Effects of Wealth Distribution on Economic Activity

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Abstract

Discussions of wealth inequality and his effects on economy activity have became more frequently, since Piketty (2014). However, wealth and wealth distribution has been often marginalized in the economic literature. To take this into account, this paper develops a theoretical three classes model, two conflicting wages and two with a propensity to consume less than one. The two richest classes allocate their wealth in money and public bonds, which generates financial income. Besides, our model also includes government sector taxing wages, profits, public bonds income and wealth. Then, this work explores, in a neo-Kaleckian Stock Flow Consistent framework, effects in income distribution, taxes, rate of interest and government expenditure on wealth distribution as well as on output.

1 Introduction¹

The distribution of income and its relation to economic growth have been studied in economic science for far a long time. Classical economists such as Smith, Ricardo, and Marx provide the importance of the division of national income among land, capital, and labor holders. More recently, neoclassical growth models determine the wage and profits endogenously, since the first is determined by the marginal productivity of labor and the second is equal to the marginal productivity of capital. Besides, this literature derives the income distribution from parameters in the aggregate production function (Stockhammer et al, 2014). In theories in which output is demand-determined, the first models were formulated by Keynes (1936) Kalecki (1942), Steindl (1952), Kaldor (1955-56). For these authors, the income distribution was determinated by the degree of market concentration and by the strength of labor unions. Later the so-called Neo-Kaleckian models were developed by Rowthorn (1982), Dutt (1984), Taylor (1985) and Bhaduri and Marglin (1990), in which functional distribution of income impacts on economic growth.

Even though income distribution has been a relevant topic in economic models of the last 80 years, wealth distribution has often been marginalized in the literature. Despite this topic has been worked on in Banerjee and Newman (1991), Galor and Zeira (1993), Benabou (1996), Aghion and Bolton (1997), in neoclassical view, and later by Barba and Pivetti (2009) and Palley (2010, 2102), by post-Keynesian literature, the importance of the quest has only became central to the scholarly debate after the release of Thomas Piketty (2014).

The first had studied the effects of human wealth distribution on the investment and its consequences to economic growth. Palley (2010,2012) explored the relation between wealth concentration of capital stock

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with capacity utilization, growth and income inequality. However, Piketty, Garbiniti, and Goupille-Lebret (2016) highlighted the importance of others ways to think wealth. In France, The financial assets represents 35% of total wealth, a rising of 75% compared to the 70s. Not to mention the housing asset that reached almost 50% of the wealth, an increasing of 70% in forty years. Moreover, 40% of the 0,1% richest wealth come from financial assets and 60% of the wealth between the fourth and nine deciles are housing assets.

In addition, Piketty and Zucman (2015) stressed on the concentration of wealth evolution in three developed countries: France, UK and USA. In France, although the value is still far from its peak, the first percentile of income has approximately 25% of total wealth, while the richest 10% possess more than 60% of it. This value is too high, especially when compared with income inequality, since this same decile appropriates 'only' 30% of the labor income. In the UK, as in France, the proportion of the richest's wealth to bottom's wealth began to drop considerably after the First World War. In the first years of the last century the first decil owned more than 90% of total wealth, but in the 1970s its possessed around 60%. However, the proportion rose again in the 1980s, reaching 80%. In the USA this proportion was considerably smaller than in Europe in the beginning of the last century. However, throughout the 60s, the proportion of richest decil of income in total wealth surpasses the same proportion in Europe and nowadays the richest own more than 80% of total wealth.

In Capital in the Twenty-First Century (2014), Piketty argues that this has happened because of changes in taxation and of the financial sector rise, initiated in the 80s, that surge the income out of capital. For this reason, the income of stocks became higher than the economic growth. As estimated by Piketty(2016), the flows that return to the top percentile are almost 6% annually, undoubtedly higher than the increasing of wages. These circumstance generates a vicious circle of more wealth and income inequality. Besides that, this inequality would impact directly the economic activity, and would be one of the sources of recent instabilities and problems of economic recovery in developed countries. (Rajan, 2010; Onaran, 2011; Palley, 2012; Stiglitz, 2012).

In addition to this question, financial sector has become an important part of economics in recent years. According to Krippner (2005), the proportion of the financial sector in the US national product was 15% in 1960 and went up to approximately 23% in 2001, surpassing manufacturing. Futhermore, the proportion of financial profit in the industry as a whole rose from 20% in 1980 to over 40% in the early 2000s.

