix} - & - \\ + & + \end{bmatrix}, \qquad \text{where } Tr(J) = J_{11} + J_{22} < 0 \text{ and } Det(J) = J_{11}J_{22} - J_{12}J_{21} > 0.$$

As explained, to rule out a saddle point we impose $|J_{12}J_{21}| > |J_{11}J_{22}|$. Since $\left|\frac{\partial s^*_{|\dot{s}=0}}{\partial g}\right| = \left|\frac{J_{21}}{J_{22}}\right|$ and $\left|\frac{\partial s^*_{|\dot{g}=0}}{\partial g}\right| = \left|\frac{J_{11}}{J_{12}}\right|$ (see Chiang and Wainwright, 2005, pp. 615–616), it follows that the $\dot{s} = 0$ isocline is steeper than the $\dot{g} = 0$ isocline $\left(\left|\frac{\partial s^*_{|\dot{s}=0}}{\partial g}\right| = \left|\frac{J_{21}}{J_{22}}\right| > \left|\frac{\partial s^*_{|\dot{g}=0}}{\partial g}\right| = \left|\frac{J_{11}}{J_{12}}\right|\right)$. The $\dot{g} = 0$ isocline remains unchanged. However, the nonlinear $\dot{s} = 0$ isocline is now downward-sloping.[13] We are interested in the case where the fixed points are to the left of the asymptote, so that the BoP is unstable. We focus on the economically more meaningful case of a positive fixed point, where the exchange rate is positive too. The dynamics around this fixed point can then be analysed graphically (Figure 2).

[13] Since we assume that $J_{22} > 0$ and $J_{21} < 0$, the slope of the $\dot{s} = 0$ isocline must be negative. This follows from the fact that $\frac{\partial s^*_{|\dot{s}=0}}{\partial g} = -\frac{J_{21}}{J_{22}}$.

Figure 2: Adjustment path after an appreciation shock,
$\boldsymbol{J_{11} < 0; J_{22} > 0; |J_{11}J_{22}| < |J_{12}J_{21}|}$



Starting from an equilibrium position, consider a negative shock to the domestic exchange rate (quadrant I). This leads to a currency appreciation feeding into investment demand via an expansionary balance sheet effect: Firms' balance sheets have improved which stimulates investment expenditures. As the domestic economy booms, profitability and aggregate demand go up. The exchange rate overshoots and keeps appreciating. The economy experiences the empirically observed pro-cyclicality of the exchange rate, where a domestic boom coincides with currency appreciation. The economic rationale for this phenomenon is that the appreciation eases the burden of interest payments on foreign debt which further reduces the demand for foreign currency. Thus, the domestic currency gains in value until the boom-induced current account deficit eventually becomes large enough to exert pressure on the domestic exchange. There is a relatively short period, in which investment keeps accelerating while the currency already depreciates (quadrant II), but this phase is quickly displaced by a long contractionary depreciation phase due to balance sheet effects (quadrant III). As a result, the downward trajectory of the current account eventually reverses until the pressure on the exchange rate is removed, and capital accumulation picks up again. Finally, the economy reaches its equilibrium again.

This trajectory describes the dynamics of emerging market business cycles featuring sustained periods of expansionary currency appreciation and contractionary depreciation, as observed in Figure 1. It thereby captures the phenomenon of pro-cyclical exchange rates and counter-cyclical trade balances in EMEs (Lane, 2003; Calderón and Fuentes, 2014; Cordella and Gupta, 2015), and provides a coherent explanation for it based on the presence of balance sheet effects that work expansionary under currency appreciation and contractionary when the exchange rate appreciates.

However, the present model is based on the assumption that the external debt-to-capital ratio remains constant over time due to conservative foreign investors who impose a ceiling on the supply of credit and firms that readily pick up the maximum amount of debt that is being supplied. This might be questioned on behavioural grounds as it requires foreign investors to possess perfect information about the domestic fundamentals. Moreover, it implies that rather than targeting a self-defined target debt ratio, foreign investors define a maximum leverage ratio based on the historically given actual debt ratio. These assumptions seem to contradict historical experience teaching that capital flows to EMEs often go through episodes of exuberance and panic during which leverage ratios change considerably. In the next section, we will thus develop an extension to the benchmark model, in which a more general credit supply function is considered, which allows for imperfect information and idiosyncratic target debt ratios on the side of foreign investors

3.3 Stability and dynamic behaviour under idiosyncratic foreign investors

Let us now suppose foreign investors are idiosyncratic and pursue, explicitly or implicitly, a self-defined, fixed target debt ratio that does not necessarily correspond to the historically given leverage ratio. Unlike in the previous scenario, we assume that foreign investors do not take the actual investment rate into account when forming their target debt ratio – either because they do possess information about the current rate of capital accumulation or because they are idiosyncratic, e.g. because their target is only an implicit consequence of their actions and they ignore domestic fundamentals. Consequently, the actual debt ratio becomes a state variable that varies over time subject to the supply of foreign credit and domestic capital accumulation. The target leverage ratio may be determined by a range of factors that are external to the domestic economy such as the liquidity (Chui et al., 2016) or risk appetite (La Marca, 2012; Nier et al., 2014; Rey, 2015) in global financial centres. We will hence treat it as an exogenous variable.

