# Endogenizing non-price competitiveness in a BoPC growth model with capital accumulation*

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**Abstract**

On a theoretical level this article revisits Thirlwall's rule (or law) proposing a new channel through which it is possible to endogenize non-price competitiveness in the BoPC framework. We develop a model that formalizes the inverted-U relationship hypothesis that non-price competitiveness rises as countries move from a primary productive structure to light manufacturers and then decreases as richer countries get locked into antiquated industrial structures. We name it *the stratification mechanism.* Finally we incorporate the supply side of the economy into the structure of the model in order to avoid the so called *inconsistency problem.*

**Keywords:** growth, Balance-of-Payments Constraint, Thirlwall´s law.

**JEL**: O10. O11, O40

## 1 Introduction

Post Keynesian models have emphasized over the years the importance of demand constraints on growth. One of the most successful empirical regularities among them is Thirlwall's rule (or law). Given that countries cannot systematically finance increasing balance-of-payments imbalances, it proposes that there is an adjustment mechanism in aggregate demand that constrains growth. Therefore, in the long-run, growth is Balance-of-Payments Constraint (BoPC).

Despite its simplicity and powerfulness in explaining the great income divergence between advanced and developing/poor economies, there are at least two main issues still to be solved. The first of them consists in the fact that foreign trade income elasticities - which represent non-price competitiveness of a country (or region) - are taken as exogenous. The second one

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is the *inconsistency* (or overdetermination) *problem* that may arise if the supply side of the economy is not properly incorporate into the model.

There have been a number of efforts trying to formally endogenize non-price competitiveness in the BoPC framework. In some cases there were also explicit concerns about the consistency of the model and the capital accumulation problem. However we are still building a consensus. Now it is well understood that distributive and technological questions matter and should be taken into account. But the subject is still open.

This paper aims to contribute to both issues by developing a model that formalizes the inverted-U relationship hypothesis that non-price competitiveness rises as countries move from a primary productive structure to light manufacturers and then decreases as richer countries get locked into old-fashioned industrial structures. We name it the *stratification mechanism*. This mechanism operates in a such a way that opposite forces that acts over economic growth are spread over time, opening space for the appearance of a falling-behind dynamics or convergence clubs.

Finally we incorporate the supply side of the economy in a dynamic fashion in order to study the interaction between capital accumulation and the external constraint avoiding the so called *inconsistency problem*. This is an innovation with respect to the contributions of Palley (2003) and Oreiro (2016) who explicitly address the over determination problem in static models.

The paper's next section brings our model of the *stratification mechanism* and its simulation exercise. In section 3 we extend it by incorporating the supply side of the economy and the capital accumulation process. Once again we run some numerical simulations. The last section brings some final remarks.

# 2 Non-price competitiveness and the *stratification mechanism*

The idea that growth is BoPC has been a crucial component of much demand-led growth theory since at least Prebisch (1959) and Thirlwall (1979). However, it is the role of demand in defining the nature of the constraint that distinguishes the approach from other growth models (Razmi, 2016). After the empirical consolidation of Thirlwall's rule during the eighties and the beginning of the nineties, the behavior of the foreign trade income elasticities came to the discussion.

In the BoPC literature the ratio of foreign trade income elasticities represents non-price competitiveness of a country (or region). In a certain sense it is comparable to "Solow's residual" since it occupies a central position in the explanation of the growth process but it is taken as exogenous to the model. After the empirical success of BoPC model in both aggregate and disaggregate versions (e.g. Bagnai, 2010; Gouvea and Lima, 2010, 2013; Lanzafame, 2014; Romero and McCombie, 2016; Bagnai, Rieber and Tran, 2016), a next and natural step consists in endogenizing the ratio of foreing trade income elasticities.

Fagerberg (1988) seems to have been the first one to formally do it based on the concept of technological competitiveness. Meliciani (2002) extended Fagerberg's model in order to take into account the role played by differences across countries in terms of technological specialization. A favorable relation between technical change and foreign trade elasticities with a Keynesian flavor can be found in Catela and Porcile (2012) as well as Cimoli and Porcile (2014). A similar route is also followed by Ribeiro, McCombie and Lima (2016) who included distributive issues into the analysis.

The possibility of positive feedbacks from growth to non-price competitiveness is further explored by Fiorillo (2001), McCombie and Roberts (2002) and Setterfield (2011). Nonetheless, those studies do not take into account capital accumulation concerns. This was done by Palley (2003) who proposed to make income elasticity of imports a positive function of the level of capacity utilization. He explicitly addressed both the endogeneity of income elasticities and the *inconsistence problem*.[1] Lately, Oreiro (2016) has recalled Palley's solution and proposed an extension putting the exchange rate at the center of stage, making income elasticity of imports a function of real exchange rate.

As this briefly revision shows, there have been a recent number of efforts trying to make non-price competitiveness an endogenous variable in a BoPC framework. In some cases there were also explicit concerns about the consistency of the external constraint and the capital accumulation problem. However, we consider that we are still far from a consensus.

The BoPC literature has taken two different routes in terms of mathematical modeling. A first group of authors such as Fiorillo (2001), Araújo and Lima (2007) and Nishi (2016) have developed multi-sectoral versions of the rule. In this approach, non-price competitiveness is in certain a sense an endogenous variable since aggregate foreign trade elasticities can change due to changes in the productive structure of the economy.[2] On the other hand a second group of scholars have incorporated the other elements of the balance-of-payments to the aggregate model (e.g. Barbosa-Filho, 2001; Moreno-Brid, 2003, Alleyne and Francis, 2008). In this paper we focus on the aggregate version of Thirlwall's rule. This allows us to present in a clearer and pedagogical way the stratification mechanisms.