In Brazil, financial wealth in 2015 was around 35% of GDP, according to data from Anbima. This represents a 9% growth, when compared to the previous year. However, this wealth was highly concentrated. About a hundred thousand people had a per capita wealth of approximately U\$ 1,5 million and five million people held per capta U\$ 29 thousand in financial wealth. The remaining sixty-six million households had, per capta, a little more than U\$3 thousand.

Nevertheless, the proportion of financial sector in emerging economies is still smaller than in developed countries (Karwowski and Stockhammer, 2016). Therefore, the potential and expected rise of this sector could have an impact on these economies in many ways. If credit was the same in BRICS, Turkey, Indonesia and Mexico as it is in developed countries, the loans would surge more than U\$ 9 trillion (PwC, 2014). This huge increase of credit could impact the investment and consumption (Barba and Pivetti, 2009; Stockhammer, 2009).

This paper will develop a neo-Kaleckian stock-flow-consistent (SFC) model which incorporates wealth and wealth distribution as well as it's allocation as crucial economic fators. Certainly, the model will not integrate all possible contributions that could be done, focusing on consumption and on financial sector. In

doing so, this article proceeds as follow: in section 2, we will document the brief summary of kaleckian and SFC literature. Moreover, some important works which also study wealth distribution developed will be discussed. In section 3, we will develop our model's framework. In section 4 and 5 the short-run and the long-run solutions, respectively, will be discussed. At last, in section 6 will be discussed the results of the previous section, and some future works and extensions are suggested.

2 Review of Literature

2.1 The Kaleckian and SFC literatures

Kalecki (1942) analyzed the effects of the functional distribution of income on the economic activity. The author believed that workers' propensity to consume is higher than capitalists'¹. For this reason, any increase in wage-share rises the household consumption. Assuming that firms sell whatever goods or services are demanded by consumers (Godley and Lavoie, 2007), Kalecki (1944) believed, therefore, that the redistribution of income could be one way for achieving full employment.

In spite of the absence of one investment theory in Kalecki, the neo-Kaleckian² function of investment was developed by Rowthrotn (1981), Dutt (1984) and Taylor (1985). According to these authors, investment is a positive function of profit rate and capacity utilization, which represents the economic activity level. According to Steindl (1952) the firms have a target of capacity utilization, less than one, in order to be prepared to any change in the demand and because of the indivisibility of plants. Therefore, if utilization is above than the target, the firms will increase the investment to rise the stock of capital.

Later, Badhuri and Marglin (1990) introduced the share of profits in the investment function³. In accordance with them, it is possible to have a constant profit rate, combining rising in the capacity of utilization and decreasing in the profit share. However, the fall in the profit share ought to have a negative impact on investment. In doing so, on one hand, the rise in proportion of wages in total income increases the consumption and, consequently the demand. On the other hand, the fall in the profit share decreases investment ⁴, dropping the demand. If the first is stronger than the latter the economy is wage-led demand. In contrast, an economy that investment effect is higher than a consumption effect is profit-led demand. Besides that, whenever the economy is profit-led demand the growth will be profit-led. But, even if demand is wage-led the growth can be profit-led or wage-led.

The Stock Flow Consistent (SFC) assumptions began to be developed by Copeland (1949) and Denizet (1967). The first was a pioneer in studies composing the real and financial flows of the economy, whereas the latter proposed the transactions flow matrix (Lavoie, 2011). However, according to Caverzasi and Godin (2014), the controversy of whom developed the first SFC models lies in James Tobin and Winne Godley. Tobin (1980) developed an empirical model of USA economy containing both financial and non-financial sides based on Copeland assumptions and Godley, in the 80s, started to develop models relating flows and stocks, as in Godley and Cripps(1983) and Godley and Zezza (1989).

¹Actually, in the first Kalecki's models, the workers' propensity to consume is equal to unity. In other words, the workers consume exactly their wages.

²According to Lavoite 2015, the neo-Kaleckians are one of the currents of Post-Keynesianism

³Lavoie(2015) calls this a post-Kaleckian investment function

⁴Actually, the neo-Kaleckians model could happen the same effect, but Blecker(2002) calls this term as "strong profit share effect'. It happens because in the neo-Kaleckians models, the effect in profit share must be highly strong, while the effect in post-Kaleckians models might be lower.

