Stagnation in the US economy: Is growth over?

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Abstract

The Great Recession of 2008 triggered a debate on the possibility of *secular stagnation*. We suggest that the relevant research question is whether growth of real GDP has indeed declined (in the US, post–WWII), and if so, *why*. We present evidence to answer the first part of this question in the affirmative. (A) Trends from wavelet decompositions of US time series and (B) the natural rate of growth estimated via Okun's Law with time–varying parameters suggest that the growth rate of real GDP has *gradually* declined from roughly 4% during the *Golden Age* to 2% during the neoliberal era. We place these insights in the context of the heterodox literature on growth and distribution. The *Goodwin growth cycle* provides fertile ground to develop further research questions. Crucial among these is whether (a) exogenous, downward shifts or (b) endogenous, demand–driven declines of labor force and labor productivity growth dominate, and (c) how either of these trends interact with short and long run distributive changes.

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1 Introduction

Growth of real GDP in the US in the (almost) ten years since the *Great Recession* has been exceedingly low. The slow recovery and overall anemic macroeconomic performance has triggered a debate on the possibility of secular stagnation.[1,2] What does secular stagnation

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[1]The slow recovery is of course a global phenomenon. We will bring international linkages into the discussion at various points, but the focus for now—and with regards to data—will be on the US economy.

[2]Larry Summers has been credited to revive the topic, see Summers (2014). In this speech, and in subsequent debate, the focus lies on the low "natural rate of interest," at which, in Wicksellian fashion, investment and savings equilibrate to generate full employment. We do *not* see historically low long–term interest rates, or the long decline in nominal and real interest rates as a relevant causal factor. At best, it is a symptom of stagnation; and hence our discussion will not focus on it. Teulings and Baldwin (2014) provide a comprehensive discussion of the relevant issues.

mean? One perspective sees stagnation as a sustained shortfall of realized GDP from the underlying supply potential. To assess such a phenomenon that *potential* needs to be estimated.

One route to do so builds on growth accounting à la Solow. The Congressional Budget Office (CBO) expends considerable effort to do this, see CBO (2001). Its estimate of real potential GDP in the US is constructed as follows. First, estimate the NAIRU based on a textbook new–Keynesian Phillips curve. Second, estimate the potential labor force, piecewise linear for peak–to–peak business cycles, and remove cyclical movements via Okun's Law, with the NAIRU from step one as the benchmark. Third, obtain total factor productivity growth from a standard Solow residual, and smooth it. Fourth, construct an index of capital services. Fifth, calculate potential output growth by assuming a Cobb–Douglas production function with a fixed capital share parameter of one third.

In a nutshell, this procedure estimates "potential" as an average of actual growth. As such, it leads us down the rabbit hole of hysteresis:[3] if realized GDP falls consistently short of potential GDP, it will likely affect exactly that supply potential. Over time, thus, such stagnation fixes itself through a reduction of potential GDP itself, and the ratio of real GDP to potential GDP returns to "normal." The resulting series describe a (cyclical) growth process conditional on socio–political conflict, evolving institutions, monetary and fiscal policies, etc—but do not pin down a hypothetical "production possibility frontier."

The CBO points to the problems in the estimation of potential in a discussion of revisions. The agency cites a reduction in labor force participation and the rise in long–term unemployment in the wake of the deep recession as the reasons for its downward revisions of potential GDP. These are, of course, the standard (supply side) forces of hysteresis. (Estimating "supply side potential" in this manner is just as limited as the search for your proverbial keys under the streetlight; a universe of unknowable counterfactuals exist in the dark.) And, while the document refers to the generally protracted nature of recovery from housing and financial crises, it does not emphasize the deleterious effects of fiscal austerity, or corruption and dysfunction in Congress.[4]

Still, and whatever the theoretical (or political) motivations may be, the resulting estimate of potential GDP is simply an average of the observed, and the projection into the future an extrapolation of the last cycle's growth trend. At the time of this writing, monetary policy is well on the way to normalization, since actual GDP has returned to that "normal." However, the *level of GDP* is much lower than a return to previous trends would have made possible, and the *growth rate of real GDP* remains depressed.

An alternative starting point to discuss growth performance (or the lack thereof) builds on

[3]Hysteresis has featured prominently in the post–Keynesian literature from a neo–Kaleckian point of view (Dutt, 1996; Lavoie, 1996). In these investigations, as is standard in the neo–Kaleckian literature, the distribution of income is treated as exogenous, so that we will not directly follow it.

[4]See CBO (2014), p.6. A similar (supply side) argument, albeit not mentioned by the CBO in this report, is that a deep and long recession will lead to capacity scrapping. When recovery occurs, firms are able to return to a desired rate of utilization at the reduced capacity. The implication is a level effect on capital stock and output, in combination with a return to the pre–crisis NAIRU. It should as well be noted that the CBO does not consider hysteresis effects in this context.

the *natural rate of growth*. Following Harrod, the natural rate of growth of real GDP is the sum of the growth rates of labor force and technological progress. Of course, the natural rate cannot be observed, either, and thus needs to be estimated. However, statistical techniques to uncover averages of the growth rates of real GDP, the labor force, and technological progress cannot, and as we will show, do not, produce significantly different answers than those from the CBO.

The relevant question about *secular stagnation* is therefore not so much whether a shortfall from potential occurs (it does, frequently), or whether it persists (it doesn't, because hysteresis removes the gap), but, rather, and simply, *whether average or natural growth has declined*, and, importantly, if so, *why*. The first question can clearly be answered in the affirmative: average or natural or potential growth of real GDP in the US has decreased from a *Golden Age* average of roughly four percent per year to the current rate of about two percent per year. Crucially, the decline in the growth rate has been gradual over the entire post–war period, and was only temporarily slowed during the "new economy" expansion of the late Nineties.

The second question is much more difficult to answer. Our intention is to discuss possible causes for stagnation from a variety of viewpoints, so as to develop research questions and testable hypotheses. Candidate causes are best illustrated on the basis of the definition of the natural rate. First, consider the 'right hand side' variables, namely the growth rates of labor force and technological progress. Obviously, demographic changes affect the growth rate of the labor force. Reductions in the population growth rate appear as an exogenous force; we will not pursue Malthusian lines of thinking.[5] Analogously, at least a fraction of the rate of labor productivity growth can reasonably be seen as exogenous. Different *types* of innovations affect the organization of production differently. This argument was put forth by Gordon (2012, 2014). He suggests that the high growth rates of the *Golden Age* were in large part due to maturation and diffusion of the innovations of successive waves of industrialization; specifically, railroads, electricity, indoor plumbing, etc.; and that the current, lower growth rates are due to a return to a much longer run "normal."

However, both labor force and labor productivity are to some extent endogenous to growth of GDP itself. First, participation rates are affected by growth and employment. The ratio of labor force to working–age–population has cyclical elements, and—following the above arguments on hysteresis—protracted shortfalls in demand can affect trend components. Further, Okun's and Verdoorn's Law suggest standard and well–documented channels through which growth rates of demand and labor productivity are causally linked. Which effects dominate? Do we observe stagnation because of exogenous downward shifts in the growth rates of labor force and labor productivity? Or do we observe stagnation because *throughout the neoliberal era, aggregate demand has been depressed*? The latter would imply that causality runs from left to right, from growth to natural growth.

[5] Similarly, increases in life expectancy and corresponding increases in dependency rates can have macroeconomic effects, but we will not focus on these either; see Rada (2012) for a discussion. A different line of thinking presumes that still too generous welfare systems, minimum wages, etc, keep "lazy folks" from working, or high taxes keep "genius folks" from expending effort. We do see such labor supply decisions at the margin as irrelevant, and will not pursue related arguments.

To allow for a more nuanced discussion, we will relate the story of *secular stagnation* to the story of the *falling labor share.* Rising inequality in the personal income distribution and a downward trend in the labor share of income have been extensively documented (Piketty and Saez, 2003; Elsby et.al., 2013; IMF, 2017), and have gained prominence as well in the new–Keynesian macroeconomic literature. For example, Eggertson and Mehrotra (2014) suggest that a rise in inequality can lead to stagnation in a DSGE framework.

Our frame of reference, however, is the heterodox macroeconomic literature on the linkages between growth and distribution. A seminal starting point in this literature is Goodwin (1967). In this paper, Goodwin develops a dynamic model in the employment rate and wage share that produces an endogenous, self–sustained, *conservative* oscillation. The driving forces are profit–driven accumulation and class conflict. Importantly, the resulting trajectories render growth and cycles one and the same phenomenon.

Evidence for these Goodwin cycles focuses on business cycle frequencies, i.e. on the short run, and, often, on developments in a single or even closed economy. Barbosa–Filho and Taylor (2006), Mohun and Veneziani (2008) and Zipperer and Skott (2011) document such short run cycles. In contrast, our question pertains to the long run, and *whether and how stagnation might emerge in the context of Goodwinian dynamics.* Evidence on *long* Goodwin cycles exists, but is mixed.

Flaschel (2009, 2015) and Tavani et.al. (2011) demonstrate that a long Goodwin–type movement in the post–WWII US economy can be observed. Barrales and von Arnim (2017) extend that investigation on the basis of wavelet decompositions; results indicate the possibility of a US long run Goodwin cycle that *collapses* during the second half of the Nineties with lower employment rates, lower income–capital ratio and a lower wage share. The implication is that systematic interaction à la Goodwin occurs at lower frequencies. (See Figure 16, and discussion further below.)

Following Flaschel, such a *long* Goodwin cycle could be classified as consisting of a *phase of prosperity*, followed by *profit squeeze and stagflation*, *stagnation and disinflation*, then *wage squeeze and recovery* to ultimately enter a new *phase of prosperity.* (See Figure 15 for a stylized picture.) According to this view, the long Goodwin cycle in the US—post-WWII "Golden Age," profit squeeze, then neoliberal recovery—should have culminated in a phase of prosperity with wage share and activity measures rising together, roughly from the mid–Nineties onwards.