Investors supply more credit whenever the actual debt ratio falls short of the target ratio:

(24) $\quad \frac{\dot{D^f}}{K} = \delta(\lambda^T - \lambda) \qquad\qquad \lambda^T > 0,$

where λ^T is the target external debt-to-capital ratio and δ the adjustment speed of the actual ratio to the desired one. This translates into the following law of motion of the external debt-to-capital ratio:

(25) $\quad \dot{\lambda} = \delta(\lambda^T - \lambda) - g\lambda.$

Note that even if foreign investors would temporarily hit the target $(\lambda^T = \lambda)$, the actual debt ratio would still change as long as the rate of investment and the actual debt ratio are different from zero. Under normal circumstance, the actual debt ratio will thus be permanently changing over time, and foreign investors keep missing their target. We consider this a more general specification as it neither imposes heroic assumptions on investor behaviour with respect to the availability of information, nor with respect to their concern for the financial fragility of the domestic economy. The idiosyncratic target ratio should be regarded as an implicit target that reflects the sentiment and risk appetite of foreign investors which can be quite detached from domestic fundamentals.

Substituting (25) into our dynamic equation for the exchange rate (18), we obtain:

(26) $\quad \dot{s} = \mu\{g\theta b_r + s[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] - \theta b_{r^f} r^f\}$

Equations (14), (25), and (26) then constitute a three-dimensional dynamic system that exhibits an intrinsically nonlinear structure due to valuation effects and normalisation of variables. Notice that this nonlinear structure emerges without having introduced nonlinearities in the behavioural functions. The Jacobian matrix of the system is given by:

(27) $$\boldsymbol{J}(g,s,\lambda) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$
$$= \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s) & 0 \\ \mu\theta b_r & \mu[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] & \mu s(i^f + \delta) \\ -\lambda & 0 & -(\delta + g) \end{bmatrix}$$

It can be shown analytically that this system has at most two fixed points (see appendix A4). We will focus on the fixed point where g, s, and λ are all positive. Given the assumptions made in this paper, we then have the following sign structure:

$$sign[\boldsymbol{J}(g,s,\lambda)] = \begin{bmatrix} - & - & 0 \\ + & + & + \\ - & 0 & - \end{bmatrix}$$

We want to examine if this system can undergo a Hopf bifurcation giving rise to a stable limit cycle. A bifurcation is a qualitative change in the behaviour of a dynamic system when a parameter of that system is varied across a critical threshold. A Hopf bifurcation is a special kind of bifurcation that gives rise to the death or birth of a limit cycle. A limit cycle is a cyclical, closed orbit of the state variables of a dynamic system around a locally unstable fixed point. Any trajectory starting in the neighbourhood of that fixed points gets drawn into the closed orbit in finite time. The steady state is locally unstable and will never be reached (unless by a fluke the economy undergoes a shock that places it exactly on this steady state). If the Hopf bifurcation creates a stable limit cycle, it is said to be supercritical. In contrast to the damped oscillations of the two-dimensional model in section 3.2, a limit cycle is periodic meaning that it persistently displays shock-independent oscillations (Gabisch and Lorenz, 1989, chap. 4).
Under certain conditions the dynamic system in (14), (25), and (26), undergoes a Hopf bifurcation as the adjustment speed of the exchange rate μ transcends a critical value μ_0 (see appendix A5). Numerically simulations (see appendix A6) suggest that the Hopf bifurcation is supercritical and gives rise to a stable limit cycle in capital accumulation, the exchange rate, and the external debt-to-capital ratio (Figures 3 and 4).

Figure 3: Limit cycle dynamics of capital accumulation, the exchange rate, and the external debt-to-capital ratio

