Harrod's Long-Range Capital Outlay as a Stabilizer of Harrodian Instability

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May 2017

Abstract

Drawing on Harrod, Kalecki and Kaldor, this paper seeks to revive the view that *ceteris paribus* firms reduce investment if they have already built up high capacities relative to their assessment of the normal potential of their markets. This reaction introduces a fundamental stabilizing mechanism into the economy. The paper adapts the idea to a growth context and applies it to the neo-Kaleckian baseline model with its Harrodian instability. It demonstrates that, in principle, a sufficiently strong feedback could stabilize the steady state.

JEL classification: C 13, E 12, E 30.

Keywords: Neo-Kaleckian framework, sentiment adjustments, long-range capacity effects, fully-adjusted positions, conflicting claims.

1 Introduction

The Kaleckian model of growth and distribution is presently a popular workhorse in heterodox macroeconomics concerned with effective demand issues. The baseline approach faces, however, a fundamental problem: either the rate of capacity utilization and thus the corresponding growth rate are endogenous variables not only in the short run but also in a long-term perspective, or, with its feedback from the firms' 'animal spirits', Harrodian instability prevails for the long-run equilibrium. Most economists criticizing the first point of view as not convincing are nevertheless not so radical as to consider the economy to be globally unstable. In a first round of the discussion, this camp has therefore to come up with concepts of how to tame the Harrodian centrifugal forces, so that the actual rate of utilization is brought back to an exogenous, so-called normal rate.

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Hein et al. (2011) is a paper that surveys a number of mechanisms that have been suggested in the literature to overcome the potential instability of the Kaleckian model. Specifically, framed in the context of local stability, the authors discuss the following proposals: the Cambridge price mechanism with its adjustments of profit margins; suitable variations in the retention rate of firms; effects of monetary policy; factors involving labour market features and/or the adjustment speed of prices; a particular form of perfect foresight of firms in their sales expectations. While Hein and his coauthors do not find any of these approaches persuasive (a statement which itself is debatable), we content ourselves with the observation that all of them signify a substantial rearming of the original model in one way or another.

Most recently, certain versions of the supermultiplier can be added to this list (Allain, 2015, and Lavoie, 2016, in the first instance). With its assumption that one component of aggregate demand grows autonomously at a constant rate, it is conceptually more parsimonious than the other designs, but on the other hand there are various immediate reasons that make it appear rather problematic (Skott, 2016; Franke, 2016). On the whole, it it thus not unfair to say that so far there is no easy and generally accepted solution to the problem of Harrodian instability.

The present paper strives for a more fundamental discussion that intends to remain within the confines of the Kaleckian baseline model with its quantity adjustments. It seeks to revive an idea on investment behaviour that can already be found in the works of Kalecki, Harrod and Kaldor. The argument is that *ceteris paribus* the investment of firms will be lower the higher the capacities they have built up in the past. In terms of a functional relationship, investment is decreasing in the capital stock (explicitly stated in Kalecki, 1935, p. 331, and Kaldor, 1940, p. 83).

Common to Kalecki's and Kaldor's seminal business cycle models is the interaction of a destabilizing and a stabilizing mechanism. In Kalecki's model, a divergent tendency is generated by the accelerator together with, as it turns out, the implementation lag of investment. In Kaldor's model divergence is brought about by the assumption of, as it is called today, Keynesian instability (this becomes perfectly clear in the reformulation of his model by Chang and Smyth, 1971). On the other hand, in both models a curb is put on an ongoing expansion, say, by the mere fact that the capital stock is continuously increasing in this phase. Therefore, investment and with it economic activity as a whole are bound to slow down again and eventually to turn around.