Caverzasi and Godin (2014) believe that the tradition descending from Godley is a specific variation of post-Keynesian models, trying to integrate the flows and stocks on all sectors of economy. Although almost all macroeconomic model is stock flow consistent, as Solow (1956), according to Dos Santos and Zezza (2008) the literature SFC has a close and natural relationship across periods. For this reason, the SFC model has procedures capable to take into account properly many variables, flows and stocks. In accordance with Macedo e Silva and Dos Santos (2011), this approach is related to Keynes (1936) and Kalecki (1971).

2.2 Post-Keynesian works with wealth distribution

According to Kalecki (1977[1954]), workers have a propensity to consume equal to the unity. Then, although in the short run it is possible that workers own some wealth, in the long-run the total wealth will belong to capitalists. In other words, if the workers do not have a positive propensity to save, the study of personal distribution of wealth does not make any sense. Then, it is necessary to change this assumption in order to study the wealth concentration. Palley (2010,2012,2016) and Dutt (2015) maintained a model with two classes, but workers save a proportion of their income. In other way, Palley (2014) introduces a third class of workers, the managers, that have a positive propensity to save. In first option, all classes conflict for the wealth distribution, while the second option just the two richest classes own wealth. Then, this subsection will present three models which some worker has a positive propensity to save, as well as the effect of variables exogenous in each model. At the end, a fourth model that does not take into account wealth distribution and even so = fundamental to our paper will also be presented.

Palley (2012) developed a neo-Kaleckian model with two classes, without government and international market. The workers earn wages and profits, while capitalists just earn profits⁵. In this economy all wealth is capital stock, then the proportion of the profits that each class receives from firms is the portion of capital-stock-wealth that each class owns. Besides , both classes have propensities to consume out of income and out of wealth, and workers' propensities are higher, considering the Kaleckian assumptions.

In the short-run, an increase in the share of capital stock by workers has two positive effects on aggregate demand: First, the consume out of income rises, because the income of workers increases. Second, the consume out of wealth also rises. Additionally, higher propensities to consume also increase the demand, i.e, the paradox of thrift holds. As Badhuri e Marglin (1990), rises in wage-share can rise or drop the demand.

The long-run is determined when the share of wealth held by workers is constant. If the economy is wage-led demand, any increase of profit share will decrease the demand, the capitalists' ownership share and the profit rate. When the economy is profit-led demand, the reverse occurs. In addition, an increase of capitalists' propensities to consume drops the demand and their ownership. Finally, an increase of workers' propensities drops their ownership, but the utilization does not change, in accordance with Pasinetti (1962).

Palley (2016) changes the model in three ways: first, the investment function is a match up of both neo-kaleckian and post-kaleckian investment functions. Second, both classes conflict wages and profits, i.e. the capitalists earn a portion of the wages as as managers. Third, the wealth just impacts the consumption indirectly, i.e, households' propensity to consume out of wealth is equal to zero. In the short-run, an increasing of the capitalists' wage bill drops the demand, since their propensity to consume is lower than workers'. The other results are the same as the Palley (2012).

In the long-run the outcomes are different. A rise of capitalists' propensity to consume increases the

⁵In the same article, Palley extends the model with managerial pay for capitalists

utilization and it is unknown what happens with their ownership share. A rise of workers' propensity to consume increases the utilization but their wealth share falls, an opposite of Pasinetti (1962) theorem. In other words, workers are able to rise their portion of wealth, but it comes with falls in economic activity. A surge in capitalists' wage bill decreases the utilization and rises the wealth concentration in their hands. Finally, an increase in the proportion of income profits has an ambiguous affect. On one hand, if the economy is profit-led, the capacity utilization increases. On the other hand, if the economy is wage-led, the capacity contracts. The outcome on wealth distribution depends on others variables.

Dutt(2015) developed a model with two classes: the top income recipients and the rest. Both earn from wages and some flows of capital. The rest receives income from property at the exogenously interest rate, while the top receives capital income in two ways: dividends from the profits and, as financiers, lending to firms with a higher interest rate. This second way captures the assumption that the top knows the bests ways to invest. Moreover, the investment function in short-run is given and it is determined by past decisions. In the long-run, the firms have a desired investment, a positive function of net profit⁶ rate, the fraction of the profits retained and capacity utilization. Then, the investment is determined by this rate and by deviations of past investment. In other words, the firms have a function that makes the investment move to its desired investment.

In the short-run, a rise in investment and in rests' share of capital increase the utilization. An increase of interest rate of the loans, a reduction of the fraction of profits retained by firms and the rise of wage-share also increase the utilization, since the investment function in short-run is given, i.e, the profit-led effect is not possible. Besides that, a surge of propensities to consume also increases the capacity utilization.