## 2.1 The model

The relation between growth and structural change is highly non-linear. In the late nineties a particular hypothesis was proposed by Thirlwall among others. Thirlwall (1997), Setterfield (1997), and McCombie and Roberts (2002) argued that non-price competitiveness – i.e. the ratio between foreign trade income elasticities - grows as the country moves from the production of primary products to manufacturing and decreases when the economy gets locked in old-fashioned industrial structures.

As a matter of fact, the empirical literature on industrial development and sectoral concentration has recently documented an inverted U-shaped pattern between diversification of the productive structure and the level of per capita income (e.g. Imbs and Wacziarg, 2003; Haraguchi and Rezonja, 2013; Romano and Traù, 2017). Bearing this in mind, consider a small open economy that is BoPC. The following set of equation summarizes the coevolution between the rate of growth and productive diversification:

$$\frac{\dot{Y}_t}{Y_t} = \rho_t z \tag{1}$$

$$\frac{\dot{\rho}_t}{\rho_t} = -\varepsilon \left( \frac{\dot{Y}_t}{Y_t} - \alpha \right) + \beta \tag{2}$$

[1]However, one should mention that Palley's hypothesis is not an issue of non-price competitiveness. He is exclusively concerned with the influence of the supply-side on a demand-determined growth path. We thank one of the referees for pointing this out to us.

[2]Fiorillo (2001) is an interesting contribution because combines a multisector model with discrete-time analysis. Moreover, one should also mention the multi-country approach developed by Nell (2003) and generalized by Bagnai, Arsene and Tran (2016).

Continuity is assumed in order to deploy standard techniques of dynamical analysis subsequently. Growth is endogenous in the sense that output's growth rate is a function of non-price competitiveness and variations in non-price competitiveness are a function of the growth rate of output. Equation (1) corresponds to Thirlwall's rule (or law), where $Y$ is domestic output, $\rho$ represents non-price competitiveness, and $z$ stands for the growth rate of the rest of the world being taken for simplification as constant. The only difference between our formulation and the original one is the subscript $t$ that follows the foreign trade elasticities ratio. Interpretation of the expression is the same.

Foreign trade income elasticities are dependent upon the level of diversification of the economy's productive structure. A low level of diversification is associated with a high marginal propensity to import which in turn implies in a high income elasticity of imports. It is also associated with low elasticity of exports, since the economy will have few different types of goods to export in face of increasing world demand. That is, a higher $\rho$ is related to a more diversified economy.

Moreover, an extensive literature on complexity has stressed the positive relation between economic complexity and productive diversification (e.g. Hidalgo et al, 2007; Hausmann et al, 2014; Hartmann et al, 2017). In fact, one of the main findings in this literature is that more-sophisticated products are located in a densely connected core whereas less-sophisticated products occupy a less-connected periphery. Economies more diversified produce more-sophisticated products. That is the main reason why $\rho$ captures non-price competitiveness of an economy.[3] Gouvea and Lima (2010; 2013), Romero and McCombie (2016) and Martins Neto and Porcile (2017) provide empirical evidence of such interpretation. The expression above is a simple but powerful instrument to explain the great divergence in income between advanced and developing/poor economies.[4]

Equation (2) is the representation of the hypothesis discussed above and we name it as the *stratification mechanism*. Shortly, the *stratification mechanism* is a formalization of the traditional Kaldorian proposition that there is a positive feedback from growth to non-price competitiveness and resembles the well-known Kaldor-Verdoorn law. Nevertheless, it comes with some worth noting novelty.

Economic development and technological opportunities press for change. Hence, countries (or regions) need to modify their institutional and productive arrangements so as to adapt to the new requirements of change ( Abramovitz, 1986; Chang, 2007; Chang, 2011). However, each economy faces specific constraints when it comes to its capacity to adapt. Such constraints further shape the development path of nations. Structural change depends on the set of institutions and capabilities that interact to boost different forms of learning in a systemic process that demands coordination (Cimoli, Lima and Porcile, 2016). For example, the capital stock of a country consists of a complex web of interlocking elements. They are built to fit together, and it is difficult to make replacements without costly rebuilding of other components.

The capacity of adaptation is deeply related to distributive issues and the struggle between reform vs *status quo*. Without entering in the matter of reforms that are "good" or "bad", once a reform is going on coordination uncertainty may arise because it is not clear who is going to be better-off afterwards (Fernandez and Rodrik, 1991; Rodrik, 1996). Consequently, change paradoxically also creates incentives to "keep things like they are".

[3] The centrality of the link between non-price competitiveness and technological performance has been recently explored in explaining Germany's success in international markets (Storm and Nastepaad, 2015).

[4] For a survey on the developments of the model as well as a further discussion of the implications behind Thirlwall's law and the BoPC growth literature, see Thirlwall (2011).