Kiefer and Rada (2015) offer a different perspective. In a time panel investigation covering OECD countries, they find evidence for (a) short run Goodwin cycles but the possibility of (b) long run stagnation with falling wage shares and rising output gaps. Further, Rada and Kiefer (2016) provide evidence for a negative impact of a globalization proxy on the wage share. Blecker (2017a,b) discusses these issues and raises the question whether it is *long run growth*—rather than a generic growth cycle, or a growth cycle with business cycle frequency—that is wage constrained. Indeed, Blecker's hypothesis is that the neoliberal era (with inflation targeting, deregulation, labor suppression and hyper–globalization) induces a contractionary bias that systematically depresses demand, which in turn weakens the profit squeeze. Such *dominant demand shifts* could explain the slow "south–west" shift of the steady state around which profit–led/profit–squeeze business cycles move.[6]

There are thus two ideas that can be usefully contrasted: a long, sixty–year Goodwin–type movement, along which business cycles follow a profit–led/profit–squeeze pattern; and a Goodwin–type movement at business cycle frequency with a shifting steady state. Our intention in this paper is to pull together data on US stagnation of *natural growth*, and discuss it in the context of these considerations of high and low frequency interactions between growth and distribution. The ultimate goal is to further develop relevant models and testable hypotheses. The centrally important question seems to be whether observed adverse changes in the components of the natural rate are exogenous, or endogenous to demand and distribution. The remainder of this paper is organized as follows. The next section presents empirical methods and data. Subsequently, we discuss the obtained trends; lastly, we conclude.

2 Data and methods

The purpose of this section is, first, to present methods used to asses long run trends of relevant time series. Two different methodologies are applied: discrete wavelet transforms allow for extraction of long run trends of a host of univariate time series; and regression analysis of Okun's Law with time–varying parameters generates an estimate of Harrod's natural rate. In a third subsection, we collect details and sources on raw data.

2.1 Wavelet transform

We begin here with a brief overview. *Wavelet decompositions* allow flexible estimation of long run trends.[7] For these decompositions, the *maximal overlap discrete wavelet transform* (MODWT) is applied to time series data. Using *multi resolution analysis*, the series can be reconstructed in a number of cyclical components (the "details") and a trend (the "smooth"). The variance of these reconstructed series sums to the variance of the original time series. The resulting trend is less sensitive to recent data than, for example, the Hodrick–Prescott filter, and, crucially, retains a degree of cyclicality. That degree depends on the number of details chosen, each of which describes cycles with periodicity $[2^j, 2^{j+1}]$. For example, detail 4 features cycles with period length between $2^4 = 16$ and $2^{j+1} = 32$ quarters, or 4–8 years. Most of the (quarterly) times series discussed here cover the years 1950–2015, or 264 quarters. We define *wavelet trends* to be the cyclical component of the series with period length greater than $2^7 = 128$ quarters, or 32 years.

[6] In this narrative, globalization appears to play a central role. Multi–country interactions are as well considered in Rezai (2011, 2015) and von Arnim et.al. (2014). These analyses suggest that a fallacy of composition or race to the bottom could occur in neo–Kaleckian economies, but do not present empirical evidence (and consider the functional distribution of income as exogenous).

[7] Wavelet methods are not very common in economics. Gallegatti and Semmler (2014) present various applications. Charpe and Bridji (2017) employ similar methods to analyze long run interaction of growth and distribution; see as well Barrales and von Arnim (2017). A complete discussion of wavelet methods for time series can be found in Percival and Walden (2000).

The MODWT for level J for a time series $\mathbf{X}$ yields highly redundant and non–orthogonal column vectors $\tilde{\mathbf{W}}_1, \tilde{\mathbf{W}}_2, \ldots, \tilde{\mathbf{W}}_J$ and $\tilde{\mathbf{V}}_J$ each of dimension N. The vector $\tilde{\mathbf{W}}_j$ contains the so called wavelet coefficients and is associated with changes in $\mathbf{X}$ on a scale of $\lambda_j \Delta t$, with $\lambda_j = 2^{j-1}$, while $\tilde{\mathbf{V}}_j$ are called scaling coefficients and are associated with variations at scales $\lambda_{J+1}\Delta t$. The wavelet and scaling coefficients are associated with zero phase filters, which improves alignment with the original series.

MODWT yields an energy decomposition

$$||\mathbf{X}||^2 = \sum_{j=1}^{J} ||\tilde{\mathbf{W}}_j||^2 + ||\tilde{\mathbf{V}}_J||^2, \tag{1}$$

where $||.||$ is the l^2–norm, and an additive decomposition called multiresolution analysis (MRA)

$$\mathbf{X} = \sum_{j=1}^{J} \tilde{\mathcal{D}}_j + \tilde{\mathcal{S}}_J, \tag{2}$$

where $\tilde{\mathcal{D}}_j$ and $\tilde{\mathcal{S}}_J$ are the j–th order *detail* and the J–th order *smooth* for $\mathbf{X}$. Each wavelet and scaling coefficient are obtained by

$$\tilde{W}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \bmod N} \quad \text{and} \quad \tilde{V}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \bmod N} \tag{3}$$

with $t = 0, \ldots, N-1$. The coefficients $\{\tilde{h}_{j,t}\}$ and $\{\tilde{g}_{j,t}\}$ are called wavelet and scaling filters with width $L_j = (2^j - 1)(L-1) + 1$. For instance, for the Haar wavelet the wavelet coefficients are $\tilde{h}_{1,0} = 0.5$ and $\tilde{h}_{1,1} = -0.5$ and scaling coefficients $\tilde{g}_{1,0} = 0.5$ and $\tilde{g}_{1,1} = 0.5$.

In matrix notation, the transform from $\mathbf{X}$ to $\tilde{\mathbf{W}}_j$ and from $\mathbf{X}$ to $\tilde{\mathbf{V}}_j$ can be expressed as

$$\tilde{\mathbf{W}}_j = \tilde{\mathcal{W}}_j \mathbf{X} \quad \text{and} \quad \tilde{\mathbf{V}}_j = \tilde{\mathcal{V}}_j \mathbf{X}, \tag{4}$$

where each row of the $N \times N$ matrix $\tilde{\mathcal{W}}_j$ has a value of $\{\tilde{h}^{\circ}_{j,t}\}$ and $\tilde{\mathcal{V}}_j$ of $\{\tilde{g}^{\circ}_{j,t}\}$, where filters $\{\tilde{h}^{\circ}_{j,t}\}$ and $\{\tilde{g}^{\circ}_{j,t}\}$ are $\{\tilde{h}_{j,t}\}$ and $\{\tilde{g}_{j,t}\}$ periodized to length N, respectively. The MRA is obtained, therefore, as

$$\tilde{\mathcal{D}}_j \equiv \tilde{\mathcal{W}}_j^T \tilde{\mathbf{W}}_j \quad \text{and} \quad \tilde{\mathcal{S}}_j \equiv \tilde{\mathcal{V}}_j^T \tilde{\mathbf{V}}_j, \tag{5}$$

so that we can write in analogy to equations (3)

$$\tilde{\mathcal{D}}_{j,t} = \sum_{l=0}^{N-1} \tilde{h}^{\circ}_{j,l} \tilde{\mathbf{W}}_{j,t-l \bmod N} \quad \text{and} \quad \tilde{\mathcal{S}}_{j,t} = \sum_{l=0}^{N-1} \tilde{g}^{\circ}_{j,l} \tilde{\mathbf{V}}_{j,t-l \bmod N}. \tag{6}$$

2.2 Estimation of the natural growth rate

2.2.1 Derivation of the natural rate of growth

Output in time t (Y_t) can be represented as follows:

$$Y_t \equiv \left(\frac{Y_t}{H_t}\right)\left(\frac{H_t}{N_t}\right)\left(\frac{N_t}{L_t}\right)(L_t) \equiv (r_t)(h_t)(e_t)(L_t) \tag{7}$$

where H_t, N_t, L_t represent hours worked, total employment, and total labour force, respectively. Therefore, $Y_t/H_t = r_t$, $H_t/N_t = h_t$ and $N_t/L_t = e_t$ represent labour productivity, hours worked per worker, and the employment rate, respectively.

Equation (7) can be expressed in growth rates as:

$$g_t = \widehat{r_t} + \widehat{h_t} + \widehat{e_t} + \widehat{l_t} \tag{8}$$

where g_t, $\widehat{r_t}$, $\widehat{h_t}$, $\widehat{e_t}$ and $\widehat{l_t}$ represent the rates of growth of actual output, labour productivity, hours worked per worker, the employment rate, and labour force, respectively.

Rearranging equation (8) yields:

$$\widehat{e_t} = g_t - \left(\widehat{r_t} + \widehat{h_t} + \widehat{l_t}\right) \tag{9}$$

On the other hand, equilibrium in the goods market requires:

$$Y_t^S = Y_t^D \tag{10}$$

where Y_t^S and Y_t^D represent the supply for goods and the demand for goods, respectively.

Therefore:

$$g_t^S = g_t^D \tag{11}$$

where g_t^S and g_t^D represent the rates of growth of the supply for goods and the demand for goods, respectively.