We view this as an argument of a most elementary kind. Situated within the sphere of pure quantity adjustments, it identifies a stabilizing mechanism that is not only historically but also logically prior to all other business cycle models and to discussions of steady state stability in general. It is thus astonishing that since then the argument has not been taken up, not even mentioned, in any of the later and more elaborated approaches to macrodynamic modelling. It seems the idea of a negative effect of rising capacities on fixed investment has fallen into oblivion.[1]

One reason for this neglect is possibly the fact that the mechanism was modelled in a stationary economy. In this way it is not necessary to specify a definite criterion according to which the firms consider their current capital stock too high or too low. Kalecki's and Kaldor's treatment was indeed somewhat mechanical in this respect. In a growth context, however, the capital stock has to be explicitly related to some benchmark, which will likewise be growing over time.

The proposal of such a concept is the basic innovation of this paper. Inspired by Harrod's (1939) famous article, it refers to a long-term perspective in the investment decisions of the firms and introduces the idea of a so-called potential, or normal, level of demand. This allows us to define as a benchmark that capital stock that would enable the firms to satisfy the potential demand by operating at the normal rate of utilization. The ratio of the actual to the benchmark capital stock can then take the role of just 'K' in a stationary economy. Instead of the volume of investment being inversely related to the level of the capital stock, the capital growth rate can be treated as a negative function of this capital ratio. Augmenting the original investment function in the Kaleckian model with this element, it will then be straightforward to show that the formerly unstable steady state can be stabilized if this effect is supposed to be sufficiently strong.

The paper is organized as follows. Section 2 recapitulates the neo-Kaleckian baseline model with its Harrodian instability. Section 3 introduces the notion of the aforementioned capital ratio and the corresponding capacity effect in the investment function. Section 4 first describes the features of the steady state, finding that because of the additional feedback it cannot generally be expected to be a 'fully-adjusted position'. Subsequently, it details the conditions for the system's local stability. Section 5 concludes, and an appendix has more to say on the consistency of possible 'conflicting claims' in the steady state.

2 Harrodian instability in the Kaleckian baseline model

Let us start out from the problem of Harrodian instability as it is, for example, laid out by Hein et al. (2011, Section 2). To keep things simple, it is considered for a closed one-good economy without taxation, government spending and capital

[1] Except in later variants of Kaldor's model itself, of course. Not the least because of its implementation lag, which poses some mathematical inconveniences, Kalecki's model remained a solitaire anyway.

depreciation, where labour is supposed to be in perfectly elastic supply. Rather than follow the practice of referring to capacity utilization as a measure of economic activity, it simplifies the notation if we directly work with the output-capital ratio u in this respect, which is the utilization rate of the capital stock in place. For short, it will be called 'utilization', too. The Kaleckian IS part of the model is thus constituted by the following four basic equations:

$$
\begin{aligned}
r &= hu && (1)\\
g^s &= sr && (2)\\
g^i &= a + \gamma_u (u - u^n) && (3)\\
g^i &= g^s && (4)
\end{aligned}
$$

In the first equation, neglecting depreciation, r is the rate of profit and h the (fixed) share of profits in total income. The function g^s in (2) represents the saving in the economy normalized by the (replacement value of the) capital stock: the firms' profits are all paid out to the rentiers who save a constant fraction s of them, while the workers consume all of their wages.

The third equation specifies the investment function, i.e. the planned growth rate of the capital stock. Within the short period it is oriented towards a rate a at which sales are expected to grow on average in the near future. Frequently, this yardstick is also introduced as the firms' 'animal spirits' (therefore the symbol 'a'). More demurely, we may refer to a as the general business *sentiment*, a term that still preserves the psychological and somewhat diffuse character of this type of expectations. The second component of investment in (3) features u^n as a normal, or desired, rate of capital utilization. In contrast to another branch of Kaleckian modelling, it is here treated as a fixed magnitude, exogenously determined by technological and institutional factors. In case of overutilization $u > u^n$ (or underutilization $u < u^n$), firms seek to close this gap by immediately increasing the capital stock at a proportionately higher (lower) rate than a (γ_u being the corresponding positive reaction coefficient).