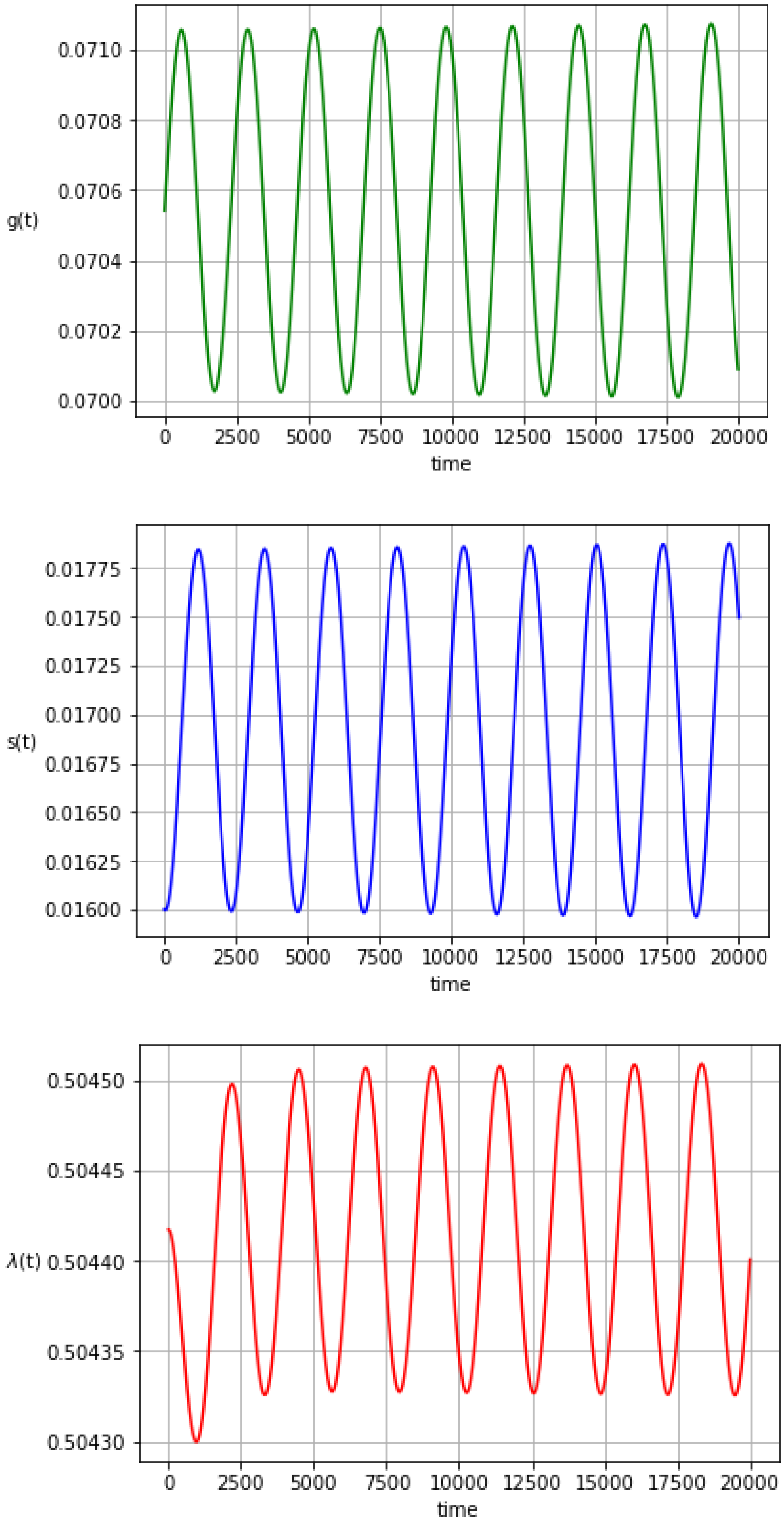
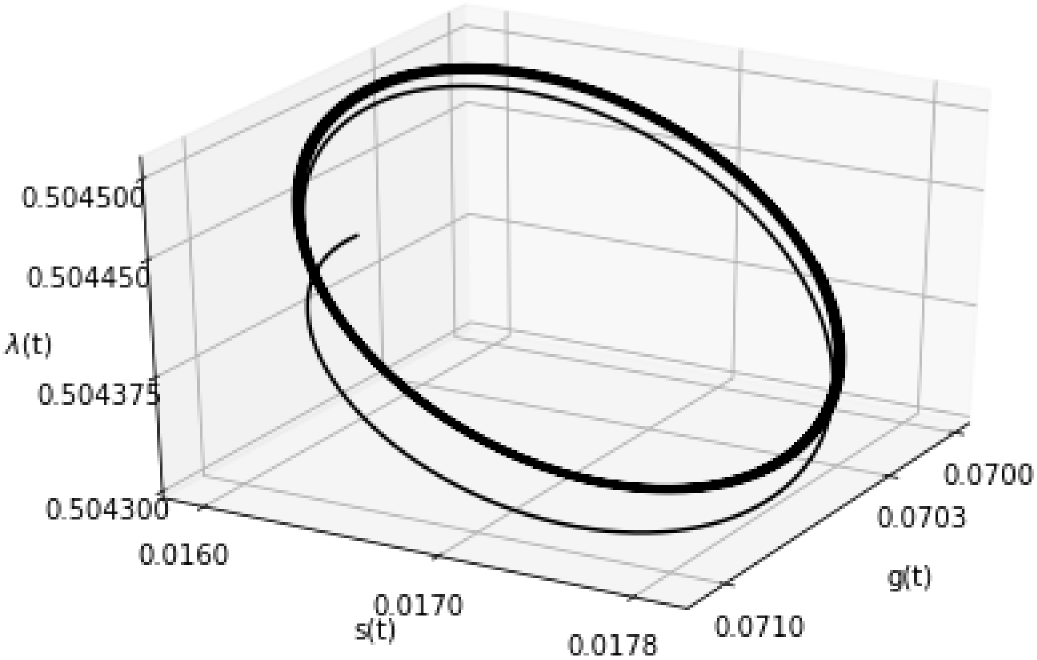


Figure 4: Phase plot of 3D limit cycle dynamics



Economically this requires first, that instability of the foreign exchange market is not excessive (i.e. the foreign interest rate has to be sufficiently low); second, that the interaction mechanism between the exchange rate and the investment rate is strong (i.e. strong balance sheet effects and a large propensity to import); third, that the adjustment speed of the exchange is sufficiently large, i.e. the financial account is open.[14] These are certainly realistic conditions for many EMEs.

Interestingly, the numerical simulations suggest that limit cycle behaviour arises for a relatively wide range of target debt ratios. The higher the target the debt ratio, the lower the critical value of the adjustment speed of the exchange rate that is needed for a limit cycle to occur. This implies that even economies with strict regulations on capital flows may be subjected to endogenous business cycle dynamics if foreign investors are strongly risk-seeking. On the other hand, for a given target leverage ratio, a sufficiently strong regulation of the financial account may prevent the occurrence of limit cycle dynamics.

We conclude that the prevalence of idiosyncratic investors turns the external debt-to-capital ratio into a dynamic variable that fluctuates jointly with the rate of capital accumulation and the exchange rate. During periods of economic expansion leverage increases, while it declines

[14] For very sluggish adjustment of the exchange ($\mu < \mu_0$), the model displays damped oscillations similar to the model with conservative investors.

when the economy contracts. Moreover, limit cycle behaviour arises for a sufficiently high speed of adjustment of the exchange rate. For a very open financial account, shock-independent cycles may emerge even for relatively low target debt ratios. If the target debt ratio increases, the economy will have to impose stricter restrictions on the financial account transactions if it wants to prevent limit cycle dynamics.