In reality, it is not possible to observe directly either g_t^S or g_t^D. However, from equation (9) it is possible to observe that, in order to maintain the e_t constant —so that $\widehat{e_t} = 0$, then $g_t = \widehat{r_t} + \widehat{h_t} + \widehat{l_t}$. In this sense, it is possible to assume that $g_t^D = g_t$ and that $g_t^S = \widehat{r_t} + \widehat{h_t} + \widehat{l_t}$. Thus, it is possible express equation (9) as:

$$\widehat{e_t} = g_t^D - g_t^S \tag{12}$$

Now, the unemployment rate in time t (u_t) can be represented as follows:

$$u_t \equiv 1 - \frac{N_t}{L_t} \equiv 1 - e_t \tag{13}$$

If we consider annual changes in equation (13), then:

$$\Delta u_t = -\Delta e_t = -e_{t-1}\left(\widehat{e_t}\right) \tag{14}$$

where Δu_t represents the change in the percentage level of the u_t and Δe_t represents the change in the percentage level of the e_t.[8]

Substituting equation (12) into equation (14) yields:

$$\Delta u_t = -e_{t-1}\left(g_t^D - g_t^S\right) \tag{15}$$

Therefore:

$$\Delta u_t = -e_{t-1}\left(g_t^D\right) + e_{t-1}\left(g_t^S\right) \tag{16}$$

$$e_{t-1}\left(g_t^D\right) = e_{t-1}\left(g_t^S\right) - \Delta u_t \tag{17}$$

$$g_t^D = g_t^S - \left(1/e_{t-1}\right)\left(\Delta u_t\right) \tag{18}$$

$$g_t^D = g_t^S - \beta_1\left(\Delta u_t\right) \tag{19}$$

$$g_t = \beta_0 - \beta_1\left(\Delta u_t\right) + \varepsilon_{1,t} \tag{20}$$

Different studies (León–Ledesma and Thirlwall, 2002; Mendieta–Muñoz, 2017) have used the first difference version of Okun's law depicted in equation (20) as a statistical device for estimating Harrod's natural rate of growth: $g_t^D = g_t$; β_1 is the Okun coefficient on unemployment; $\varepsilon_{1,t}$ is the error term that satisfies the standard statistical properties; and $g_t^S = \beta_0 = \widehat{r_t} + \widehat{h_t} + \widehat{l_t}$ represents the natural rate of growth, that is, the sum of the rates of growth of labour productivity $(\widehat{r_t} + \widehat{h_t})$ and of the labour force $(\widehat{l_t})$ that are independent of aggregate demand fluctuations.

In other words, the natural rate of growth is associated with that rate of output growth (g_t) consistent with a stable unemployment rate: when the u_t is constant —that is to say, when $\Delta u_t = 0$, then output is growing at its potential or natural rate (g_n) since this estimate represents the minimum level of output growth needed to reduce u_t given labour force and labour productivity growth.

2.2.2 Estimating a time-varying natural rate of growth

We estimate the time-varying natural rate of growth by using a time-varying parameter model (TVPM) (Kim and Nelson, 1999), composed of the observed variables Δu_t and g_t, and of the unobserved parameters $\beta_{0,t}$ and $\beta_{1,t}$:

$$g_t = \beta_{0,t} - \beta_{1,t}(\Delta u_t) + e_{1,t}, \quad e_{1,t} \sim i.i.d.\mathrm{N}(0, \sigma_e^2) \tag{21}$$

$$\beta_{i,t} = \beta_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim i.i.d.\mathrm{N}(0, \sigma_{\varepsilon,i}^2), \quad i = 0, 1 \tag{22}$$

where $\beta_{1,t}$ represents the time-varying Okun coefficient on unemployment; and $\beta_{0,t}$ represents an estimate of a time-varying natural rate of growth.

[8] To see this, note that $\widehat{e_t} = \Delta e_t / e_{t-1}$. Thus, $\Delta e_t = e_{t-1}\left(\widehat{e_t}\right)$, and $-\Delta e_t = -e_{t-1}\left(\widehat{e_t}\right)$.

One problem with the estimation of the TVPM is that the regressor Δu_t may be correlated with $e_{1,t}$ in equation (21) since both output and unemployment are endogenous variables to a complex system. The estimation of equation (21) through the conventional Kalman filter via Maximum Likelihood (ML) cannot be performed because a successful application of the latter critically depends upon the assumption that the regressors are uncorrelated with the disturbance terms (Kim, 2006). In other words, the Kalman filter provides us with invalid inferences of the model if the regressors are endogenous.

In order to correct the possible endogeneity problem, we employ the Heckman-type two-step approach developed by Kim (2006), which allows us to obtain consistent estimates of both hyper-parameters and time-varying coefficients. We use Instrumental Variables (IVs) assuming that the relationship between the endogenous regressor Δu_t and the vector of IVs (z_t) is given by:

$$\Delta u_t = \delta(z_t) + \mu_t, \quad \mu_t \sim i.i.d.\mathrm{N}(0, \sigma_\mu^2) \tag{23}$$

where δ is a vector of constant parameters.[9]

It is possible to decompose Δu_t into its predicted component $(\mathrm{E}\left[\Delta u_t|\psi_{\mathbf{t-1}}\right])$ and its prediction error component (v_t):

$$\Delta u_t = \mathrm{E}\left[\Delta u_t|\psi_{\mathbf{t-1}}\right] + v_t \tag{24}$$

$$v_t = \sigma_v v_t^*, \quad v_t^* \sim i.i.d.\mathrm{N}(0,1) \tag{25}$$

where $\psi_{\mathbf{t-1}}$ denotes the available information in $t-1$; σ_v is the standard deviation of v_t; and v_t^* is the standardised prediction error of v_t.

If we denote the correlation between v_t and $e_{1,t}$ by the constant correlation coefficient ρ, the joint distribution of v_t^* and $e_{1,t}$ is the following:

$$\begin{bmatrix} v_t^* \\ e_{1,t} \end{bmatrix} \sim i.i.d.\mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_e \\ \rho^{-1}\sigma_e & \sigma_e^2 \end{bmatrix}\right) \tag{26}$$

Therefore, a Cholesky decomposition of the covariance matrix results leads us to decompose $e_{1,t}$ in equation (26) into:

$$e_{1,t} = \rho\sigma_e v_t^* + \omega_t^*, \quad \omega_t^* \sim i.i.d.\mathrm{N}(0, \sigma_\omega^2), \quad \sigma_\omega^2 = (1-\rho^2)\sigma_e^2 \tag{27}$$

Equation (27) shows the two components of $e_{1,t}$: the v_t^* component, which is correlated with Δu_t; and the ω_t^* component, which is not correlated with Δu_t. Substituting equation (27) into equation (21) results in:

$$g_t = \beta'_{0,t} - \beta'_{1,t}(\Delta u_t) + \rho\sigma_e v_t^* + \omega_t^* \tag{28}$$

[9] We also estimated equation (23) assuming that δ follows a random walk. However, it was not possible to find a global solution —that is, the solution went singular— when this specification was used. We also employed different starting values (using the OLS estimates as initial parameters); however, it was not possible to improve the results obtained using a time-varying specification for this equation.

where ω_t^* is not correlated with v_t^* or Δu_t; and both the new time-varying potential output growth rate ($g'_{n,t} = \beta'_{0,t}$) and the new time-varying Okun coefficient on unemployment ($\beta'_{1,t}$) can be generated assuming that the time-varying coefficients follow a random walk:

$$\beta'_{i,t} = \beta'_{i,t-1} + \varepsilon'_{i,t}, \quad \varepsilon'_{i,t} \sim i.i.d.\mathrm{N}(0, \sigma^{2'}_{\varepsilon,i}), \quad i = 0,1 \tag{29}$$

To summarise, the TVPMs with bias correction terms are estimated via ML in two steps:

1. Equation (23) is estimated through ML and the standardised one step-ahead forecast errors are obtained.
2. Equations (28) —which incorporates the standardised prediction errors, v_t^*, as bias correction terms— and (29) are estimated based on the prediction error decomposition and the Kalman filter.

2.2.3 Estimation results

We used annual data for the period 1950-2016 obtained from the Federal Reserve Economic Database of the St. Louis Fed (Fred). Besides the estimation of a time-varying natural rate of growth, we also estimated TVPMs using the rate of growth of GDP per capita as the dependent variable in order to generate a time-varying per capita natural rate of growth. Regarding the IVs employed for Δu_t, we used different combinations of the lags (up to two) of: Δu_t; the rate of growth of labour productivity measured as GDP per hour worked ($\widehat{r_t}$); and the rate of growth of hours worked per person employed ($\widehat{h_t}$). The series for r_t and h_t were extracted from the Total Economy Database (TED) of the Groningen Development Centre. The final combination of instruments ($\widehat{h_{t-1}}$ and $\widehat{h_{t-2}}$, as shown in Table 1) was selected because the latter satisfied the exogeneity condition (*i.e.*, instruments were uncorrelated with the error term) according to Hansen's J-statistic[10] and because the estimation of equation (23) using these instruments did not present misspecification problems. The first two lags of the rate of growth of hours worked per person employed ($\widehat{h_{t-1}}$ and $\widehat{h_{t-2}}$) can be considered as relevant instruments for the Δu_t since we expect that changes in labour market outcomes can be influenced by the number of hours worked in previous periods.[11]

In all cases we proceeded as follows. We first ran the Kalman filter in order to obtain the respective innovation variances and the initial values of the parameters to be estimated in the different equitations. In the subsequent step, the Kalman filter was run again with the preceding estimates of the innovation variances, the initial values of the parameters and their respective variance-covariance matrices in order to obtain the evolutionary coefficients of the models. Table 1 presents the estimates of the innovation variances for the different state-space models.

[10]Hansen's J-statistic is a test for over-identifying restrictions that is consistent in the presence of heteroskedasticity and autocorrelation (Hayashi, 2000). The p-value associated with Hansen's J-statistic was 0.92, so that the joint null hypothesis of Hansen's J-test (that is, the instruments are valid and the excluded instruments are correctly excluded from the estimated equation) was not rejected.

[11]The possibility of finding more relevant instruments for Δu_t is left for future research.

Firstly, following Kim and Nelson (1999), we corroborated the appropriateness of the specified models checking for the lack of serial correlation and of heteroskedasticity in the standardized one-period-ahead-forecast errors of the different estimations. The estimations of the different models do not present problems of serial correlation (up to order 3) or heteroskedasticity (up to order 2) at the 5% level, which suggests no evidence of model misspecification.[12]

Secondly, regarding the possible endogeneity problem of the regressor Δu_t, from Table 1 it is possible to observe that the estimated coefficient on the correction term bias, ρ, is statistically non-significant in all estimations. Hence, it is possible to ignore the endogeneity problem, and we considered equations (21) and (22) in order to retrieve the natural output growth rates estimates from the TVPMs.