Thus, parameter $\alpha$ captures the maximum growth rate the economy is able to "support", so to speak. That is, for growth rates up to $\alpha$ the economy is able to adapt to the new conditions imposed by economic development and translate them into a more diversified productive structure. On the other hand, very high growth rates imply in dramatic changes in the economic system. If such changes are above a certain limit given by $\alpha$, we shall expect on the contrary a negative impact on non-price competitiveness. Therefore, one could think of $\alpha$ as a threshold for increasing returns or in Abramovitz terms a measure of social capabilities.

McCombie and Roberts (2002) considered that poor previous growth rates can be expected to give rise to pressure for reform, while high previous growth rates have the opposite effect. It may sound at first implausible that structural change is induced by an economy with low growth. Actually, we would argue that growth bellow the capacity of adaptation of the economy generates productive diversification through traditional Kaldorian channels. Up to $\alpha$ the cumulative causation mechanism operates unrestricted. Furthermore, it is not that high growth rates inhibit structural change. Nevertheless, growth above the capacity of adaptation of the economy might have negative effects in the productive structure.

All this process is intermediated by the stratification coefficient (or elasticity), $\varepsilon$, that corresponds to the speed at which changes induced by growth are translated to non-price competitiveness. Thus, while a higher $\alpha$ is associated with countries that possess human and technical capabilities to easily adapt to change, it is not clear the relation between higher (or lower) values of $\varepsilon$ and such capabilities. On the one hand, it is reasonable to suppose that a higher $\varepsilon$ is associated with countries with complex and diversified productive structures because the economy would improve faster. However, for low values of $\alpha$, a higher $\varepsilon$ actually might indicate that the economy is running faster to a low-income trap. Therefore, the most we can say at this point is that both parameters should be interpreted together. Finally and because the stratification mechanism does not intend to account for the whole (nor even most part) process of structural change, $\beta$ aggregates all forces not represented.

Substituting equation (1) in (2) and rearranging, we obtain:

$$\dot{\rho}_t = \rho_t \left( \varepsilon\alpha + \beta - \varepsilon z \rho_t \right). \tag{3}$$

The expression above depicts an inverted-U relationship between variations and levels of non-price competitiveness. In the beginning an increase in $\rho$ increases competitiveness itself up to a limit when there is a stratification of the economic structure. The speed of this process and the final level of the elasticity ratio depend on the parameters of the economy.

Despite its economic intuition, equation (2) and consequently equation (3) may still sound too ad hoc. Therefore, we provide some empirical evidence for them. We use the Economic Complexity Index (ECI) from the Observatory of Economic Complexity as a proxy for non-price competitiveness. Data covers 125 countries between 1964 and 2015. First, we apply the *lowest* nonparametric procedure to obtain a flexible form for the relationship linking variations and levels of ECI. Going further, equation (3) is explicitly estimated controlling for country fixed effects (FE).[5] Figure 1 displays the fitted curves:

[5] Because ECI might exhibit persistence over time and we are dealing with long enough series we also performed panel Unit-root tests. Estimates indicate that series are stationary. Results are available under request.

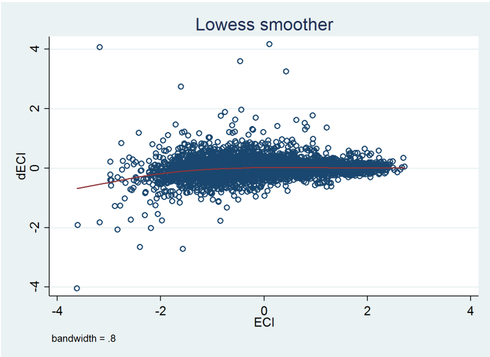


Figure 1a: Estimated curve (nonparametric)

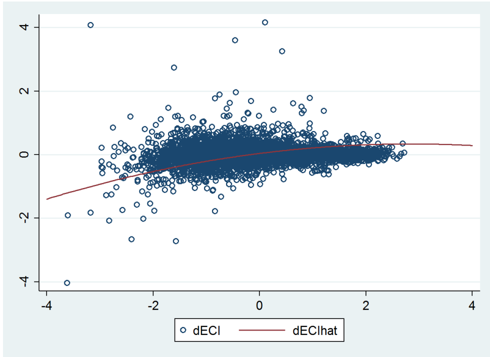


Figure 1b: Estimated curve (FE Panel)

Table 1 reports the estimates of the FE regression analysis:

| Dependent variable: dECI | | |
|---|---|---|
| ECI | 0.2039739*** | 0.204589*** |
| $ECI^2$ | -0.0587923*** | -0.0584514*** |
| c | 0.0649078*** | 0.0814449*** |
| Year dummies | No | Yes |
| Observations | 5655 | 5655 |
| $R^2$ within | 0.1725 | 0.1729 |

Table 3: FE Panel Structural Change. *, **, ** indicate 10%, 5% and 1% of significance

Results indicate that the inverted U-shaped formulation adopted in this paper do find some empirical support and allow us to continue exploring the dynamic properties of the model. When the system comes to apparent rest (or steady-state) $\dot{\rho}_t = 0$. We have one non-trivial solution given by:

$$\rho^* = \frac{\alpha + \beta/\varepsilon}{z} \tag{4}$$

with $\frac{\partial \rho^*}{\partial \varepsilon} < 0$.

As Thirlwall's rule itself, the result above is very simple but with a straight message. A higher stratification elasticity implies lower non-price competitiveness. We shall expect that economies with higher $\varepsilon$ would lock-in themselves faster, conditional to a given threshold. This brings us to our second point. Higher threshold implies in higher foreign trade elasticities. There is a direct and positive relation between $\beta$ and the equilibrium level of non-price competitiveness. Finally, notice that a country (or region) may greatly enhance growth but if the stratification elasticity is high enough, non-price competitiveness will be low. That is, $\alpha$ and $\varepsilon$ push $\rho^*$ in opposite directions.