Equation (4) sets up the temporary IS equilibrium, where market clearing is brought about by quantity variations. Utilization is then given by[2]

$$u = u(a) = \frac{a - \gamma_u u^n}{sh - \gamma_u} \qquad (5)$$

Stability of the underlying ultra short-run quantity adjustment process requires the denominator to be positive, that is, investment must not be too sensitive to changes

[2]Since no other situations than IS are considered, we abstain from earmarking these values by an asterisk or something similar.

in utilization. This is the so-called Keynesian stability condition, which we will not call into question. It will be noted that normal utilization $u = u^n$ implies $a = g^i = g^s = shu^n$ for the expected growth rate (in fact, $u(shu^n) = u^n$). Certainly, $u(a) > 0$ in (5) as long as a is not dramatically smaller.

What happens if $a \neq shu^n$, so that $u \neq u^n$? The utilization gap in (3) tells the firms that they have misread the economic situation, that they have under- or overestimated the current growth of demand. They consequently revise their expectations, which means that the sentiment a is a dynamic variable. Specifically, a is supposed to rise (fall) if u happens to be above (below) its normal level u^n. Translating this idea into formal language, it will be convenient to work in continuous time. Thus, denoting the speed of these adjustments by $\eta_u > 0$, we have,[3]

$$\dot{a} \;=\; \eta_u \left[u(a) - u^n \right] \tag{6}$$

The process can only come to a halt and the economy is on a balanced growth path if normal utilization is achieved, which, as has just been seen, is the case for $a = a^o = g^o = shu^n$ (steady state values are indicated by a superscript 'o').[4] Suppose some news in this state induces the firms to become more optimistic about their future sales, that is, the sentiment variable jumps to a value $a > a^o$. The correspondingly higher investment raises aggregate demand and the multiplier causes utilization to increase above its normal level, $u(a) > u^o = u^n$. The positive utilization gap leads to a further increase in a, which in turn widens the utilization gap, which in turn increases a, etc. Hence we have a process of cumulative causation that drives the economy more and more away from its long-run equilibrium. The mathematical argument is, of course, $d\dot{a}/da = \eta_u \; du/da > 0$ in (6). Equations (1) – (6) are an elementary way to formalize the Harrodian instability problem and prepare the ground for a rigorous discussion on how it may be overcome.

3 Long-range capital outlay in the investment function

The conclusion of the analysis within the common framework of the preceding section is obvious. In Harrod's (1939, p. 26) own words, "the instability principle seems quite secure." He nevertheless does not stop at this point. Although

[3]A dot above a dynamic variable x designates its derivative with respect to time, $\dot{x} = dx/dt$, and a caret (further below) its growth rate, $\hat{x} = \dot{x}/x$.

[4]This steady state is often referred to as a 'fully-adjusted position'. This term, however, neglects that in mature countries and in the very long run, the associated growth rate may be in conflict with the growth rate of the labour force.

(as hinted at in the Introduction) many proposals for additional mechanisms have been advanced in the literature that may be able to put a curb on the centrifugal forces, none of them starts out from the most elementary idea that Harrod himself had sketched on the following pages. While we will not try an exegesis of Harrod's original text with its analytical focus on the warranted rate of growth (which represents kind of a moving equilibrium concept), we can refer to his article from 1939 as a fruitful stimulation to develop our own specifications, along the lines of today's modelling practices.

In the next sentence after the statement just quoted, Harrod continues, "The force of this [instability] argument, however, is somewhat weakened when *long-range capital outlay* is taken into account" (emphasis added). Two paragraphs later he writes, "It is now expedient to introduce further terms into our equation to reduce the influence of the acceleration principle. Some outlays of capital have no direct relation to the current increase of output. They may be related to a *prospective long-period increase of activity*, and be but slightly influenced, if at all, by the current increase of trade" (emphasis added; cf. also Harrod, 1939, §13 on p. 27).

In this respect, it is also interesting to recall Kaldor's seminal "model of the trade cycle". After introducing, with their nonlinear features, his saving and investment schedules as functions of economic activity x, Kaldor proceeds with the remark, "Both $S(x)$ and $I(x)$ are 'short-period' functions—i.e., they assume the total amount of fixed equipment in existence" (Kaldor, 1940, p. 83). With respect to investment, he then supposes that in stages when activity is high, so that the level of investment is high and correspondingly "the total amount of equipment gradually increases", "the [investment] curve gradually falls." Conversely, when activity is low and capital gradually decumulates, the investment curve will shift upward (pp. 83–85).