Thirdly, as regards the estimates of the standard errors associated with the time-varying natural rates of growth ($\sigma_{\varepsilon,0}$), Table 1 shows that these are statistically significant at the 10% level. The standard errors associated with the time-varying Okun coefficients on unemployment ($\sigma_{\varepsilon,1}$) were found to be statistically non-significant. However, the Likelihood Ratio tests calculated assuming the respective models with constant parameters reject the null hypothesis of constant parameters at the 5% level. In this sense, the LR tests also suggest that it is preferable to consider the models with time-varying parameters instead of assuming constant parameters.[13]

The time-varying natural rate of growth and the time varying per capita natural rate of growth are presented in Figures 1 and 2, respectively. These figures also plot the respective actual growth rates, and the measure of potential output annual growth rate calculated by the Congressional Budget Office (CBO).[14] On the other hand, Figures 3 and 4 show the time-varying Okun coefficients obtained from the estimation of the natural and the per capita natural rates of growth. All figures show the smoothed estimates of the respective variables, together with their respective 90% confidence intervals.[15]

It is worth noting that: 1) the CBO's potential estimates lie within our estimated 90% confidence intervals during the period of study; and 2) the time-varying Okun coefficients estimates are fairly similar.[16]

[12]A detailed report is available on request.

[13]The estimation of the models assuming constant parameters and the respective LR tests are also available on request.

[14]To the best of our knowledge, there are no measures of potential output per capita growth, so that it is not possible to compare our results with other measures.

[15]Depending upon the information set used, it is possible to find the basic filter and smoothing filter. The former refers to an estimate of the time-varying coefficients based on information available up to time t; whereas the latter refers to an estimate of the time-varying coefficients based on all the available information in the sample through time T. The smoothed values provide a more accurate inference about the time-varying parameters (see Kim and Nelson (1999) and Kim (2006) for a description).

[16]It is also possible to say that the results obtained corroborate previous findings: using a penalized regression spline estimator for the period 1981–2011, Mendieta–Muñoz (2017) reports a reduction in the Okun coefficient in the US (from around -2.0% to around -1.4%) during the period 1981–2011; whereas Daly et al. (2014) use rolling regressions (40-quarter rolling window) during the period 1949Q1-2014Q1 for the US economy, finding a reduction in the Okun coefficient on unemployment from around -2.1% to around -1.9%.

2.3 Time series data

Here we provide an overview of the data series and sources. Key sources are the Bureau of Economic Analysis' National Income and Product Accounts (NIPA) and Fixed Assets Accounts (FAA), the Bureau of Labor Statistics (BLS), and Federal Reserve Economic Data (FRED).

- Real GDP growth is the q/q–growth rate of quarterly level of US real GDP at an annualized rate in chained 2009 USD. The source is FRED series GDPC1. We label the wavelet trend $g(Y)$.[17]

- Real potential GDP growth is the q/q–growth rate of quarterly level of US real potential GDP at an annualized rate in chained 2009 USD. The FRED series signifier is GDPPOT, which obtains the series from the Congressional Budget Office (CBO). The CBO estimates potential GDP based on factor supply growth and technological progress in a neoclassical production function framework. We label the wavelet trend $g(Y^*)$.

- The output gap is the ratio of the level of US real GDP (FRED series GDPC1) to said CBO estimate of the level of US real potential GDP (FRED series GDPPOT). Both are in chained 2009 USD. We label the wavelet trend Y/Y^*.

- Labor productivity growth is defined as the q/q–growth rate of real GDP per hour. We label the wavelet trend $g(Y/H)$. The data is from Groningen's growth project database.

- Labor force and population growth is the q/q–growth rate of the civilian labor force, which we obtained from FRED series CLF16OV. The wavelet trend is $g(L)$. We further consider growth rates of civilian non–institutional population P (FRED series CNP16OV), working–age population W (15-64 years old, FRED series LFWA64TTUSM647N), and total non–farm employment N (FRED series PAYEMS), and their respective trends.

- The annual natural growth rate of real GDP, $g_n(Y)$, and of real GDP per capita $g_n(Y/P)$, are based on time–varying coefficents regressions; see the preceding note and appendix 2.2 for details. (The quarterly series are interpolations of the annual estimates.) We compare the latter series to the wavelet trend of **real GDP per capita growth**, $g(Y/P)$, which is defined as the q/q–growth rate of real (chained 2009 USD) GDP per person, and based on FRED series A939RX0Q048SBEA.

- The employment rate is defined as the remainder to 1 of the quarterly civilian unemployment rate. The FRED series is UNRATE. The NAIRU is taken from the CBO's estimate of this rate; the FRED series code for the long term natural rate of unemployment is NROU.

- The income–capital ratio is calculated as the ratio of a nominal net income measure to a nominal net fixed asset measure at current (replacement) cost. The fixed asset estimates are available only on a annual basis. To construct a quarterly series, we interpolate the annual estimates such that the year–end observations coincide with fourth quarter values. We consider the ratio of corporate net value added (BEA/NIPA Table 1.14, line 3) to

[17] These series labels refer to the labels in the figures. Other sections use different notations, as of yet.

the interpolated series of corporate net fixed assets (BEA/FAA Table 6.1, line 2); and the ratio of national income (BEA/NIPA Table 1.7.5, line 16) to the interpolated series of private net fixed assets (BEA/FAA Table 6.1, line 1).

- The growth rate of the capital stock is defined as the ratio of net private domestic investment in billions of USD (BEA/NIPA Table 5.2.5. line 6) to net private fixed assets in billions of USD (BEA/FAA Table 6.1 line 1).
- The wage share is constructed from corporate sector data, and adjusted using data on the wage income distribution from Piketty and Saez (2003). The unadjusted wage share is calculated as the ratio of compensation of employees in the corporate sector relative to corporate net value added. It thus includes wages and salaries as well as supplements. The wage share is then adjusted by a quarterly interpolation of the annual series on the bottom 99% of the wage income distribution from Piketty and Saez (2003), so as to exclude the top sliver of wages and salaries—which are more akin to capital income. For a more detailed discussion, see Barrales and von Arnim (2017).

3 Discussion

In this section, we seek to provide a critical discussion of the data obtained from empirical analysis in the previous section, and place it in the context of our motivating considerations in section 1. This discussion is quite preliminary, and should be read as such. We begin here with an overview of key univarate time series.

3.1 The growth record: US post–WWII

Our empirical results indicate that the growth rate of US real GDP has, roughly, halved over the post–WWII era, from about 4% to about 2%. As already mentioned in the subsection on estimation results (2.2.3, p. 10), and shown in Figure 1, the *natural rate of growth*, estimated on the basis of Okun's Law with time–varying parameters, gradually declined from 1950 to 2016. Even if we exlude the crisis of 2008 and its aftermath, growth has declined considerably: the decadal average of this estimate pre–*Great Recession*—which includes the boom of the late–Nineties—represents only two–thirds of the decadal average beginning in the mid–Fifties. The ratio of the twenty year average 1996–2006 to 1950–1970 amounts to 0.55; that of 1980–2000 to 1950–1970 to 0.75.

The trends obtained from wavelet decompositions corroborate this finding. Figure 5 plots three trend growth rates of real GDP together: first, $g(Y)$ is the wavelet trend of real GDP, second, $g(Y^*)$ is the wavelet trend of the CBO's series of quarter–over–quarter growth of potential GDP, and $g_n(Y)$ is the estimate of natural growth (from Figure 1).[18] We obtain the trend of the CBO's potential rate of growth, because its quarterly series retains significant cyclicality; see as well Figure 1, which plots the corresponding annual series together with

[18]Recall from Section 2.1 that we define these wavelet trends as the *cyclical* components of the original time series with periodicity larger than 32 years.

the estimate of the natural rate of growth. As can be seen, the piecewise–linear regressions based on peak–to–peak growth trends of business cycles in the CBO's estimate of potential plays a crucial role. Smoothing this across cycles, by applying the wavelet decomposition, produces Figure 5.

Importantly, *all three series display the same trajectory.* One is based on a neoclassical production function, one on a time–varying intercept in Okun's Law, and one on a statistical procedure on the original series. All show the *gradual* decline from roughly 4% to roughly 2% over the entire post–WWII period. Especially the comparison of the two theoretically motivated estimates of natural or potential growth to the simple wavelet trend of realized GDP should be emphasized: what import do these estimates of the evolution of the supply side constraints on growth have *if they are indistinguishable from, in so many words, a moving average of actual demand*? The evolution of the capital stock shows similar dynamics. Figure 6 shows the growth rate of the capital stock for the private sector in the US over the post–war period, together with its wavelet trend. This trend as well displays a significant deterioration from the *Golden Age* to today, comparable to that of growth of real GDP.

Further, how do these results relate to growth of real GDP *per capita*? If growth of real GDP per capita remained steady across the sample period, one could infer that the slowdown in the rate of natural growth is due, simply, to the slowdown in the growth of the labor force, which is in turn driven by the decline in birth rates. To illustrate, consider first Figure 2. It presents the time–varying intercept of Okun's Law, with real GDP per capita as the dependent variable in the regression (see Section 2.2.3 for details). Further, Figure 7 shows that estimate together with the wavelet trend of the quarter–over–quarter growth of real GDP per capita. Clearly, both series show similar *declines* in this trend growth rate; the trend has not remained steady across the post–war era. The decline in the estimate of the natural rate of growth per capita appears roughly comparable to that of the natural rate of growth: the ratio of the twenty–year average 1996–2016 to the average 1950–1970 is only 0.52, compared to the 0.55 mentioned above. The ratio of the average 1980–2000 to that of 1950–1970 is 0.77, compared to 0.75 above. These results might indicate that (exogenous) changes in the rate of growth of the population do not play a significant role in the determination of natural growth.

It is difficult to pin down how these statistics on growth relate to an economy in its "normal" state. In heterodox macroeconomic growth theory, cycles and trend are often considered the same phenomenon. As mentioned above, Goodwin (1967) represents a foundational model in this literature, since it generates self–sustaining oscillations which incorporate growth itself. There is no trend independent of the cycle; see Velupillai (1998) for an appreciation of this line of thinking. Further seminal publications in this tradition include Skott (1989) and Flaschel (2009, 2015). The latter models generate limit cycles in stock–flow or ratio variables. These define a locally unstable steady state, around which the economy fluctuates perpetually. GDP and the capital stock, however, grow *on average* at a steady rate; and steady state growth satisfies Kaldor's stylized facts.