Therefore the BoPC growth rate is given by:

$$\frac{\dot{Y}_t}{Y_t} = \rho^* z = \alpha + \beta/\varepsilon. \tag{5}$$

Equation (5) allowed us to decompose growth process in two elements. The first one is the capacity of adaptation of the economy. The second one is the sum of all other factors that explain changes in non-price competitiveness divided by the stratification elasticity. This in turn implies that increases in $\varepsilon$ make growth to be more dependent on $\alpha$. Inversely, lower values of $\varepsilon$ are associated with an economy that depends less on the capacity of adaptation threshold. We would further explore this property in the numerical simulations. Also notice that even though the original expression for Thirlwall's rule depends on the growth rate of the rest of the world, once the stratification mechanism is taken into account this t does not occurs anymore. However this is a property that comes from the functional form adopted in (2).

## 2.2 Numerical Simulations

We shall proceed to some numerical simulations in order to study the dynamic properties behind the model so far developed. The software employed was the E&F Chaos in its 2012 version. This exercise can be understood as a comparative static analysis. It intends to show how the system responds to changes in some crucial parameters of interest. When choosing their values and initial conditions we followed the magnitude of the coefficients estimated in the previous subsection. In addition, we tried to provide outcomes with economic meaning.

First, we represent a situation in which the world economy growths at a rate of $z = 3\%$, the stratification elasticity is $\varepsilon = 1$, threshold growth rate $\alpha = 1.5\%$, and the autonomous component of non-price competitiveness growth is $\beta = 1.5\%$. Under this conditions 50% of the growth rate depends on the capacity of adaptation and 50% depends on other factors weighted by $\varepsilon$. We set it as a benchmark case. Two different initial conditions are established, namely, $\rho_0 = 1.5$ and $\rho_0 = 0.5$. Results are plotted in figure 2:

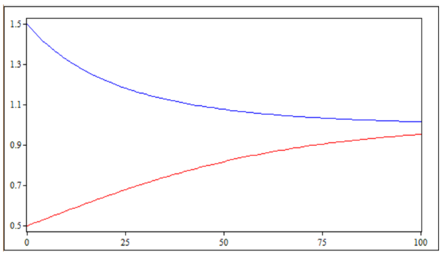
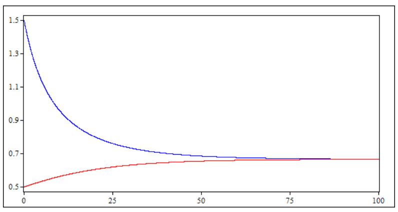


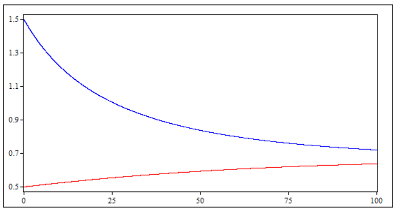
Figure 2: Benchmark economy

For both initial conditions the model shows a slow convergence process. The blue line corresponds to an economy that use to growth faster than the rest of the world but is converging to the same growth rate. The red line represents the opposite situation. After one hundred interactions both economies still have not reached the equilibrium.

A second case corresponds to an economy whose equilibrium configuration is a falling behind path, that is, it grows less than the rest of the world. We keep almost all the parameters and the initial conditions constant. The exceptions are the stratification elasticity and the capacity of adaptation. In the diagram of the left we adopted $\varepsilon = 3$ and $\alpha = 1.5\%$. The diagram of the right plots $\varepsilon = 1$ and $\alpha = 0.5\%$. Consequently we have in both cases that $\rho^* \approx 0.67$. Results are plotted in figure 3.



$\varepsilon$=3; $\alpha$=1.5%



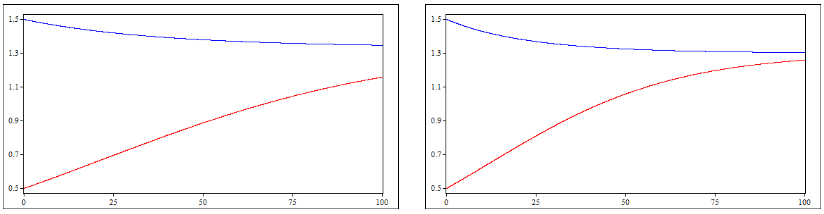
$\varepsilon$=1; $\alpha$=0.5%

Figure 3: Falling behind economy

Convergence to equilibrium in the first case happens in 50 periods. Therefore an increase in the stratification elasticity not just reduces the level of elasticities but also increases the speed of convergence. On the other hand, we can see that changes in $\alpha$ only produce a change in steady-state equilibrium. The blue line represents the path of an economy that used to grow faster than the rest of the world but after 10 periods starts falling behind. The red line corresponds to an economy that is able to increase its growth rate but not enough to overcome the falling behind process. It is also worth noting that *ceteris paribus* the contribution of the capacity of adaptation to growth increased to 75% in the first case and fell to 25% in the second.