In order to make Kaldor's model mathematically tractable, Chang and Smyth (1971) have thirty years later translated this idea into the concept of an investment function that (with certain nonlinearities) is increasing in output and decreasing in the capital stock, and many authors with their subsequent elaborations of the model followed them suit.[5]

It will be expected, and Chang and Smyth's (1971, p. 40) stability analysis confirms it, that a sufficiently strong negative effect of the capital stock on investment could stabilize the equilibrium, which would otherwise be locally repelling. Our intuition is that a similar feedback in the otherwise unstable Harrodian dynamics should work out in a similarly efficient way. Accordingly, the challenge is

[5] An investment function of this kind can also be found earlier in Kalecki's business cycle model (e.g., Kalecki, 1935, p. 331), although strictly speaking, the way in which it is derived does not allow one to treat the reaction coefficients on output and capital as independent parameters.

to incorporate the basic idea into the specific framework of eqs (1) – (6). This is not quite straightforward, mainly because the equations are formulated in a growth context (in contrast to Kaldor's stationary economy).

Kaldor's argument for the inverse relationship between investment and the capital stock is that high levels of the latter restrict the range of available investment opportunities, while these opportunities gradually accumulate in times of low levels of capital and investment (Kaldor, 1940, pp. 83–85).[6] We will not follow this (rather sketchy) reasoning but propose another and more direct story. It starts out from Harrod's and implicitly also Kaldor's distinction of a 'short-period' and 'long-range' perspective for the firms' investment decisions, where we will prefer to speak of the medium term and long term. For concreteness, the medium term may be identified with a period of up to three years and the long term with a period longer than that (with a grain of salt, of course). With this distinction, the well-established investment function (3) is interpreted as being derived from medium-term considerations.

Let us suppose that, in addition, firms have an idea of the size of their equipment that would be appropriate in a long-range perspective. All firms have their marketing experts and other specialists who try to assess a steady evolution of the demand that would normally be directed to them. This means they largely abstract from an ongoing expansion or contraction of the business cycle and other not easily predictable demand fluctuations. Similar to the notion of 'potential' or 'normal' output, this projected demand may be called potential, or normal, demand, designated D^n. An increase of D^n may be viewed as a counterpart to Harrod's formulation of a "prospective long-period increase of activity", in order to satisfy D^n. Thus, investment attributed to this argument corresponds to Harrod's "long-range capital outlay".

The capital stock K^n that would be desirable, or normal, in a long-term perspective is given by $K^n = D^n/u^n$. With respect to this argument, the capital growth rate is increased (decreased) if $K^n > K$ (if $K^n < K$). In finer detail, $g - a$ is supposed to be proportional with factor γ_k to the percentage deviations $(K^n - K)/K^n$. This idea induces us to relate the current equipment K to the normal capital stock K^n and introduce the ratio

$$k \;:=\; \frac{K}{K^n} \;=\; \frac{K}{D^n/u^n} \tag{7}$$

as a second dynamic state variable. For short, we call k the capital ratio. Thus, $k = 1$ if K coincides with K^n, and $(K^n - K)/K^n = (1 - k)$. Combining the medium-term

[6] Kaldor (1940, p. 83) also mentions a possibly opposite effect, but states that it is bound to be less powerful after a time.

with the long-term perspective, the investment function (3) generalizes to

$$g^i \;=\; a \,+\, \gamma_u\,(u - u^n) \,-\, \gamma_k\,(k-1) \tag{8}$$

where loosely speaking the coefficient γ_k in proportion to γ_u represents a weight that the firms attach to the long-term relative to the medium-term factors.