4 Conclusion

This paper has been concerned with boom-bust-cycles in emerging market economies. It has aimed to provide a Minskyan approach to business cycles in EMEs, incorporating the stylized facts of pro-cyclical exchange rates and significant levels of foreign currency-denominated corporate debt. Having demonstrated that several EMEs underwent counter-clockwise cycles in economic activity and the exchange rate in the last one and a half decade, we developed a simple Minskyan open economy model that explains this business cycle pattern. The model proposes a causal mechanism that not only explains the observed pro-cyclicality of exchange rates, but also shows how they interact with foreign-currency debt to generate business cycle dynamics. Two distinct assumptions about foreign investor behaviour and their consequences for the nature of business cycles were examined: conservative investors that use their perfect information about domestic fundamentals to prevent historically given leverage ratios from rising, and idiosyncratic investors that target a self-defined debt ratio under imperfect information or ignorance of domestic fundamentals.

Under the first type of investor behaviour, exchange rate shocks lead to currency appreciation which improves the balances sheets of foreign currency indebted firms inducing an investment boom. Throughout the boom phase, the exchange rate continues to appreciate due to an unstable foreign exchange market and the current account position worsens. Pressures on the domestic exchange rate mount until it eventually depreciates. Contractionary balance sheet effects then set in as domestic firms face a drop in their nominal net worth. Business cycles assume the form of damped oscillations towards a locally stable equilibrium.

Under the more general case of idiosyncratic investor behaviour with imperfect knowledge, the model retains its main mechanisms and outcomes but the external debt-to-capital ratio changes over time as foreign investors constantly fail to reach their target. Under certain conditions, the system gives rise to shock-independent limit cycles around a locally unstable equilibrium via a Hopf bifurcation. For a given degree of regulation in the financial account, this is more likely to happen when foreign investors target a high debt ratio. On the other hand, economies could avoid getting drawn into limit cycle dynamics by imposing stricter regulations on the financial account.

The model captures the key role of exchange rate and balance-of-payments dynamics in emerging market business cycles that has been highlighted in the structuralist and Minskyan literature (Ocampo et al., 2009, chap. 5 and 7; Harvey, 2010; Ocampo, 2016). Business cycle dynamics are caused by an interaction mechanism between the financial sphere represented by the exchange rate and foreign currency debt, and the real economy. However, unlike previous models (Foley, 2003; Taylor, 2004a, chap. 10; Gallardo et al., 2006) it shifts the focus from interest rate issues and currency crises towards exchange rate dynamics and balance sheet effects. We consider this an important step forward given that the majority of EMEs presently follow some form of floating.

The model has interesting policy implications. It predicts an association between exchange rate volatility and fluctuations in real activity. This link is expected to be stronger, the larger the stock of foreign currency-denominated debt in the private sector. In order to curb emerging market boom-bust-cycles, the model identifies three points for interventions: first, a more active exchange rate policy can seek to smoothen exchange rate fluctuations. In fact, this is already being done by many emerging market central banks who manage their exchange rate via sterilised foreign exchange intervention (Menkhoff, 2013). Moving away from the ultimately inconclusive debate on fixed versus flexible exchange rates, managed floating has also gained growing theoretical support among structuralist and post-Keynesian authors (Ocampo, 2002; Palley, 2003; Frenkel, 2007; Ferrari-Filho and De Paula, 2008). A second point of intervention is the reduction of foreign currency-denominated external debt. Strengthening the domestic banking system may help shift credit demand towards domestic lenders. This could involve investment-oriented macro-prudential regulations. China and the East Asian Tigers have shown how public and development banks that selectively provide cheap credit for long-term investment can play an important role for economic development (Herr and Priewe, 2005; Stiglitz and Uy, 1996). Third, capital controls can not only prevent firms from taking on foreign debt in the first place, they may also prevent the emergence of limit cycle dynamics and thereby reduce macroeconomic fluctuations.

The present model indeed has limitations. While it highlights exchange rate dynamics, it neglects other factors. Interest rates and their effect on capital flows can play an important role too, especially in inflation-targeting regimes. Monetary policy that raises interest rates during a boom due to inflationary pressures may attract more capital inflows which can further fuel the boom (Williamson, 2005, chap. 2). Similarly, market interest rates may be endogenous to economic activity due to risk premia as argued in some post-Keynesian models (Kohler, 2016; Nikolaidi and Stockhammer, 2017). Lastly, exchange rate dynamics are presently modelled in a highly simplified way. Post-Keynesians have argued that it is mostly capital not trade flows that determine the exchange rate, and that expectations play a key role in investor behaviour (Harvey, 2009, chap. 5). The recent behavioural literature on exchange rate determination has highlighted the role of heterogenous agents in the foreign exchange market (Westerhoff, 2009; De Grauwe and Kaltwasser, 2012). Future research could introduce these aspects into the present framework.

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Appendix

A1 Symbol definitions

Roman letters		
Symbol	**Mathematical Definition**	**Conceptual Definition**
b	$\frac{X - sM}{K}$	Net export rate
b_r		Sensitivity of net exports w.r.t. domestic rate of profit
b_{r^f}		Sensitivity of net exports w.r.t. foreign rate of profit
b_s		Sensitivity of net exports w.r.t. exchange rate
C		Consumption
C^P		Planned consumption
c	$\frac{C}{K}$	Consumption rate
D^f		Foreign currency-denominated corporate bonds
$\dot{D}^f$	$\frac{dD^f}{dt}$	Rate of change of foreign currency-denominated corporate bonds
g	$\frac{I}{K}$	Investment rate
$\dot{g}$	$\frac{dg}{dt}$	Rate of change of investment rate
g^d		Desired investment rate
g_0		Animal spirits
g_r		Sensitivity of capital accumulation w.r.t. profit rate
g_s		Sensitivity of capital accumulation w.r.t. exchange rate
I		Investment
I^P		Planned investment
i^f		Interest rate on foreign currency-denominated corporate bonds
M		Imports (denominated in foreign currency)
M^P		Planned imports
R		Profits
R^{Net}	$R - si^f D^f$	Net profits
r	$\frac{R}{K}$	Profit rate
r^f		Foreign rate of profit
s		Spot exchange rate (units of domestic currency per unit of foreign currency)
$\dot{s}$	$\frac{ds}{dt}$	Rate of change of exchange rate
t		Time
u	$\frac{Y}{K}$	Rate of capacity utilisation
W		Wage bill
X		Exports
X^P		Planned exports
Y		National income
Y^D		Aggregate demand
Z		Foreign reserves
$\dot{Z}$	$\frac{dZ}{dt}$	Change in foreign reserves