In the long-run, the rise of wage share increase or decrease the utilization, since the change in profit share increases the desired investment. In other words, in spite of the economy in short-run is wage-led demand, in the long-run the economy can be wage-led or profit-led demand. Besides that, the effect in wealth distribution is also ambiguous. It happens because the accumulation might increase more than the rise in rests' income, raising the top's capital further than the rests' capital. An increase in the fraction of profits retained and in the interest rate of loans have the same effect as the rise in wage-share, i.e, it is ambiguous.

Although Godley and Lavoie (2007) do not a model of wealth distribution, it will be considered as such, not only because their importance of SFC literature (Caverzasi and Godin,2014) but because the similarities with our model. The model developed in chapter 4 has one class, the households who earn income. The households consume a portion of this income, and allocate the savings in money and treasury bills⁷. It assumes that each bill has a price of one unity, and this price does not change. In other words, the importance of the bill for the households is an increase of the income, but it makes impossible capital gains. In doing so, the government issues these bills to finance its expenditure. Moreover, the households' wealth is the sum of the previous wealth plus the actual savings. Finally, the authors assume an economy of pure service, i.e, the model has no circulating and fixed capital and all the bills issued by the government that households do not want to hold, the central bank purchases.

In the short-run, any increase in the rate of interest or in the propensities to consume will rise the demand, since the former surges the income and the latter the consumption. Moreover, an increase of the tax rate drops the demand as result of the decrease in households' income. Moreover, a rise of government's expenditure increases the demand.

⁶The net profit is the output of economy, without the wages and the payments to the loans.

⁷Actually, before the households' decision of consumption and savings, the government taxes a portion of the income.

The long-run state occurs when the government balance is, in equilibrium, without issues of new bills and when the households do not save. Thus, the propensities to consume do not impact directly the households' consumption, since this is equal to disposable income. However, accumulation of wealth is a negative function of the propensities to consume, an increase of any of them will drop the consumption out of wealth. Then, even though the paradox of thrifts holds in short-run, it does not hold in long-run. In spite of that, the other variables have the same effect in the long-run as in the short run.

3 Accounting Framework

The accounting framework of this model is inspired in Palley (2012) and Godley and Lavoie (2007). Unlike the first, our model has three classes: two classes of workers and the capitalists. The workers are separated by their skills, and it impacts their wages, W_i . Actually, the wages are related with the productivity of the sector where he or she works⁸. In this model proposed in this paper, we will call low skilled worker and high skilled worker. This difference allows us to understand the limits of redistribution of wages in demand, as in income and wealth distribution.

Besides that, in our model, as in Dutt (2015), the capital stock is owned by capitalists. So, differently from Palley(2012), all redistributed profits are destined to capitalists. Likewise Godley and Lavoie (2007), our model introduces public bonds, B_i , which every time generated financial income by an interest rate, r. These bonds are shared by the class of high skilled workers and the capitalists. Then the wealth distribution is characterized by financial wealth, bonds and money, D_i .

In addition, the model considers a government sector that taxes wages, income of bonds, profits and wealth with the rates, τ_W , τ_B , τ_{π} and τ_V , respectively. The government, unlike Godley and Lavoie (2007), uses these taxes in order to decides its expenditure. Finally, the model has a simple international market.

The accounting framework of the model is given by the balance sheet and transactions matrices shown in tables 1 and 2. The subscripts l, h and c denote respectively the low skilled worker, the high skilled worker and the capitalist. For example, the second column in table 2 says that the low skilled workers, L_l , earn wages, W_l , and his all their disposable income is consumed, C_l . The fourth and sixth columns in table 2 indicate that high skilled workers, L_h and capitalists have two sources of income: both earn from their public bonds, but the first earns wages, W_h , and the latter earn profit, πY . These two classes save out of income, accumulating financial wealth each period, as it can see in table 1.

Table 1: Sheet Matrix

		Households				
Stocks	Low Skilled Worker	High Skilled Worker	Capitalists	Firms	Government Sector	Total
Money	0	ΔD_h	ΔD_c		- ΔD	0
Bonds	0	ΔB_h	ΔB_c		- ΔB	0
Capital				$K - \xi$		$K - \xi$
Total		V_h	V_c	$K - \xi$	- $(\Delta D + \Delta B)$	$K - \xi$

⁸Remark that here we are not talking about managerial class.