The consequences for utilization and the capital growth rates in the IS equilibria are straightforward. They are now given by

$$\begin{aligned} u \;=\; u(a,k) \;&=\; \frac{a - \gamma_k\,k - \gamma_u\,u^n + \gamma_k}{s\,h - \gamma_u} \\ g \;=\; g(a,k) \;&=\; s\,h\,u(a,k) \end{aligned} \tag{9}$$

Regarding the general business sentiment, i.e. the benchmark rate a in the investment function, we allow for a slight extension of the adjustment equation (6). In forming their expectations about the future growth of demand, firms may be aware of the negative effect of the capital ratio on investment and thus total output and income. Correspondingly, higher values of k could reduce their expected growth rate a somewhat. By how much precisely is represented by a coefficient $\eta_k > 0$, so that the sentiment adjustments become,

$$\dot{a} \;=\; \eta_u\,(u - u^n) \,-\, \eta_k\,(k-1) \tag{10}$$

It might seem somewhat unappetizing that the same variables, with the same sign, enter investment g^i, a level variable, and the change in the sentiment a, which in turn feeds back on g^i.[7] This is true formally, but conceptually the arguments are different. In the investment function (8), individual firms react to their own utilization rates and capital ratios (which would be explicitly heterogeneous in a more ambitious, agent-based modelling). The changes (10) in a, on the other hand, are concerned with a general climate in the economy, which is supposed to react to aggregate data (for example, similar to the framework proposed by Franke, 2012). In short, with respect to u and k (8) refers to the micro level and (10) to the macro level.

It remains to determine the normal demand D^n. In reality and at the micro level, its path and the expectations formed about it will depend on a multitude of fundamental, firm-specific and also subjective aspects. For our present purpose, however, we are not interested in the temporary variations of D^n that may thus occur. We limit ourselves to considering just one fundamental factor determining its projected growth. This is the number of future customers and their purchasing power, that is, in growth terms, the sum of the growth rate of the labour force

[7] Although, such a misgiving was never articulated within the original framework (1) – (6).

and the rate of technical progress. Both of them are treated as exogenously fixed magnitudes, their sum constituting the natural rate of growth, g^n.[8] The projected growth of normal demand thus reads,

$$\dot{D}^n / D^n \quad = \quad g^n \tag{11}$$

This equation completes the augmentation of the Kaleckian baseline model.

4 The steady state and its stability

The motions of the capital ratio are readily obtained from logarithmic differentiation, $\hat{k} = \hat{K} - \hat{D}^n$. Thus, together with (11) and the IS solution (9), our extended Harrodian economy is a two-dimensional dynamic system in the business sentiment and the capital ratio,

$$\begin{aligned} \dot{a} &= \eta_u [u(a,k) - u^n] - \eta_k (k-1) \\ \dot{k} &= k[g(a,k) - g^n] \end{aligned} \tag{12}$$

An equilibrium of (12) is obtained from setting its right-hand sides equal to zero. $\dot{k} = 0$ immediately tells us that, unlike the baseline model, the economy is sure to grow at its natural rate, $g^o = g(a^o, k^o) = g^n$. Using $g = sh\,u$ from (1), (2), the same equality can be solved for the steady state utilization u^o. Subsequently, k^o can be calculated from $\dot{a} = 0$, whereupon $u(a^o, k^o) = u^o$ in (9) can be solved for a^o. The resulting expressions are collected in the following proposition.

Proposition 1

An equilibrium position of (12) is uniquely determined by

$$\begin{aligned} g^o - g^n &= 0 \\ u^o - u^n &= (g^n - sh\,u^n) / sh \\ k^o - 1 &= (\eta_u / \eta_k)(u^o - u^n) \\ a^o &= sh\,u^n + (sh - \gamma_u + \eta_u \gamma_k / \eta_k)(u^o - u^n) \end{aligned}$$

[8] "Long-period anticipations" may also be "influenced by the present state of prosperity or adversity" (Harrod, 1939, p. 27), which could be captured by an additional influence of the sentiment variable a. While this extension is straightforward, we abstain from it for simplicity. Suffice it to mention that a strong influence of the sentiment may lead to saddle point instability, a phenomenon that is not possible in the simpler setting.