A typical example of interrupted catching up and/or falling behind behavior would be the South America experience and in particularly the case of Argentina. The so called "Argentine paradox" it is a unique episode of a country that had achieved advanced development in the early 20th century but experienced a slowly reversal in the last eighty years. In the light of the model discussed in this paper we would argued that Argentina is a combination of very low $\alpha$ and moderate $\varepsilon$. A low $\alpha$ reflects the low capacity of adaptation of the Argentine economy to the new conditions imposed once the cycle of exploitation of the rich land of the pampas came to an end. A moderate $\varepsilon$ naturally responds forth the speed of the process.

Finally we consider the case of an economy that in equilibrium growths faster than the rest of the world. It could be a developing country in catching up or an already developed economy differentiating itself from the leading group. Once more we keep the same parameters except the stratification elasticity and the capacity of adaptation. In the diagram of the left we adopted $\varepsilon = 0.6$ and $\alpha = 1.5\%$. The diagram of the right plots $\varepsilon = 1$ and $\alpha = 2.4\%$. Non-price competitiveness at equilibrium approximates $\rho^* \approx 1.3$. Results are plotted in figure 4.




$\varepsilon$=0.6; $\alpha$=1.5% $\varepsilon$=1; $\alpha$=2.4%

Figure 4: Catching up/differentiating economy

The first figure shows that convergence takes more time and goes to higher values of non-price competitiveness. This is because of the lower values adopted for the stratification elasticity. The second diagram depicts the effect of an increase in $\alpha$ on the steady-state values of non-price competitiveness. The blue line corresponds to an economy that maintains its growth rate above the rest of the world. The red line represents an economy that was falling behind but after 50 periods is able to start catching up.

It is difficult not to think in the experience of East Asia as a successful case of catching up. Even though the sustainability of China's growth rates might be a reason for controversy, the region started an uninterrupted process of modernization from the sixties that continues nowadays. In the light of the model presented here we would argued that a combination of high capacity of adaptation and a low stratification elasticity might be part of the explanation.

Disaggregating growth between the capacity of adaptation and other factors we have that in the first case $\alpha$ accounted for 37.5% of all growth while in the second case its contribution was increased to 60%. Increases in $\varepsilon$ make growth more dependent on the capacity of adjustment threshold. Therefore, the stratification elasticity operates as way through which divergent forces on growth among countries are diluted over time, opening space for the appearance of falling-behind behavior or convergence clubs. Even though it allows us to go one step further in the study of the dynamics behind non-price competitiveness in a BoPC growth framework, one might ask how it interacts with the rest of the productive structure. This issue is discussed in the next section.

## 3 Incorporating the supply side

It is necessary to incorporate the supply side of the economy in order to avoid the *inconsistence* (or over determination) *problem.* All adjustment mechanisms presented concern the long run. Keep also in mind that we are dealing with a small open economy without government. We begin stating the following production function:

$$Y_t = \min \{a_t K_t u_t; q_t L_t\}, \tag{6}$$

where capital, $K$, and labor, $L$, are weighted by their respective productivity coefficients, $a$ and $q$. Variable $u$ stands for the level of capacity utilization and it is equal to the ratio of current output, $Y$, and potential output, $Y^*$. If inputs are used efficiently then the economy must be operating with a level of output that satisfies the following condition:

$$Y_t = a_t K_t u_t = q_t L_t. \tag{7}$$

Assume for simplicity that capital productivity is constant, $a_t = a$. Taking logarithms and time derivatives:

$$\frac{\dot{u}_t}{u_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{K}_t}{K_t} \tag{8}$$

$$\frac{\dot{q}_t}{q_t} = \frac{\dot{Y}_t}{Y_t} - \frac{\dot{L}_t}{L_t}. \tag{9}$$

From equation (8) we have that changes in the level of capacity utilization are given by the difference between the growth rate of output and capital accumulation. It is reasonable to consider that if the economy is growing faster than the increase in productive capacity we will observe an increase in the level of capacity utilization. Inversely, if capital accumulation is growing ahead of aggregate demand the economy then we will observe a reduction in the level of capacity utilization. Equation (9) on the other hand states that the growth rate of labor productivity is obtained as a residual of the difference between the growth rate of output and labor. Both expressions directly come from the Leontief efficiency condition.

Turning back to equation (7) it follows that:

$$\frac{I_t}{K_t} = a\frac{I_t}{Y_t}u_t, \tag{10}$$

with $I$ standing for aggregate investment. Defining

$$h_t = \frac{I_t}{Y_t} \tag{11}$$

as the ratio between investment and output (or the propensity to invest), and disregarding capital depreciation so that $I_t = \dot{K}_t$, we have:

$$\frac{\dot{K}_t}{K_t} = a h_t u_t. \tag{12}$$

An increase in the investment-output ratio or in the level of capacity utilization directly increases capital accumulation. This comes again from the efficiency condition. In the limit, if there is no investment or no capacity utilization is being used, obviously there is no capital accumulation.

Now, we are dealing with an economy that is BoPC. This means that in the long run

$$\frac{\dot{Y}_t}{Y_t} = \rho_t z$$

because aggregate demand follows the external constraint, i.e. Thirlwall's rule. The reader may ask which mechanism brings aggregate demand in line to the BoPC growth rate. Unfortunately we do not tackle this issue here since it is a topic for a different paper.[6] For us it is enough to say that output's growth rate follows aggregate demand which in turn faces the external constraint. In appendix 1 we show under which conditions growth must be BoPC.