Table 2: Transactions Matrix											
Flow	Households Firms Government Sector Tot						Total				
	Low Skilled Worker High Skilled Worker			Capitalists							
	Flow	Capital	Flow	Capital	Flow	Capital	Flow	Capital	Flow	Capital	
Consumption	- C _l		- C _h		- C _c		+ C _s				0
Investment						+ I	- I				0
Government Expenditure							+ G		- G		0
Salaries	$+ W_l L_l$		$+ W_h L_h$				$-W_lL_l - W_hL_h$				0
Profits					$(1 - \delta)\pi Y$		$-(1 - \delta)\pi Y$	$\delta \pi Y$			0
Interest			rB_h		rB_c				$-r(B_h + B_c)$		0
Taxes	- $W_l L_l \tau_W$		$-\tau_W W_h L_h$ - $\tau_B r B_h$ - $\tau_V h$		- $\tau_{\pi}(1 - \delta)\pi Y$ - $\tau_{B})rB_{c}$ - $\tau_{V}V_{c}$				$W_l L_l \tau_W + \tau_W W_h L_h \tau_{\pi} (1 - \delta) \pi + \tau_B (B_h + B_c) \tau_V (V_h + V_c)$		0
Money			$-\Delta D_h$	$+\Delta D_h$	$-\Delta D_c$	$+\Delta D_c$			ΔD		0

4 The Model

4.1 Firms

$$I = \gamma_0 K + \delta \gamma_1 \pi Y + \gamma_2 Y \tag{1}$$

As in Rowthorn (1981) and Dutt (1984), the investment is a positive function of the capital stock, the profit rate, and the output. However, the stock capital depreciates by ξ rate.

Then,

$$I = \gamma_0 (1 - \varphi_B) V + \gamma_1 (1 - \psi_W) Y + \gamma_2 Y \tag{2}$$

So that, φ_B is the proportion of financial wealth, $V_h + V_c$, of the total wealth, V. Besides that, ψ_W is the wage share. Finally, $K = (1 - \varphi_B)V$ and $(1 - \psi_W) = \pi$.

Then, the growth is:

$$\frac{I}{K} = g = \gamma_0 + [\gamma_1(1 - \psi_W) + \gamma_2]u \tag{3}$$

So that, $u = \frac{Y}{K} e^{\frac{Y^{FC}}{K}} = 1$

4.2 Households

The income is conflicted between the workes, who receive wages, $Y_W = W_l L_l + W_h L_h$ and capitalists who earn profits, $Y_{\pi} = \pi Y$. Thus, $Y = Y_W + Y_{\pi}^{-9}$

4.2.1 Low Skilled Workers

$$Y_{Dl} = C_l = (1 - \tau_W)\psi_W\psi_L Y \tag{4}$$

So that, ψ_L is the fraction of low skilled workers wages in the total income out of wages, Y_W .

The low skilled worker receives income from wages and fully consumes it, compatible with the classical assumption. Nevertheless, part of the wage income is appropriated by the government through a tax rate, τ_W .

⁹According to Godley and Lavoie (2007), interest payments on government debt are transfer payments, not part of income.

4.2.2 High Skilled Workers

$$Y_{Dh} = (1 - \tau_W)(1 - \psi_L)\psi_W Y + r(1 - \tau_B)\psi_{Bh}B - \tau_V \varphi_B \varphi_{Bh}V$$
 (5)

So that, ψ_{Bh} is the proportion of total public bonds owned by high skilled workers. Moreover, φ_{Bh} is the proportion of the high skilled workers' financial wealth on the total financial wealth, i.e. $\varphi_{Bh} = \frac{V_h}{V_h + V_c}$.

$$C_h = \beta_h Y_{Dh} + \theta_h \varphi_B \varphi_{Bh} V \tag{6}$$

So that, $\beta_h + \theta_h < 1$.

$$\frac{B_h}{V_h} = (1 - \lambda_0) + \lambda_1 r - \lambda_2 \frac{Y_{Dh}}{V_h} \tag{7}$$

So that $\lambda_0 > 0$, $\lambda_1 > 0$, $\lambda_2 > 0$

The high skilled workers represent the workers who have a higher salary, that is, $W_2 > W_1$, because they are in more productive jobs. Due to this higher income, this class consumes part of the disposable income, i.e, they have propensity out of disposable income, β_h , less then one. Therefore, this class accumulates wealth over time, allocating it in public bonds B_2 , which yields interest, r, at each period, and in money, D_2 . The government taxes the income of wages and of public bonds. Aside from it, the government also taxes the financial wealth, V_h , decreasing the income disposable of high skilled workers. Finally, according to empirical studies of Case et al (2001), Catte et at (2004) and Giroud et al (2006), consumption decisions are sensible, θ_h , to personal wealth.