It follows that both utilization and the capital ratio are on target if, and only if, the product of the saving propensity s and the 'normal' rate of profit hu^n happens to be equal to the natural growth rate g^n. In this case, $a^o = g^n$, too. Otherwise, both $u^o - u^n$ and $k^o - 1$ are either positive or negative. That is, the steady state is no longer a 'fully-adjusted position' (but notice footnote 4 regarding the usage of this term). With respect to the sentiment adjustments (10), it could be said that the firms trade off one target against the other.[9]

By virtue of the (quasi-) linearity of system (12), the stability of the steady state is independent of possible deviations of u^o from u^n. The determinant of the Jacobian matrix is easily seen to be unambiguously positive, which rules out saddle-point instability. Examining the trace leads to the following conclusion.

Proposition 2

> *Defining $\gamma_k^H := \eta_u / k^o sh$, the equilibrium of (12) is locally asymptotically stable if $\gamma_k > \gamma_k^H$, while it is repelling if $\gamma_k < \gamma_k^H$. For γ_k sufficiently close to γ_k^H, the trajectories are of a cyclical nature.*

It follows that Harrodian instability can be overcome if only the counteracting effect of the capital ratio in the investment function (8) is sufficiently strong. In detail, however, this does not depend on the other coefficient γ_u in the same function but on the impact of utilization in the sentiment adjustments (10), that is, stability depends on whether γ_k is sufficiently high relative to the adjustment speed η_u. This is not obvious without a mathematical analysis.

The variable D^n could be viewed as long-run autonomous demand, the constant growth rate of which determines the overall growth of the economy. This term may be reminiscent of the approach of taming Harrodian instability by the concept of the so-called supermultiplier, which has recently received some attention (Allain, 2015; Lavoie, 2016). In these models, however, it is a component of current demand that is supposed to grow at an exogenous rate, whereas in the present model it is a projected path of total demand which serves to put the current level of productive capacity into a long-range perspective. The autonomous growth effect is therefore of an indirect nature.[10]

[9] The situation that even in a long-run equilibrium position firms may have "conflicting claims" has been most prominently pointed out by Dallery and van Treeck (2011). To highlight this feature, they also call it an "equilibrium without equilibrium" (p. 191), meaning that "the economy as a whole may be in 'equilibrium' (i.e. a steady state) without all the actors being in 'equilibrium' (i.e. some of their objectives remain unsatisfied)." As many economists are rather skeptical about the logical consistency of such a situation, the appendix will have to say something more on this.

[10] In particular, the present model is not affected by one criticism that was put forward against (at least) the simple two-dimensional versions of the supermultiplier. In a succinct manner, Franke

In Harrod's view, situations that invalidate the instability principle "might well arise in certain phases of the trade cycle" (Harrod, 1939, p. 28), but they are not bound to last. So his discussion could be read as an interaction of stabilizing and destabilizing forces giving rise to the ups and downs of the economy. The present model can in principle support this notion, as the last statement in Proposition 2 reveals a scope for cyclical behaviour if γ_k is not too far away from the benchmark γ_k^H. It was, however, numerically checked that the cycles remain bounded and do not die out if, and only if, $\gamma_k = \gamma_k^H$.[11] That is, the nonlinearity in the second equation of (12) is so weak that the system behaves just as a perfectly linear dynamics. On the whole, the model can be seen as providing the core of a growth cycle model taking up some of Harrod's elementary arguments, where it would require a true nonlinearity to generate self-sustaining endogenous oscillations (for such an endeavour, see Franke, 2017).

5 Conclusion

The paper sought to revive Harrod's concept of long-range capital outlay, i.e., investment expenditures which are largely independent of the current state of the economy and rather related to long-period anticipations. Concentrating on the firms' projections of sales in a more distant future, the paper adapted the idea to the growth context of the neo-Kaleckian model with its Harrodian instability. As in Harrod's reasoning, this component in investment demand proved to be able to reduce the destabilizing influence of the acceleration principle. If theoretically the effect is assumed to be sufficiently strong, it can even stabilize the steady state. While this result would not conform to Harrod's point of view, it helps understand the dynamic potential of the model.