In the context of ongoing debates regarding stagnation, it might be justified to consider the relevance of the concept of a steady state. A growth model requires a constant income–capital ratio in steady state, as well as a constant profit rate, which implies constant distributive shares. We will return to these issues further below, when we move on to two dimensional narratives. For now, consider Figures 8 and 9. First, Figure 8 shows a precipitous decline in the income–capital ratios of the corporate and total private sectors in the US. *If* the full capacity income–capital ratio is constant, the realized income–capital ratio should as well be constant. The observed (private sector) income–capital ratio in the US, however, is clearly non–stationary.[19] Figure 9 shows related series: the output gap, a proxy for the employment rate and the CBO's estimate of the NAIRU. The output gap and the employment rate show—like the income–capital ratio—a decline over the post–war period, though it is less significant. Only the NAIRU is *steady* in the sense that it appears to show "long run mean reversion."

Figures 10 and 11 further complicate the picture. The *actual* employment rate—not based on the official measure of the civilian unemployment rate, but as the ratio of those employed to those in the labor force—shows a strong and sustained increase from the early Fifties to the late Nineties. This of course reflects the rise in the participation rate of women. The decline since is driven to a large extent by the decline in employment and participation of *men*. It is not immediately clear how to incorporate such shifts (and the underlying structural changes across sectors; i.e. from manufacturing to health care) in the stylized models referenced here.

However, Figure 12 points to a relevant question: how important is the *exogenous* decline in the working–age to total population ratio in the slowdown of growth? The figure clearly shows the tremendous increase in the labor force related to boomers being born and women entering the labor market. After the year 2000, however, the growth rate of the working–age population falls short of total population—reflecting beginning retirement of that generation.

Figure 13 sheds further light on the components of *natural growth*: labor force and labor productivity. The figure show the wavelet trends of growth of output per hour, growth of hours per employee, and growth of the labor force. *Assuming a steady employment rate*, these sum to the trend of growth of real GDP. First, note that the growth rate of hours per employee have shrunk until, roughly, the *Great Recession*, and now are near zero. The trend of the growth rate of the labor force is as in the previous figure. The trend of the growth rate of labor productivity—output per hour—shows the decline across the Seventies (the first productivity slowdown), followed by the IT–driven recovery, and the second slowdown—or, according to Gordon, the return to normal—since 2000. The figure illustrates that during the *Golden Age*, high but declining productivity growth combined with rising labor force growth to produce high natural growth; during the neoliberal era, productivity growth has been insufficient to make up for the shortfall of the continued decline in the growth labor force.

In summary, these measures suggest that something is rotten with the state of this economy.

[19]In the (neo–)Marxian literature, the ratio has been interpreted as a measure of "capital productivity." Its decline, if unchecked by rising profit shares, would drive a decline in the profit rate, and thus possibly trigger crisis and stagnation. See Basu and Vasudevan (2013) for a discussion. The relationship between the technical coefficient, i.e. the full capacity income–capital ratio, and realized output is similar to that of supply–side potential GDP and realized GDP, and we will thus leave it aside for the moment.

Growth has declined significantly, as has growth per capita. Income–capital ratios have declined across the post–war period, and employment and participation rates have declined from their peaks. Variables that are *constructed* to be stationary are of little help.

Lastly, we present here as well a measure of the wage share; see Figure 14. The measure includes supplements, but excludes the top 1% of the wage distribution, based on data from Piketty and Saez (2003). Thus, this wage share peaks in 1980, and the wavelet trend displays the rise and fall generally associated with narratives on secular profit squeeze and neoliberal recovery of profitability. See Barrales and von Arnim (2017) for a detailed discussion. The interaction between the wage share and growth, in turn, brings as back to the Goodwin model.

3.2 Goodwin, reconsidered

This section (very briefly, for now) presents key questions regarding the possibility of stagnation in a Goodwin–type framework. First, it should be noted that this question builds on the age–old research agenda regarding the dynamic interaction of growth and distribution. Second, however, the stylized picture drawn up in Goodwin (1967) matters greatly: regular counter–clockwise *business* cycles in measures of economic activity and the labor share occur across OECD countries.

The relevant issue here concerns the long run interaction. To set the stage, consider a comparison of *Golden Age* growth and distribution, and current state of affairs. The immediate post–war combination saw high growth, high income–capital ratios, high productivity growth, high employment rates, *and a rising wage share.* The current period, of course, is mired in stagnation, in combination with a *collapsed wage share.* Thus, two possibilities arise.

First, the short run profit–led/profit–squeeze cycles evolve around a shifting equilibrium. In other words, a variety of shocks or changes in exogenous variables alter the steady state of the model. This possibility is raised by Kiefer and Rada (2015). The authors estimate a time panel on OECD countries in output gap and labor shares, and obtain short run Goodwin cycles with a moving steady state; see Figure 17, reproduced from their paper. How does the steady state of the model move towards lower economic activity, and a lower wage share? Blecker (2017b) clearly delineates the relevant hypothesis: demand must be depressed, and lead to a move along the distributive curve. Alternatively, the distributive locus is shifted downwards, but *less so* than the demand locus.

Second, the short run profit–led/profit–squeeze cycles evolve around a long run cycle, which itself is of Goodwin–type. This possibility is raised by the longer run phase trajectories already discussed in the introduction. Figure 16, reproduced from Barrales and von Arnim (2017) shows the data. The figure combines 4–8 year cycles, obtained from a detail of the wavelet decompositions, and the trend, i.e. cycles with periodicity larger than 32 years. As can be seen, the sixty year Goodwin cycle "almost" completes; except that in the late Nineties, the wage share and economic activity should have begun rising together. Granger–causality tests on the wavelet decompositions indeed indicate that there is systematic interaction between growth and distribution at all frequencies. Still, it is not at all clear how one might further investigate such a sixty year cycle: for econometric methods to detect patterns, one needs more than one (incomplete) cycle.[20]

Both these possibilities raise similar questions. First, we clearly observe an increase in inequality and fall in the labor share. The reasons are generally seen in the relative gain of power by capital—through globalization, financialization, and technical change, or all of the above. One might hypothesize that these changes affect the distributive curve either by weakening the profit squeeze, or adversely affecting the "intercept." However, in (either a short or long) Goodwin cycle, these changes should lead to an acceleration of economic activity.

Except it hasn't: activity has declined more strongly than distribution. Which exogenous and endogenous factors affect growth? One would expect globalization, financialization and technical change to exert a positive effect on (potential) growth. Smithian specialization, availability of credit and the not–yet–matured IT revolution are all very real, principally positive forces on (potential) growth. Thus, we return to Blecker's hypothesis that it is conservative, neoliberal policy itself that depresses demand, and *shifting isoclines.*

An alternative hypothesis could be drawn up on the basis of explicit treatment of expectations. The Goodwin model's lower turning point is provided by the falling labor share. That, in the original supply–constrained version with a fixed income–capital ratio, triggers the increase in investment. However, in a demand–driven version—à la Flaschel or Taylor—the decline in real unit labor costs is only *one* argument; the other is, of course, the endogenous activity variable. Now, *if capitalists do not expect demand to rise* , the labor share could fall to zero, and investment would still not come forth. In other words: suppose capitalists observe a weakening profit squeeze, technological unemployment, and the increasingly precarious lives of their customers. Would they conclude that expansion of capacity is not warranted, even at falling labor costs?

A third hypothesis focuses on *induced technical change.* If technological progress is strongly driven by increases in real unit labor costs, labor suppression and the resulting fall in the labor share might be a causal factor in the weakening of labor productivity growth. Is it conceivable that long run growth is wage–constrained in this sense, along Marx–Verdoorn/Kennedy–Weizsäcker lines of thinking? (INCLUDE CITATIONS)

These questions need to be formulated more clearly. Our intention is to develop testable hypotheses along these lines, in order to ultimately provide insight on the role of demand and distribution in the slowdown of growth. Gordon's story just does not seem satisfactory.

[20]Even leaving the data limitation aside, conceptual problems arise. Are there two different models that drive the short and long run; or is it one model that operates at different frequencies? Would the low frequency (component of the) model evolve around a long run steady state?

4 Conclusions

In this paper, we have sought to offer a discussion of stagnation in the US from a heterodox perspective. Along a variety of dimensions, economic performance in the neoliberal era has been lackluster. Crucially, growth of real GDP has decreased from an average of 4% in the immediate post–war period to an average of 2% currently. This weakness has been accompanied by a decline in the wage share since 1980.

Our preliminary conclusion is that research on US (and global) economic stagnation should more carefully account for the detrimental effects of low growth of demand on productivity growth and labor force, as well as the interaction of activity with distribution.

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Table 1: Estimation of the hyper-parameters for the time-varying parameter models

Hyper-parameters	*Natural rate of growth*	*Per capita natural rate of growth*
Models without bias correction terms: equations (21) and (22)[a]		
$\sigma_{\varepsilon,0}$	0.266*	0.234*
	(0.150)	(0.120)
$\sigma_{\varepsilon,1}$	0.071	0.073
	(0.114)	(0.106)
σ_{e}	0.997***	1.000***
	(0.090)	(0.105)
L[b]	-120.239	-120.133
Instrumental variable estimation: equation (23)[a,c]		
σ_{μ}	1.000***	1.000***
	(0.065)	(0.065)
L[b]	-120.877	-120.877
Models with bias correction terms: equations (28) and (29)[a]		
$\sigma'_{\varepsilon,0}$	0.246*	0.229*
	(0.142)	(0.120)
$\sigma'_{\varepsilon,1}$	0.076	0.076
	(0.114)	(0.102)
$\sigma_{\omega*}$	0.938***	0.946***
	(0.083)	(0.090)
ρ	-0.418	-0.436
	(0.366)	(0.355)
L[b]	-113.305	-113.221

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood; [c]Instruments employed for Δu_t: h_{t-1} and h_{t-2}. Note that the same instrumental variable estimation was employed to estimate the natural and the per capita natural rates of growth with bias correction terms.
*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively.