Substituting Thirlwall's rule and equation (12) in equation (8), we obtain:

$$\frac{\dot{u}_t}{u_t} = \rho_t z - a h_t u_t. \tag{13}$$

Changes in aggregate demand have lagged effects on investment that may give rise in the long run to the Harrodian instability problem. Therefore, we model it as:

$$\frac{\dot{h}_t}{h_t} = \gamma \left(u_t - u_n\right) \tag{14}$$

where $u_n$ is the planned, desired or normal level of capacity utilization and $\gamma$ is an adjustment parameter. Equation (14) may sound too *ad hoc* but it is a fair representation of Harrodian dynamics and fits what we need to continue the exercise without overcomplicating it under the risk of missing its point. If the level of capacity utilization is above its normal rate, capitalists increase investment above demand. On the other hand if we are below the planned level, there is a reduction in investment that reduces $h$.[7]

The dynamic non-linear system is formed by equations (3), (13) and (14). When the economy comes to a state of apparent rest, $\dot{\rho}_t = \dot{u}_t = \dot{h}_t = 0$. In this situation, for $\rho_t$, $u_t$, and $h_t \neq 0$, we have that:

$$\beta = \varepsilon \left(\rho_t z - \alpha\right), \qquad \text{(Non-price competitiveness curve)}$$

$$a h_t u_t = \rho_t z, \qquad \text{(Capacity utilization curve)}$$

and

$$\gamma \left(u_t - u_n\right) = 0. \qquad \text{(Harrodian curve)}$$

The non-price competitiveness curve is built on the equality of the effect of growth on competitiveness with its autonomous component. On the other hand the capacity utilization

[6] Dávila-Fernández and Sordi (2017) provide a theoretical model and some preliminary empirical evidence of such adjustment mechanism.

[7] Notice that from the Harrodian dynamics and the definition of $h$ we have that $\frac{\dot{h}_t}{h_t} = \frac{\dot{I}_t}{I_t} - \frac{\dot{Y}_t}{Y_t} = \gamma \left(u_t - u_n\right)$ which implies that $\frac{\dot{I}_t}{I_t} = \gamma \left(u_t - u_n\right) + \rho_t z$. That is, investment follows aggregate demand and deviations of capacity utilization from its normal rate.

curve corresponds to the combination of endogenous variables that equalize aggregate demand (that follows the external constraint) and capital accumulation growth rates. From the Harrodian curve it is straightforward that in equilibrium the level of capacity utilization equals its normal rate.

Equilibrium values are defined and given by:

$$\rho^* = \frac{\varepsilon\alpha + \beta}{\varepsilon z}, \tag{15}$$

$$u^* = u_n, \tag{16}$$

$$h^* = \frac{\rho^* z}{a u_n}. \tag{17}$$

All equilibrium values are positive. An increase in non-price competitiveness has no permanent impact on the level of capacity utilization but implies in a higher investment-output ratio. It is an interesting result because while the level of capacity utilization accommodates differences between demand growth and capital accumulation, the investment-output ratio is the variable that accommodates in steady state variations in non-price competitiveness.

## 3.1 Local stability analysis

To investigate the local stability of the system we linearized it around the equilibrium values. So that:

$$\begin{bmatrix} \dot{h}_t \\ \dot{u}_t \\ \dot{\rho}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & J_{12} & 0 \\ J_{21} & J_{22} & J_{23} \\ 0 & 0 & J_{33} \end{bmatrix}}_{J} \begin{bmatrix} h_t - h^* \\ u_t - u^* \\ \rho_t - \rho^* \end{bmatrix}, \tag{18}$$

$$J_{12} = \gamma h^* > 0, \tag{18a}$$

$$J_{21} = -a\left(u^*\right)^2 < 0, \tag{18b}$$

$$J_{22} = -a h^* u^* < 0, \tag{18c}$$

$$J_{23} = z u^* > 0, \tag{18d}$$

$$J_{33} = -\varepsilon z \rho^* < 0. \tag{18e}$$

The characteristic equation of the Jacobian matrix is:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{19}$$

where $\lambda$ denotes a characteristic root. Each coefficient of equation (19) is given by:

$$\begin{aligned} a_1 &= -\operatorname{tr} J = -\left(J_{22} + J_{33}\right), \\ a_2 &= \begin{vmatrix} J_{22} & J_{23} \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & J_{12} \\ J_{21} & J_{22} \end{vmatrix} = J_{22}J_{33} - J_{12}J_{21}, \\ a_3 &= -\det J = J_{12}J_{21}J_{33}. \end{aligned}$$

The necessary and sufficient condition for the local stability is that all characteristic roots of the Jacobian matrix have negative real parts, which, from Routh–Hurwitz condition, is equivalent to the following inequalities:

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0 \text{ and } a_1a_2 - a_3 > 0.$$

Let us examine whether these inequalities hold. It is easy to check that $J_{22}$ and $J_{33}$ are negative, therefore $a_1 > 0$, and the first inequality is always satisfied. It is also straightforward that $J_{22}J_{33} > 0$ and $J_{12}J_{21} < 0$. Therefore $a_2 > 0$, and the second inequality is always satisfied. Moreover, $J_{12}J_{21}J_{33} > 0$ so that $a_3 > 0$, and the third inequality is also fulfilled. Finally we have to check if $a_1a_2 - a_3 > 0$. Through direct computation we have that:

$$a_1a_2 - a_3 = -\left(J_{22} + J_{33}\right)\left(J_{22}J_{33} - J_{12}J_{21}\right) + J_{22}J_{12}J_{21} > 0.$$

Therefore the system is stable around the equilibrium values.