It was briefly indicated that the interaction of the stabilizing and destabilizing forces could constitute the core of a modern business cycle modelling. The role of the capital stock as a stabilizer *via* its negative effect on fixed investment would in fact be historically prior to all other conceptions of putting a curb on a model's centrifugal forces. Such an approach may also be said to be logically prior, in the sense that it still remains within the sphere of quantities reacting to quantities, where it contents itself with straightforward specifications.

(2016) points out that if an additional mechanism turns η_u in eq. (6) into a negative and thus, when taking on its own, stabilizing coefficient, then the supermultiplier turns its originally stable (under $\eta_u > 0$) steady state into a saddle point. Checking our model's Jacobian matrix with $\eta_u < 0$, it is easily verified that stability is not turned upside down in this way.

[11] Mathematically speaking, γ_k^H constitutes a Hopf bifurcation (therefore the superscript 'H'), though of a degenerate nature.

Appendix: Considering the steady state notion

Many economists seem rather skeptical about the internal consistency of a steady state with ‘conflicting claims’. They feel there should be other forces which will eventually unbalance such a position. But what forces could this be in the present framework? To begin with, the firms would note that normal demand D^n is persistently distinct from the actual sales Y. Because everything grows steadily at the same rate g^n, they should conclude that their assessment of the level of D^n is wrong.

One conceivable consequence is that the firms reset D^n at another value. Whatever it is, the capital ratio will thus be perturbed from k^o. The economy jumps out of the steady state, but its law of motion (12) remains the same. Hence, if stability prevails, the system will return to the same type of inconsistency as before.

Another attempt deviates from this crude scenario and concedes the firms that also outside the steady state they will be aware of possible divergencies of D^n from the trend of aggregate demand. To correct for them, they slightly increase (decrease) the growth rate of D^n if $Y > D^n$ (if $Y < D^n$). Correspondingly, the assumption (11) may be modified as

$$\dot{D}^n/D^n \;=\; g^n + \mu\,\frac{Y-D^n}{D^n} \;=\; g^n + \mu\,\Big[\,\frac{uk}{u^n}-1\Big]$$

($\mu > 0$ but relatively small). The corresponding change of the derivative of the capital ratio in (12) affects the equilibrium values u^o, k^o. Unfortunately, in the wrong direction. Suppose that $u^o > u^n, k^o > 1$ originally for $\mu = 0$. Considering $\dot{k}/k = shu - g^n - \mu(uk/u^n - 1)$ when μ is slightly increased from zero, the expression is negative if u^o, k^o are maintained. Substituting $k = k^o$ from Proposition 1, it is seen that $u = u^o$ has to increase rather than decrease in order to reestablish $\dot{k} = 0$.

In a more principled way, it may be argued that the entire problem is ill-posed. The point is that its reasoning refers to homogeneous firms, whereas system (12) may be viewed as a description of the aggregate dynamics of firms that are heterogeneous at the micro level. To make this objection more concrete, suppose the individual firms expect their normal demand to grow at either a high rate a^+ or a low rate a^-. The sentiment a is the average of them, weighted by their population shares. The firms switch between a^+ and a^- with certain probabilities that are influenced by their monitoring of aggregate statistics. The resulting changes in a can be modelled in a canonical fashion similar to Franke (2012, 2014), and eq. (10) can be conceived of as a linear approximation.

According to this approach, the firms have different growth rates of their capital stocks, and thus also different utilization rates and capital ratios. Moreover, their variables are continuously changing, beginning with the fact that even when $\dot{a} = 0$, a certain fraction of firms is always switching in both directions. Not only these changes, but also the resulting changes in the individual output-capital ratios average out such that (by the law of large numbers) the aggregate variable stays constant. The general notion is that the economy is at rest at the macro level, while the microeconomic variables do not cease moving. As a consequence, the individual firms are permanently reacting to new situations, so that the question whether they keep on making obvious systematic errors does not even arise.

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