Greek letters		
Symbol	**Mathematical Definition**	**Conceptual Definition**
γ		Adjustment speed of investment rate
δ		Adjustment speed of external debt-to-capital ratio
θ	$\frac{1}{1+b_r}$	Composite parameter
λ	$\frac{D^f}{K}$	External debt-to-capital ratio
$\dot{\lambda}$	$\frac{d\lambda}{dt} \equiv \frac{\dot{D^f}}{K} - g\lambda$	Rate of change of external debt-to-capital ratio
λ^T		Target external debt-to-capital ratio
μ		Adjustment speed of exchange rate (Hopf bifurcation parameter)
μ_0		Critical value of Hopf bifurcation parameter

A2 Mathematical condition for oscillations in two-dimensional systems of differential equations

Consider the characteristic equation of a two-dimensional Jacobian matrix (Chiang and Wainwright, 2005, pp. 599–601):

$$\lambda^2 - \lambda Tr(J) + Det(J) = 0.$$

The characteristic roots of this equation are given by:

$$\lambda_{\pm} = \frac{Tr(J) \pm \sqrt{Tr(J)^2 - 4Det(J)}}{2}.$$

Oscillatory behaviour occurs when the characteristic roots are a pair of complex conjugates (Chiang and Wainwright, 2005, pp. 522–527). This, in turn, requires that the discriminant $\Delta = Tr(J)^2 - 4Det(J)$ becomes negative. This condition can be simplified as follows:

$$\begin{aligned}
\Delta = \; & Tr(J)^2 - 4\,Det(J) < 0 \\
& = (J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{12}J_{21}) < 0 \\
& = (J_{11} - J_{22})^2 + 4J_{12}J_{21} < 0 \\
& = (J_{11} - J_{22})^2 < |4J_{21}J_{12}|
\end{aligned}$$

A3 Fixed points of the two-dimensional system

The fixed points of the system in (14) and (20) can be found as follows. First, we determine the fixed points of s by setting equations (14) and (20) equal to zero and solving for g:

$$g^*_{|\dot{g}=0} = \frac{s(\theta b_s g_r - g_s) + g_0 + \theta b_{rf} r^f g_r}{1 - \theta g_r}; \qquad \frac{\partial g^*_{|\dot{g}=0}}{\partial s} = \frac{\theta b_s g_r - g_s}{1 - \theta g_r} < 0$$

$$g^*_{|\dot{s}=0} = \frac{s(i^f \lambda - \theta b_s) - \theta b_{rf} r^f}{s\lambda - \theta b_r}; \qquad \frac{\partial g^*_{|\dot{s}=0}}{\partial s} = \frac{\theta\left[b_r(\theta b_2 - i^f \lambda) + \lambda b_{rf} r^f\right]}{(s\lambda - \theta b_r)^2} \lesseqgtr 0.$$

Setting these two equations equal and solving for s yields:

$$s^2(\theta b_s g_r - g_s)\lambda + s\left[\left(g_0 + \theta b_{rf} r^f g_r\right)\lambda - (\theta b_s g_r - g_s)\theta b_r + (i^f \lambda - \theta b_s)(\theta g_r - 1)\right] + [1 - \theta g_r]\theta b_{rf} r^f - \left(g_0 + \theta b_{rf} r^f g_r\right)\theta b_r = 0$$

Let us define:

$$\alpha_s = (\theta b_s g_r - g_s)\lambda < 0$$

$$\beta_s = \underbrace{\left(g_0 + \theta b_{rf} r^f g_r\right)\lambda}_{+} - \underbrace{(\theta b_s g_r - g_s)\theta b_r}_{-} + \underbrace{(i^f \lambda - \theta b_s)(\theta g_r - 1)}_{-} \lesseqgtr 0$$

$$\varsigma_s = \underbrace{[1 - \theta g_r]\theta b_{rf} r^f}_{+} - \underbrace{\left(g_0 + \theta b_{rf} r^f g_r\right)\theta b_r}_{+} \lesseqgtr 0.$$

This is an inverted U-shaped parabola. Its roots, which are the two fixed points of s, are given by:

$$\text{(A.1)} \quad s_{1,2} = \frac{-\beta_s \pm \sqrt{{\beta_s}^2 - 4\alpha_s \varsigma_s}}{2\alpha_s}.$$

If the discriminant $({\beta_s}^2 - 4\alpha_s \varsigma_s)$ is positive, this curve will have two roots. An economically meaningful equilibrium requires the exchange rate to be positive. A necessary condition is that the first term of (A.1) is positive, which in turn requires b_s to be positive. This corresponds to the assumption made throughout this paper that the feedback effects of capital accumulation

and the exchange rate on themselves are comparatively small. We shall assume that both fixed points exhibit a positive exchange rate.
Second, we obtain the two fixed points of g by taking (22) and (23),

$$(22)\quad s^*_{|\dot{g}=0} = \frac{g(\theta g_r - 1) + g_0 + \theta b_{rf} r^f g_r}{g_s - \theta b_s g_r}$$

$$(23)\quad s^*_{|\dot{s}=0} = \frac{\theta\left(b_{rf} r^f - g b_r\right)}{(i^f - g)\lambda - \theta b_s},$$

setting them equal and solving for g:

$$g^2\lambda(1-\theta g_r) + g\left[(\theta g_r - 1)(i^f\lambda - \theta b_s) - \lambda\left(g_0 + \theta b_{rf} r^f g_r\right) + \theta b_r(g_s - \theta b_s g_r)\right] + \left(g_0 + \theta b_{rf} r^f g_r\right)(i^f\lambda - \theta b_s) - \theta b_{rf} r^f (g_s - \theta b_s g_r) = 0$$