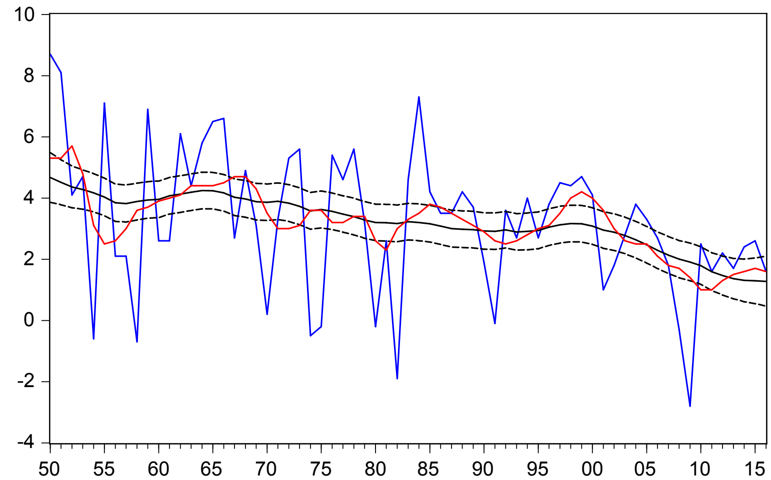

Figure 1: **Time-varying natural rate of growth; annual 1950–2016**. *The figure shows the estimated annual time-varying natural rate of growth (black straight lines) with 90% confidence interval (black dotted lines). The actual GDP growth rate (blue straight line) and the CBO's potential output growth rate (red straight line) are also plotted.*

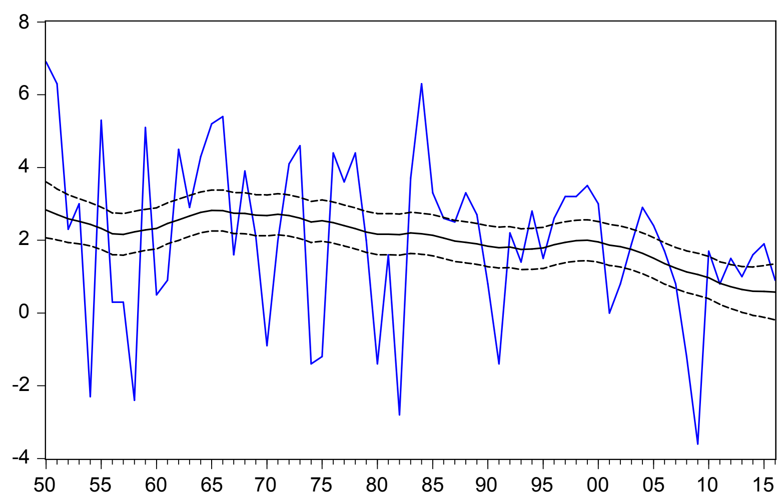

Figure 2: **Time-varying per capita natural rate of growth; annual 1950–2016**. *The figure shows the estimated annual time-varying per capita natural rate of growth (black straight lines) with 90% confidence interval (black dotted lines). The actual GDP per capita growth rate (blue straight line) is also plotted.*

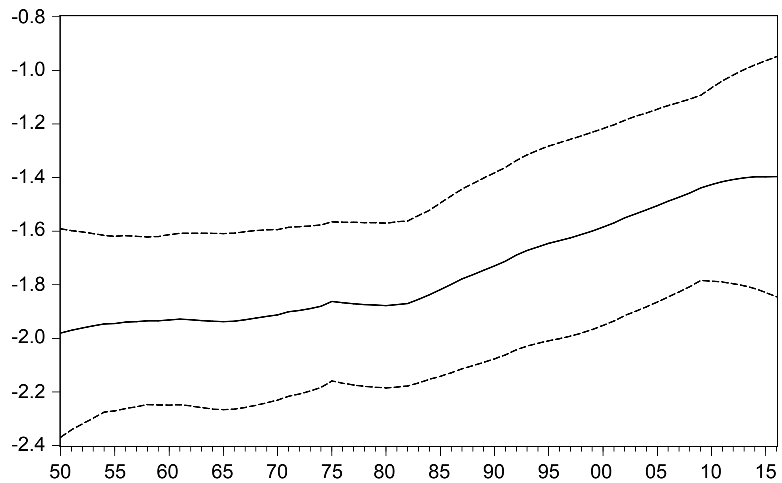

Figure 3: **First time-varying Okun coefficient on unemployment; annual 1950–2016**. *The figure shows the estimated time-varying Okun coefficient on unemployment (black straight lines) obtained from the estimation of the time-varying natural rate of growth with 90% confidence interval (black dotted lines).*

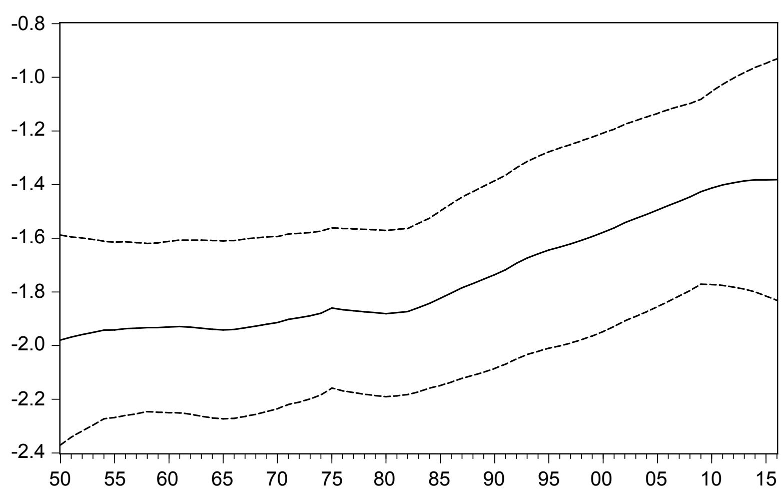

Figure 4: **Second time-varying Okun coefficient on unemployment; annual 1950–2016**. *The figure shows the estimated time-varying Okun coefficient on unemployment (black straight lines) obtained from the estimation of the time-varying per capita natural rate of growth with 90% confidence interval (black dotted lines).*

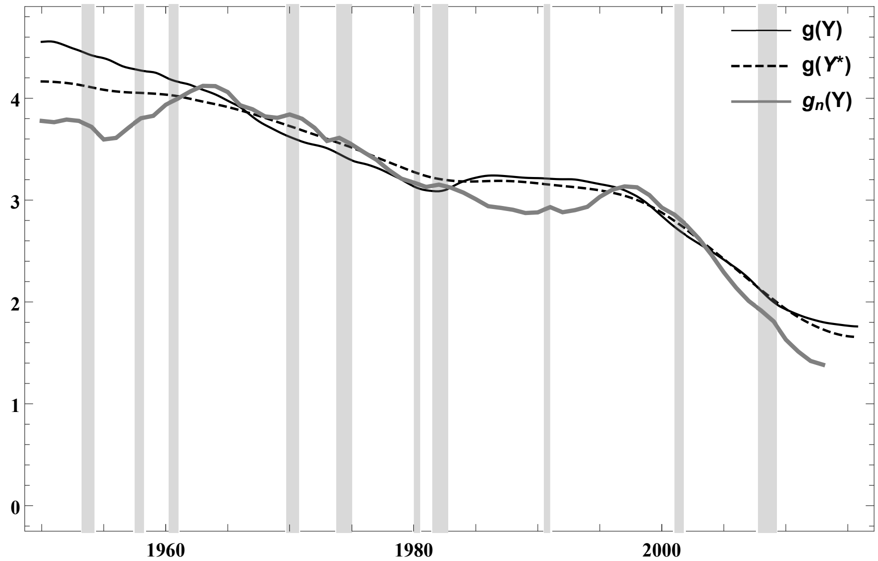

Figure 5: **Trend growth rates: US real output; quarterly, 1950–2015**. *The figure shows trends for three measures of real output growth. These are based on the wavelet trends of the growth rates of real GDP, $g(Y)$, of real potential GDP, $g(Y^*)$, as well as the estimate of the natural growth rate, $g_n(Y)$. See section 2.3 for further discussion on all series. Shaded areas indicate NBER recession dates.*

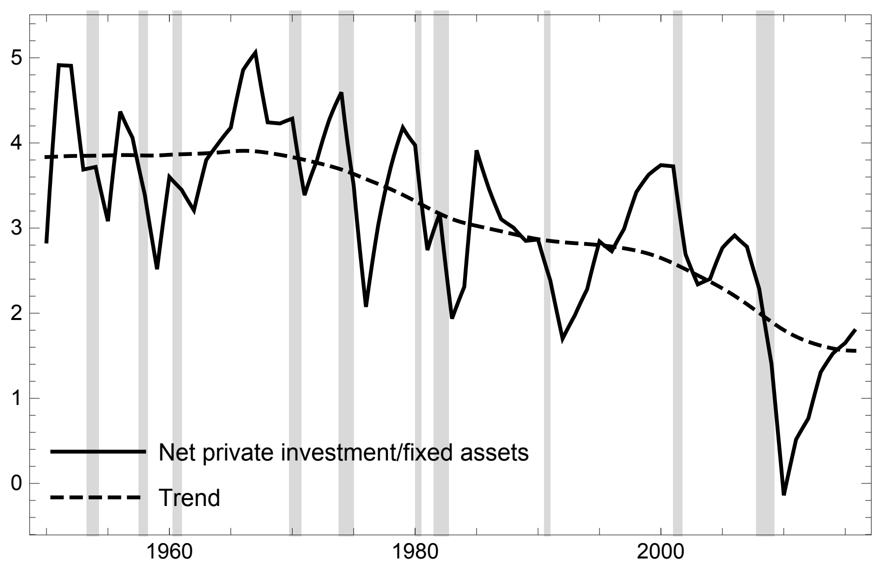

Figure 6: **Trend growth rates: US private capital stock; 1950–2015**. *The figure shows the growth rate of the US capital stock and its trend. The numerator of the series is net private domestic investment, the denominator net private fixed assets. See section 2.3 for further discussion on all series. Shaded areas indicate NBER recession dates.*