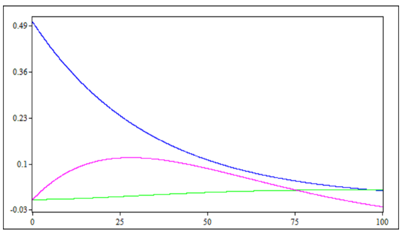
For an exogenous growth rate of labor force, labor productivity growth is directly obtained as a residual from equation (9) so that there is no *inconsistency problem.* Moreover if we state that labor productivity follows a Kaldor-Verdoorn rule - as it is done by Palley (2003) and Oreiro (2016) - labor growth adjusts to the difference between output and productivity growth rates, and once again there is no inconsistency.

## 3.2 Numerical Simulations

In the second section we showed how the stratification mechanism operates in the BoPC framework for a given stratification coefficient while in section 3 we showed how it interacts with the supply side of the economy. We shall proceed to some numerical simulations in order to study the dynamic properties behind this broader structure. Since from the previous section changes in $\alpha$ only changed equilibrium values while changes in $\varepsilon$ had an impact in both speed of convergence and steady-state equilibrium, in what follows we will focus in the dynamics behind the stratification elasticity.

First we represent a situation in which the world economy grows at a rate of $z = 3\%$, the stratification elasticity is $\varepsilon = 1$, threshold growth rate $\alpha = 1.5\%$, and the autonomous component of non-price competitiveness growth is $\beta = 1.5\%$. Normal level of capacity utilization $u_n = 0.7$. Capital productivity coefficient $a = 0.2$. Finally the adjustment coefficient of investment $\gamma = 0.02$.

Our simulations are done in terms of deviations from the equilibrium values. As initial conditions we use the equilibrium values of $h$ and $u$, and a deviation of 0.5 for $\rho$. From the combination of the parameters above we obtain $h^* \approx 0.215$, $u^* = 0.7$ and $\rho^* = 1$. We set it as a benchmark case. Results are plotted in figure 5.



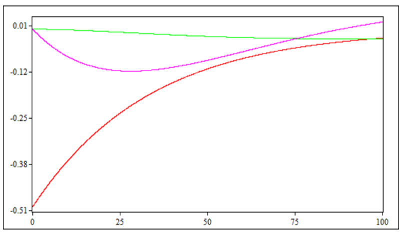




Figure 5: Benchmark economy

For both initial conditions the model shows slowly convergence. Figure (5a) depicts an economy that used to grow faster than the rest of the world but is converging to the same growth rate. The blue line is non-price competitiveness while the green and purple lines are investment and capacity utilization, respectively. Because non-price competitiveness is initially above its equilibrium value, demand grows faster than capital accumulation and we observe an increase in the level of capacity utilization and of the investment ratio. This increase of investment increases capital accumulation. The stratification mechanism slowly reduces non-price competitiveness so that BoPC and capital growth rates come closer and the system reaches equilibrium.

Figure (5b) represents the opposite situation, that is, an economy that used to grow less than the rest of the world but that is converging to the same growth rate. Because non-price competitiveness is below its equilibrium value, demand grows less than capital accumulation and we observe a decrease in the level of capacity utilization and of the investment ratio. This in turn reduces capital accumulation. However, the stratification mechanism increases non-price competitiveness bringing BoPC and capital growth rates to equilibrium.

We continue simulating an economy that is falling behind over its equilibrium path, that is, growing less than the rest of the world. We keep constant all parameters and initial conditions except the stratification elasticity that now equals to $\varepsilon = 3$. Consequently we have that $h^* = 0.14$, $u^* = 0.7$, and $\rho^* \approx 0.65$. Results are plotted in figure 6.

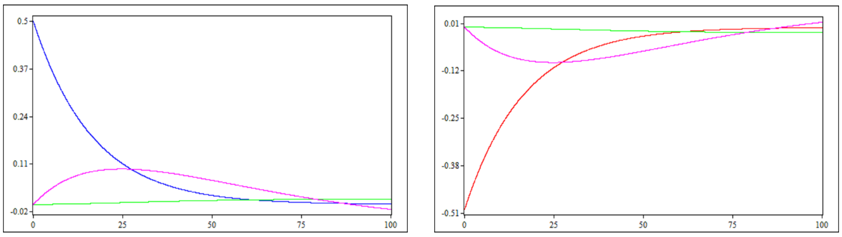


Figure 6: Falling behind economy

Diagrams are remarkably similar to the last simulation. Figure (6a) corresponds to an economy that used to grow faster than the rest of the world but after some periods starts falling behind. Figure (6b) on the other hand represents an economy that starts a catching up process but interrupts it before it is complete. Because the stratification coefficient is higher we observe a faster convergence of $\rho$ to its equilibrium value. Moreover, the speed of convergence of the other variables does not seem to have changed even though the magnitude of their variation is slightly lower.

Finally, we consider the case of an economy that in equilibrium grows faster than the rest of the world. It could be a developing country in catching up or an already developed economy differentiating itself from the leading group. All parameters were kept the same except the stratification elasticity that now equals $\varepsilon = 0.6$. Equilibrium values are $h^* \approx 0.28$, $u^* = 0.7$ and $\rho^* \approx 1.3$. Results are plotted in figure 7.