If we define

$$\alpha_g = \lambda(1-\theta g_r) > 0$$

$$\beta_g = \underbrace{(\theta g_r - 1)(i^f\lambda - \theta b_s)}_{-} - \underbrace{\lambda\left(g_0 + \theta b_{rf} r^f g_r\right)}_{+} + \underbrace{\theta b_r(g_s - \theta b_s g_r)}_{+} \gtreqless 0$$

$$\varsigma_g = \underbrace{\left(g_0 + \theta b_{rf} r^f g_r\right)(i^f\lambda - \theta b_s)}_{+} - \underbrace{\theta b_{rf} r^f (g_s - \theta b_s g_r)}_{+} \gtreqless 0.$$

This is a U-shaped parabola. Its roots, the fixed points of g, are given by:

$$(\text{A}.2)\quad g_{1,2} = \frac{\beta_g \pm \sqrt{{\beta_g}^2 - 4\alpha_g\varsigma_g}}{2\alpha_g}.$$

We shall assume that the discriminant is positive to ensure the existence of two fixed points. It is more difficult to make a statement of the sign of these two fixed points because the sign of b_g is ambiguous. We will make the assumption that there is a negative and a positive fixed point for g and focus our attention on the positive fixed point.

A4 Fixed points of the three-dimensional system

We want to find the fixed points of the three-dimensional system, reproduced here for convenience:

(14) $\dot{g} = \gamma\left[g(\theta g_r - 1) + s(\theta b_s g_r - g_s) + g_0 + \theta b_{r^f} r^f g_r\right].$

(25) $\dot{\lambda} = \delta(\lambda^T - \lambda) - g\lambda.$

(26) $\dot{s} = \mu\left\{g\theta b_r + s[i^f \lambda - \delta(\lambda^T - \lambda) - \theta b_s] - \theta b_{r^f} r^f\right\}$

First, we note that (14) is linear in s and independent of λ. We set (14) equal to zero and solve for g:

(A.3) $g = \frac{s(\theta b_s g_r - g_s) + g_0 + \theta b_{r^f} r^f g_r}{(1 - \theta g_r)}$

In order to avoid clutter, we introduce the following composite parameters:

$$\Phi_0 = \frac{g_0 + \theta b_{r^f} r^f g_r}{(1 - \theta g_r)} > 0$$

$$\Phi_1 = \frac{(\theta b_s g_r - g_s)}{(1 - \theta g_r)} < 0,$$

which allows us to re-write (A.3) as:

(A.4) $g = \Phi_0 + \Phi_1 s$

By substituting our solution for g into (26) and (27), we can examine these two equations as a nonlinear two-dimensional system. Substituting (A.4) into (25) yields, setting $\dot{\lambda} = 0$, and solving for λ:

(A.5) $\lambda = \frac{\delta\lambda^T}{\delta + \Phi_0 + \Phi_1 s}$

By the same token, we substitute (A.4) into (26), set $\dot{s} = 0$, and solve for λ:

(A.6) $\lambda = \frac{s(\delta\lambda^T + \theta b_s - \Phi_1\theta b_r) + \theta b_{rf} r^f - \Phi_0\theta b_r}{s(i^f + \delta)}$

The general structure of (A.6) and (A.7) is:

(A.5') $\lambda = \frac{\rho_0}{\rho_1 + \Phi_1 s}$, where $\rho_0 = \delta\lambda^T > 0$; and $\rho_1 = \delta + \Phi_0 > 0$.

(A.6') $\lambda = \rho_2 + \frac{\rho_3}{s\rho_4}$,

where $\rho_2 = \frac{\delta\lambda^T + \theta b_s - \Phi_1\theta b_r}{i^f + \delta} > 0$; $\rho_3 = \theta b_{rf} r^f - \Phi_0\theta b_r \lesseqgtr 0$; and $\rho_4 = i^f + \delta > 0$.

We note that (A.5) and (A.6) are rectangular hyperbolas in the (s, λ)-space. We find the s-coordinates of the intersection points by setting (A.5') and (A.6') equal and solving for s:

$$s^2\Phi_1\rho_2\rho_4 + s(\rho_1\rho_2\rho_4 + \Phi_1\rho_3 - \rho_0\rho_4) + \rho_1\rho_3 = 0$$

The roots of this quadratic equation are given by:

$$s_{1,2} = \frac{-(\rho_1\rho_2\rho_4 + \Phi_1\rho_3 - \rho_0\rho_4) \pm \sqrt{(\rho_1\rho_2\rho_4 + \Phi_1\rho_3 - \rho_0\rho_4)^2 - 4\Phi_1\rho_2\rho_4\rho_1\rho_3}}{2\Phi_1\rho_2\rho_4}$$

If $(\rho_1\rho_2\rho_4 + \Phi_1\rho_3 - \rho_0\rho_4)^2 > 4\Phi_1\rho_2\rho_4\rho_1\rho_3$, this equation has two roots. Although the sign of ρ_3 is generally ambiguous, it is likely to be negative. In this case, two roots occur. If, moreover, $\rho_1\rho_2\rho_4 + \Phi_1\rho_3 - \rho_0\rho_4 < 0$, these two roots must be positive. In our numerical example (see appendix A6), this is the case.