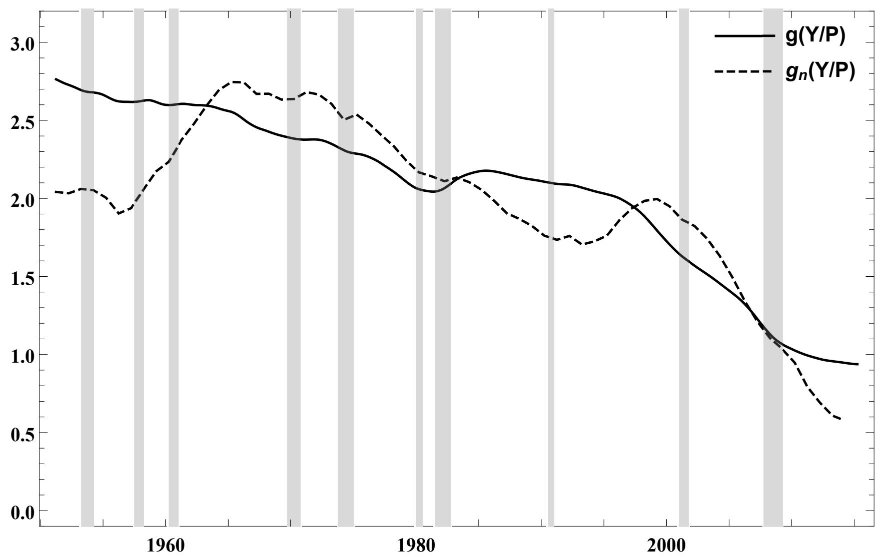

Figure 7: **Trend growth rates: US real GDP per capita, 1950–2015**. *The figure shows trends for two measures of real output growth. These are based on the wavelet trends of the growth rates of real GDP per capita, $g(Y/P)$, as well as the estimate of the natural growth rate, $g_n(Y/P)$. See section 2.3 for further discussion on all series. Shaded areas indicate NBER recession dates.*

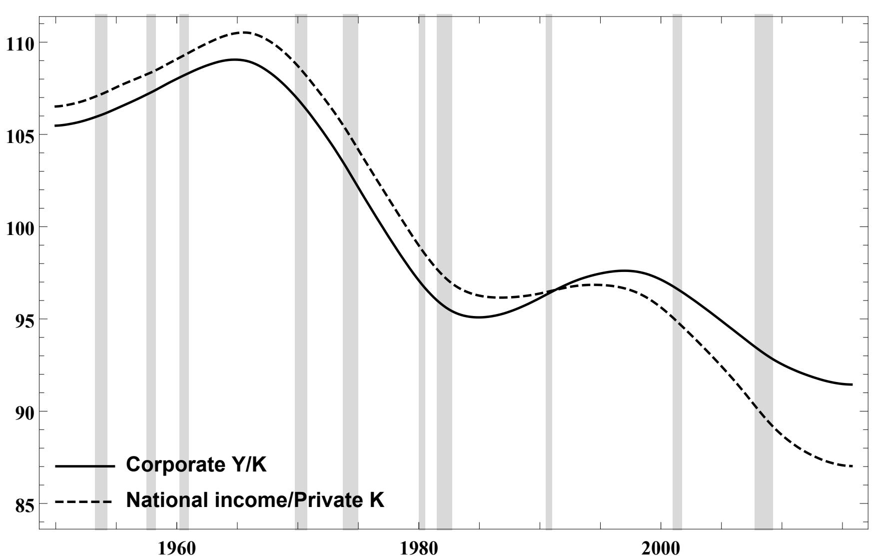

Figure 8: **Income–capital ratios; quarterly, 1950–2015**. *The figure shows wavelet trends for two measures of the US income–capital ratio. The solid line shows the wavelet trend of corporate net value added relative to the wavelet trend of current–cost corporate net fixed assets. The dashed line shows the wavelet trend of national income relative to the wavelet trend of current–cost private net fixed assets. Both series are normalized by their sample mean. See section 2.3 for further details and discussion on the series. Shaded areas indicate NBER recession dates.*

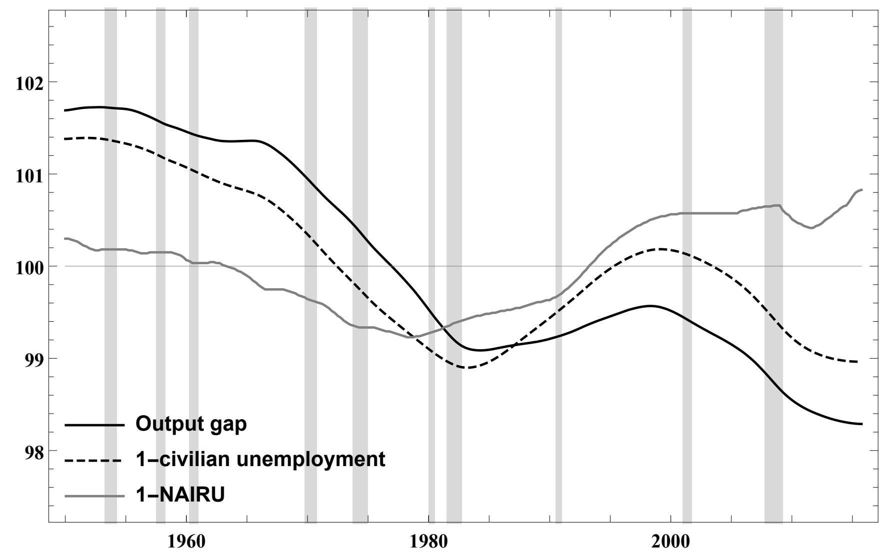

Figure 9: **Output gap, employment rate, and NAIRU; quarterly, 1950–2015**. *The figure shows wavelet trends for the output gap, defined as the ratio of real GDP to real potential GDP, the employment rate, defined as the remainder to 1 of the civilian unemployment rate, and the remainder to 1 of the CBO's estimate of the NAIRU. All three series are normalized by their sample mean. See section 2.3 for further details on the series. Shaded areas indicate NBER recession dates.*

Figure 10: **Employment and participation, I**. *The figure shows wavelet trends of employment rate, participation rate and working–age–to–population ratio. All three series are normalized by their sample mean. See section 2.3 for further details on the series. Shaded areas indicate NBER recession dates.*

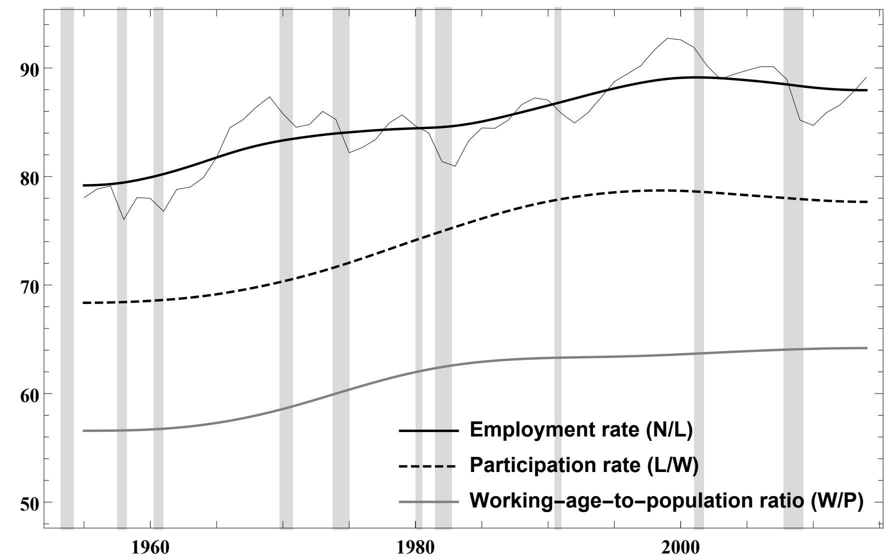

Figure 11: **Employment and participation, II**. *The figure shows the employment rate, AND wavelet trends of employment rate, participation rate and working–age–to–population ratio. See section 2.3 for further details on the series. Shaded areas indicate NBER recession dates.*

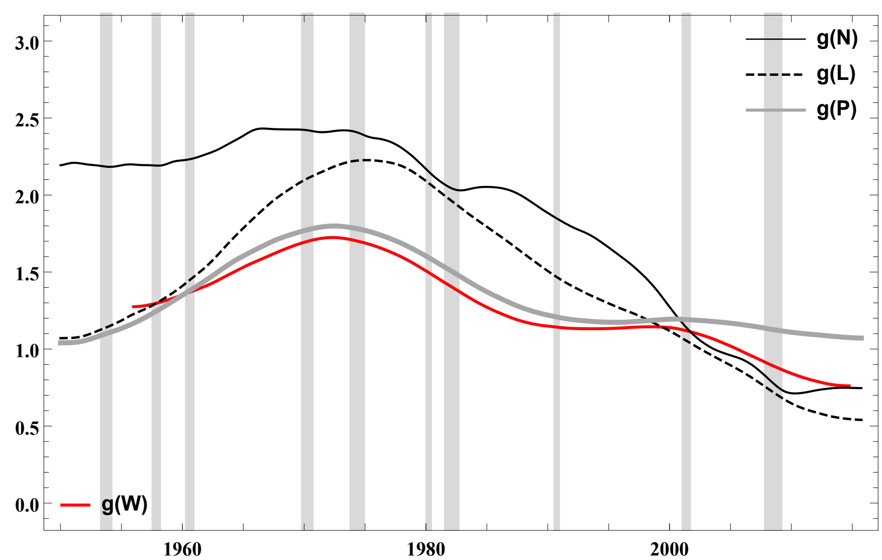

Figure 12: **Trend growth rates: US employment and labor force; quarterly, 1950–2015**. *The figure shows wavelet trends of the growth rates of population* $g(P)$*, working–age population* $g(W)$*, for the years 1956–2014, labor force* $g(L)$*, and employment* $g(N)$*. See section 2.3 for further details on the series. Shaded areas indicate NBER recession dates.*