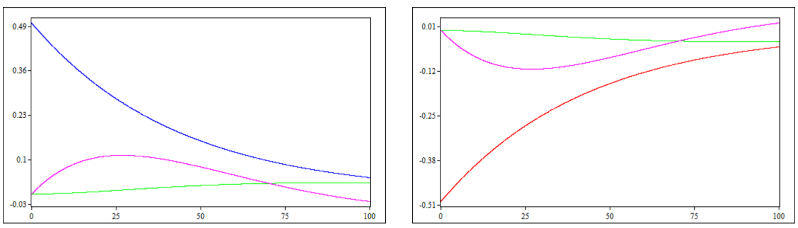


Figure 7: Catching up/differentiating economy

Once again the results are similar to previous simulations. For lower values of the stratification mechanism convergence is slower. However it is also possible to observe that the magnitude of the variation in capacity utilization and investment necessary to accommodate misalignments between demand and capital accumulation growth rates is higher. This suggests that the stratification mechanism influences not just the final values of investment and non-price competitiveness but also their path.

# 4 Final considerations

Even though there have been recent efforts in order to endogenize non-price competitiveness in the BoPC framework, we could say that we are still building a consensus about this issue. It is well understood that distributive and technological questions matter and should be taken into account. But the subject is still open.

This paper aims to contribute to the discussion. We developed a model that formalizes the inverted-U relationship hypothesis that non-price competitiveness rises as countries move from a primary productive structure to light manufacturing and then decreases as richer countries get locked into old-fashioned industrial structures. The inverted U relationship was first proposed by Thirllwall himself and discussed by authors like Setterfield and McCombie. We name it the *stratification mechanism*. We also furnished some empirical evidence that justifies our modelling choice. Finally we incorporate the supply side of the economy into the structure of the model in order to explicitly avoid the over determination problem.

Our simulation exercises provided some insights in terms of comparative statics. In section 2 it is clear how variations in the capacity of adaptation and in the stratification elasticity impact non-price competitiveness in different scenarios. A lower capacity of adaptation and a higher stratification coefficient implies in lower non-price competitiveness and faster convergence to equilibrium. Moreover, the higher the stratification elasticity the more dependent the economy on its own capacity of adaptation.

Section 3, on the other hand, showed how this mechanism depends on the productive structure of the economy. Furthermore, while the level of capacity utilization accommodates differences between demand growth and capital accumulation, it is the investment-output ratio that accommodates to variations in non-price competitiveness in steady-state.

We believe that our exercise was able to formalize the *stratification mechanism* under not so restrictive assumptions. Growth is as complex as fascinating and we hope that above allthis paper helps to enlighten a glance of it.

# Appendix 1

From the so called fundamental identity:

$$S_t - I_t = X_t - E_t M_t,$$

where $S$ stands for total savings, $X$ corresponds to exports, $M$ is imports, and $E$ is the exchange rate.

Assume $\frac{S_t - I_t}{X_t}$ as constant and exogenously given. Taking logarithms and time derivatives:

$$\frac{\dot{X}_t}{X_t} = \frac{\dot{E}_t}{E_t} + \frac{\dot{M}_t}{M_t}. \tag{A1.1}$$

Following the standard BoPC growth model we define:

$$\frac{\dot{X}_t}{X_t} = -\psi \frac{\dot{E}_t}{E_t} + \varphi_t z \tag{A1.2}$$

and

$$\frac{\dot{M}_t}{M_t} = \delta \frac{\dot{E}_t}{E_t} + \pi_t \frac{\dot{Y}_t}{Y_t}, \tag{A1.3}$$

where $\psi$ and $\delta < 0$ are price elasticities, and $\varphi$ and $\pi > 0$ are income elasticities.

Under constant terms of trade and substituting equations (A2.3) and (A2.4) in (A2.2):

$$\frac{\dot{Y}_t}{Y_t} = \rho_t z,$$

where $\rho_t = \frac{\varphi_t}{\pi_t}$. That is Thirlwalls rule (or law).

Assume now $\frac{S_t - I_t}{Y_t}$ as constant and exogenously given. As one of the referees pointed out, this is the "correct" Kaldorian closure because $\frac{I}{Y}$ and $\frac{S}{Y}$ are balanced. Taking logarithmics and time derivatives of the fundamental identity we have that:

$$\theta_t \frac{\dot{X}_t}{X_t} + (1 - \theta_t)\frac{\dot{Y}_t}{Y_t} = \frac{\dot{E}_t}{E_t} + \frac{\dot{M}_t}{M_t} \tag{A1.4}$$

where $\theta_t = \frac{X_t}{X_t + I_t - S_t}$.

Under constant terms of trade and using equations (A2.3) and (A2.4) we obtain once again:

$$\frac{\dot{Y}_t}{Y_t} = \rho_t z,$$

with $\rho_t = \frac{\theta_t \varphi_t}{\pi_t + \theta_t}$.

Summarizing, as long as:

1. $\frac{S_t - I_t}{X_t}$ or $\frac{S_t - I_t}{Y_t}$ is constant and exogenously given;
2. Terms of trade are constant, and
3. Exports and imports given by (A2.3) and (A2.4)

then, growth must be given by Thirlwall's rule (or law). The result is driven from the fact that the fundamental identity must always hold.

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