To find the λ-coordinates of the intersection point, we solve (A.5') and (A.6') for s,

(A.7) $s = \frac{\rho_0 - \lambda\rho_1}{\Phi_1\lambda}$,

(A.8) $s = \frac{\rho_3}{\rho_4(\lambda - \rho_2)}$,

and set the resulting equations equal to obtain a quadratic equation for the fixed points of λ:

$$-\lambda^2\rho_1\rho_4 + \lambda(\rho_1\rho_2\rho_4 + \rho_0\rho_4 - \Phi_1\rho_3) - \rho_0\rho_2\rho_4 = 0$$

The roots of this quadratic equation are given by:

$$\lambda_{1,2} = \frac{(\rho_1\rho_2\rho_4 + \rho_0\rho_4 - \Phi_1\rho_3) \mp \sqrt{(\rho_1\rho_2\rho_4 + \rho_0\rho_4 - \Phi_1\rho_3)^2 - 4\rho_1\rho_4\rho_0\rho_2\rho_4}}{2\rho_1\rho_4}$$

If $(\rho_1\rho_2\rho_4 + \rho_0\rho_4 - \Phi_1\rho_3)^2 > 4\rho_1\rho_4\rho_0\rho_2\rho_4$, this equation has two roots, and if ρ_3 is negative, these two roots must be positive. Again, in our numerical example, this is the case. Finally, we find the two fixed points for g by substituting the two solutions s_1 and s_2 successively into (A.4). Thus, since (A.4) is a linear function of s only, we can conclude that there are at most two fixed points for the system (14), (25), and (26).

A5 Hopf bifurcation in the three-dimensional system

The characteristic equation of the Jacobian matrix (27) is:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0,$$

where

$$a_1 = -Tr(\boldsymbol{J})$$
$$a_2 = Det(\boldsymbol{J_1}) + Det(\boldsymbol{J_2}) + Det(\boldsymbol{J_3})$$
$$a_3 = -Det(\boldsymbol{J}).$$

The Routh-Hurwitz stability conditions for three-dimensional are $a_1, a_2, a_3 > 0$ and $a_1a_2 - a_3 > 0$, or, in terms of trace and determinants of the Jacobian,

(RH.1) $Tr(\boldsymbol{J}) = -a_1 < 0$
(RH.2) $Det(\boldsymbol{J_1}) + Det(\boldsymbol{J_2}) + Det(\boldsymbol{J_3}) = a_2 > 0$
(RH.3) $Det(\boldsymbol{J}) = -a_3 < 0$
(RH.4) $-Tr(\boldsymbol{J})[\sum_{i=1}^{3} det(\boldsymbol{J_i})] + det(\boldsymbol{J}) = a_1a_2 - a_3 > 0.$

where $\boldsymbol{J}_i$ is the 2x2 principal minor obtained by deleting row and column i from the Jacobian matrix systems (Gabisch and Lorenz, 1989, p. 166). We want to examine, if this system can undergo a Hopf bifurcation giving rise to a stable limit cycle. For a Hopf bifurcation to occur, the condition (RH.4) has to switch from positive to zero when the Hopf bifurcation parameter assumes the critical value. In this case, the characteristic equation of the Jacobian matrix exhibits a negative real root and a pair of pure imaginary eigenvalues (Gabisch and Lorenz, 1989, pp. 162–166).

Let us examine these four conditions successively.

1) The trace is:

$$\underbrace{\gamma(\theta g_r - 1)}_{-} + \underbrace{\mu[i^f\lambda - \theta b_s - \gamma(\lambda^T - \lambda)]}_{+} - \underbrace{(\delta + g)}_{+} \lesseqgtr 0,$$

which becomes negative if $|J_{11} + J_{33}| > |J_{22}|$. This is likely to be the case, unless the BoP becomes highly unstable.

2) Next, consider the sum of the three principal minors:

$$\underbrace{\begin{vmatrix} \mu[i^f\lambda - \theta b_s - \delta(\lambda^T - \lambda)] & \mu s(i^f + \delta) \\ 0 & -(\delta + g) \end{vmatrix}}_{-} + \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & 0 \\ -\lambda & -(\delta + g) \end{vmatrix}}_{+} +$$

$$\underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s) \\ \mu\theta b_r & \mu[i^f\lambda - \theta b_s - \delta(\lambda^T - \lambda)] \end{vmatrix}}_{+/-} \lesseqgtr 0$$

Again, the sign is ambiguous, but provided that the last term is positive, it is likely to be positive.

3) Using a Laplace expansion along the third column, the determinant is given by:

$$\underbrace{-\mu s(i^f + \delta)}_{-} \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s) \\ -\lambda & 0 \end{vmatrix}}_{-} -$$

$$\underbrace{(\delta + g)}_{+} \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s) \\ \mu\theta b_1 & \mu[i^f\lambda - \theta b_s - \delta(\lambda^T - \lambda)] \end{vmatrix}}_{+/-} \lesseqgtr 0$$

The sign of the second determinant is ambiguous, but in line with what has been assumed in the section 3.2, we assume it is positive (the interaction effect between the exchange rate and the growth rate is stronger than the independent dynamics of these two variable). In this case, the overall sign of the determinant is still ambiguous. We must assume that the second term is larger in absolute value to ensure that the determinant is negative.