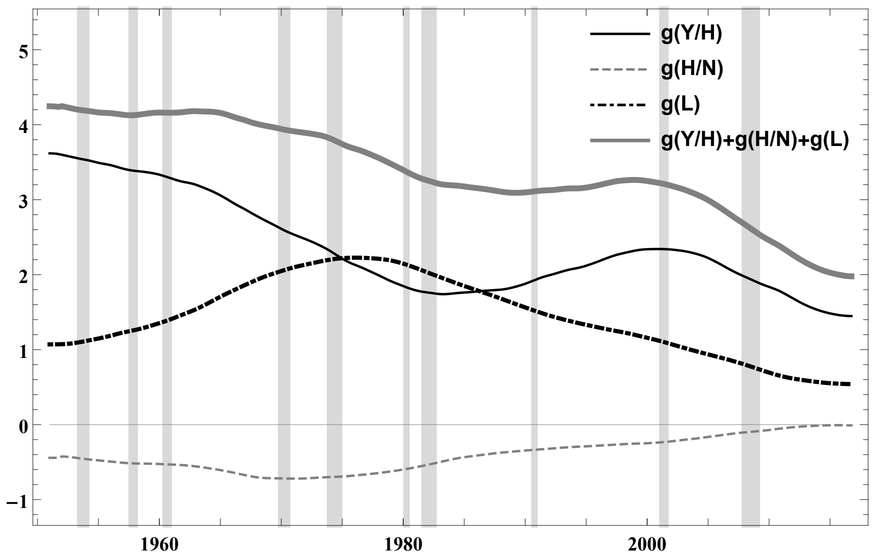

Figure 13: **Trend growth rates: natural growth, labor force and productivity**. *The figure shows wavelet trends for the components of the natural growth rates; growth of real GDP per hour $g(Y/H)$, growth of hours per employee $g(H/N)$, and growth of the labor force $g(L)$, as well as the sum of these items. Compare to Figure 5. See section 2.3 for further details on the series.*

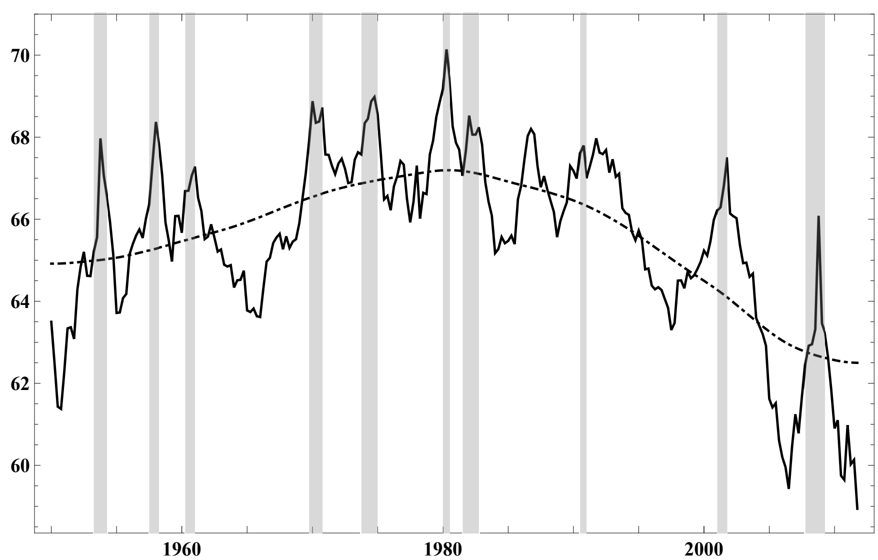

Figure 14: **Wage share; quarterly, 1950–2011**. *The figure shows the wage share and its wavelet trend. The wage share is adjusted to exclude the top 1% of the wage income distribution. See section 2.3 for further details on the series. Shaded areas indicate NBER recession dates.*

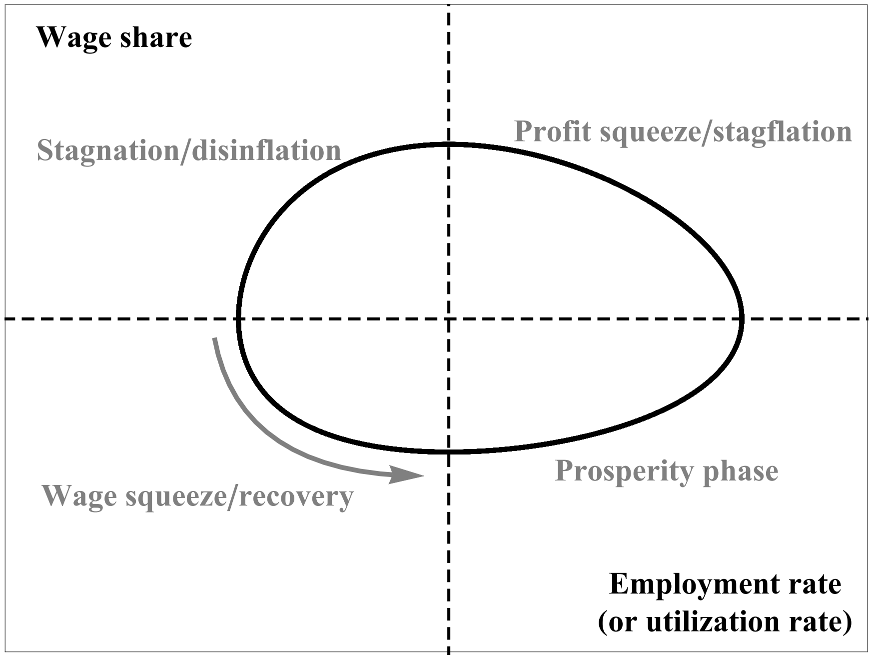

Figure 15: **A long Goodwin cycle.** *The figure gives a stylized picture of the Goodwin growth cycle (Goodwin, 1967). The labels are adapted from Flaschel (2009, 2015). See section 3.2 for further discussion.*

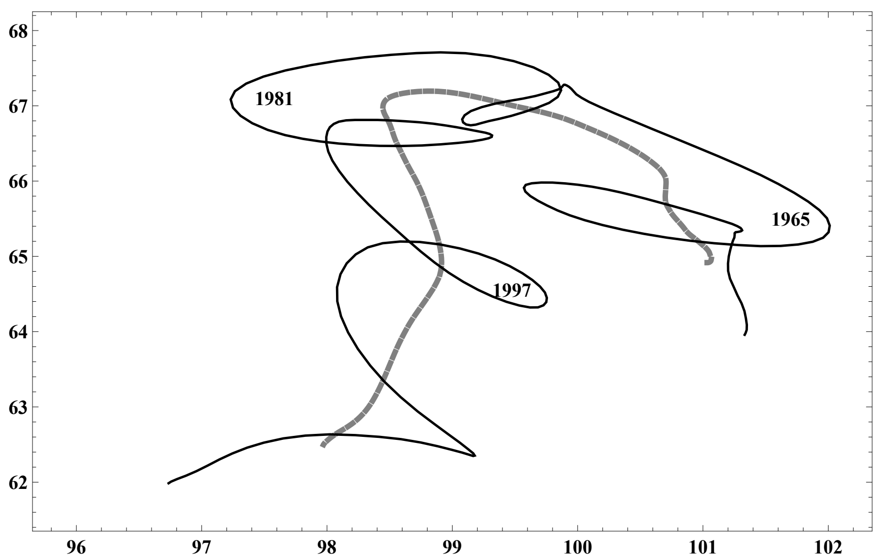

Figure 16: **A long US Goodwin trajectory in output gap and wage share; quarterly 1950–2011**. *The figure shows long run wavelet trends (dashed gray line, with periodicity greater than 32 years) and business cycles (solid black line, with periodicity between 4–8 years) together in an empirical phase diagram. The trend for the output gap is shown in Figure 9; the trend for the wage share is shown in Figure 14. The figure reproduces the left panel of Figure 10 in Barrales and von Arnim (2017), p. 214.*

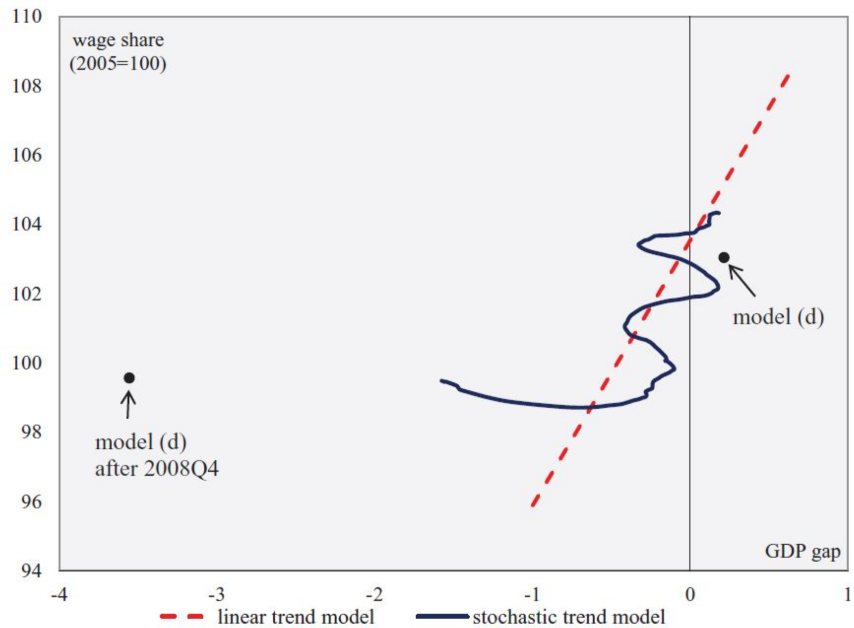

Figure 17: **Kiefer/Rada CJE, Figure 6**