4) Lastly, we have:

$$-Tr(\boldsymbol{J})[\textstyle\sum_{i=1}^{3} det(\boldsymbol{J_i})] + det(\boldsymbol{J}),$$

which must become zero for a Hopf bifurcation to take place. We will use μ as the Hopf bifurcation parameter as it has the convenient property that it does not affect the steady state values. Let us write the three parameters of the characteristic equation as functions of μ:

$$a_1 = \mu\kappa_1 + \kappa_2$$
$$a_2 = \mu\kappa_3 + \kappa_4$$
$$a_3 = \mu\kappa_5.$$

We can then rewrite (RH.4) as:

$$\beta_1\beta_2 - \beta_3 = f(\mu) = \kappa_1\kappa_3\mu^2 + (\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5)\mu + \kappa_2\kappa_4 = 0.$$

This expression must become zero for a Hopf bifurcation to occur.
The coefficients κ_1 to κ_5 have the following signs:

$$\kappa_1 = -[i^f\lambda - \theta b_s - \delta(\lambda^T - \lambda)] < 0$$
$$\kappa_2 = \gamma(1 - \theta g_r) + (\delta + g) > 0$$

$$\kappa_3 = \underbrace{[i^f\lambda - \theta b_s - \delta(\lambda^T - \lambda)]}_{+} \underbrace{[\gamma(\theta g_r - 1) - (\delta + g)]}_{-} - \underbrace{\theta b_r \gamma(\theta b_s g_r - g_s)}_{-} \lesseqgtr 0$$

$$\kappa_4 = \gamma(1 - \theta g_r)(\delta + g) > 0$$

$$\kappa_5 = \underbrace{s(i^f + \delta)\lambda\gamma(\theta b_s g_r - g_s)}_{-} + \underbrace{(\delta + g)}_{+} \left\{ \underbrace{[i^f\lambda - \theta b_s - \delta(\lambda^T - \lambda)]}_{+} \underbrace{\gamma(\theta g_r - 1)}_{-} - \right.$$

$$\left. \underbrace{\theta b_r \gamma(\theta b_s g_r - g_s)}_{-} \right\} \lesseqgtr 0$$

κ_3 is likely to be positive given that we assume strong interaction effects and weak independent dynamics. κ_5 can be positive or negative. We then have

$$\text{(A.9)} \quad f(\mu) = \underbrace{\kappa_1\kappa_3}_{-}\mu^2 + \left(\underbrace{\kappa_1\kappa_4}_{-} + \underbrace{\kappa_2\kappa_3}_{-} - \underbrace{\kappa_5}_{-}\right)\mu + \underbrace{\kappa_2\kappa_4}_{+} = 0.$$

This parabola is opened downward. If its discriminant is positive, the equation has two real roots. This requires the expression $\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5$ to be sufficiently large in absolute value. At these two roots, a Hopf bifurcation occurs:

$$\mu_\pm = \frac{-(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5) \pm \sqrt{(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5)^2 - 4\kappa_1\kappa_3(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5)}}{2\kappa_1\kappa_3}$$

A6 Numerical simulation of the three-dimensional model

The numerical simulation is based on the following parameterisation:

Parameter	g_0	g_r	g_s	$b_0 r^f$	b_r	b_s	i^f	λ^T	γ	δ
Value	0.04	1.3	1.5	0.05	0.7	-0.02	0.09	0.54	1	1

The system has two fixed points, one at a negative and one at a positive investment rate for this parameterisation. We will focus on the second equilibrium, which is given by:

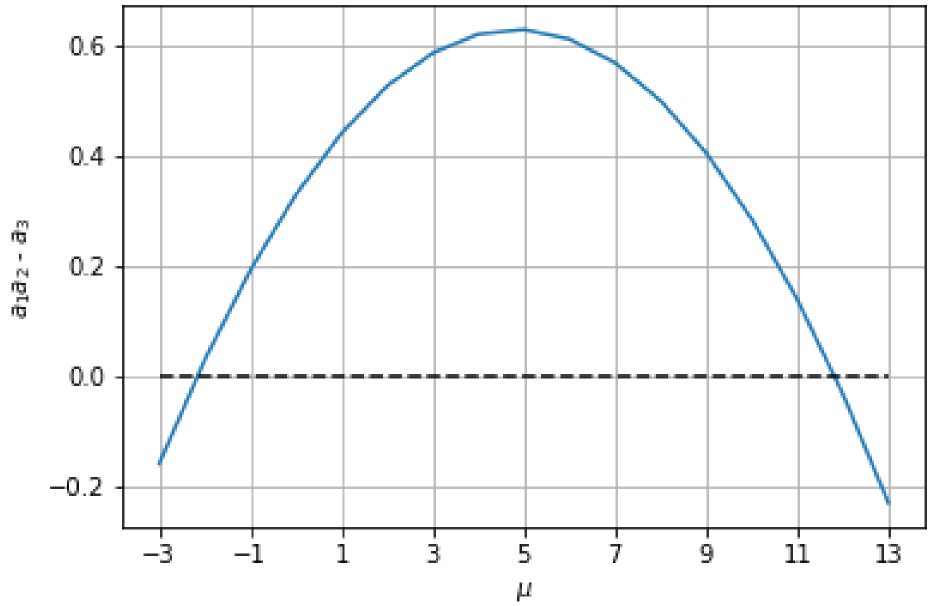
(g*, s*, λ*) = (0.0705, 0.017, 0.504).

We use the following initial values:

(g(0), s(0), λ(0)) = (0.0705, 0.016, 0.504).

Under this parameterisation, we obtain the following quadratic function for the Hopf bifurcation parameter μ (equation A.9):

Figure A1: $a_1a_2 - a_3 = 0$ as a function of μ



Thus, a Hopf bifurcation arises at the points $\mu_- = -2.165$ and $\mu_+ = 11.818$. We focus on the economically meaningful positive critical value $(\mu_+ = \mu_0)$. The simulation results presented in Figures 3 and 4 are based on $\mu = 11.82 > \mu_0$.