As in Godley and Lavoie (2007), the high skilled workers' wealth is the sum of the wealth accumulated in the previous period with current saving. The allocation of wealth, V_2 , in money, D, and in public bonds, B, is reviewed at each period and depends on the interest rate, r, of the worker's income and wealth, and on exogenous parameter.

4.2.3 Capitalists

$$Y_{Dc} = (1 - \tau_{\pi})(1 - \delta)(1 - \psi_W)Y + r(1 - \tau_B)(1 - \psi_{Bh}B - \tau_V\varphi_B(1 - \varphi_{Bh})V$$
(8)

So that, δ is the retained profits by firms.

$$C_c = \beta_c Y_{Dc} + \theta_c \varphi_B (1 - \varphi_{Bh}) V \tag{9}$$

$$B_c = B - B_h \tag{10}$$

So that, $\beta_c + \theta_c < 1$.

The capitalist has a higher income than high skilled worker, then, his propensity out of disposable income is less then the former class, i.e, $\beta_c < \beta_h < 1$. Therefore, this class accumulates wealth over time, allocating it in public bonds B_2 , which yields interest, r, at each period, and in money, D_2 . The government taxes the unretained profits and the income of public bonds as a tax rate T_{π} and T_{π} , respectively. Aside from it, the government also taxes the financial wealth, T_{π} , decreasing the income disposable of capitalists. In the same

way as in the high skilled workers, the size of capitalist's wealth has a psychological effects on his consume, represented by θ_c , which is lower than θ_h .

Capitalists will hold all bonds that the high skilled workers do not want to carry. Thus, the allocation of the capitalist's wealth in public bonds, B_c , is determined by the difference in the total of public bonds issued by the government and the choice of the allocation by the high skilled workers¹⁰.

4.3 Government

$$G + r(B_h + B_c) = T + \Delta B \tag{11}$$

$$G = \omega Y + T \tag{12}$$

$$G = \tau_W \psi_W Y + \tau_\pi (1 - \delta)(1 - \psi_W) + r \tau_B (B_h + B_c) + \tau_V (V_h + V_c) + \omega Y$$
(13)

Government's expenditure plus his payments in interest out of public bonds need to be compensated by taxed or by issued of new bonds. Besides that, the decision of the government expenditure is determined by an exogenous parameter, ω , which represents a target of a proportion of the fiscal deficit and the total income.

4.4 Trade Balance

The trade balance, X, is an exogenous variable.

5 Short-run solution

$$DA = Y = C + I + G = C_l + C_h + C_c + I + G + X$$
(14)

The short-run solution involves solving the output, Y, and the growth, g, supposing goods market equilibrium. Aside from the capital stock, K, and the total wealth, V, which are endogenous variables, the others variables are exogenous. Then the output and the growth are determined by fiscal deficit, ω , by the function distribution of income, ψ_W , by the proportion of the low skilled wages in the total wages, ψ_L , by the proportion of the high skilled worker's financial income in the total of financial income, ψ_{Bh} , by the proportion of financial wealth in the total wealth, φ_B , the distribution of financial wealth, φ_{Bh} , by the taxes rate of wages, τ_W , of profits, τ_{π} , of financial income, τ_B , of financial wealth, τ_V , by the propensities to consume out of disposable income, β_h and β_c , by the propensities to consume out of wealth, θ_h and θ_c , by the parameters in the investment function, γ_0 , γ_1 and γ_2 , by the proportion of retained profits, δ , the rate of interest, r, the public bonds issued, B, and by the net exports. The table 3 summarizes the effects of shocks on the exogenous variables in the output and growth.

¹⁰Therefore, differently than Godley and Lavoie (2007) our model do not have a central bank. In their model, the central bank purchases all bills issued by the government that the households do not want to hold. In our model, all the bonds are held by households high skilled workers or capitalists.

$$Y^* = f(\omega, \psi_W, \psi_L, \psi_{Bh}, \varphi_B, \varphi_{Bh}, \tau_W, \tau_B, \tau_\pi, \tau_V, \beta_h, \beta_c, \theta_h, \theta_c, \gamma_0, \gamma_1, \gamma_2, \delta, r, B, V.X)$$

$$\tag{15}$$

$$g^* = g(\omega, \psi_W, \psi_L, \psi_{Bh}, \varphi_B, \varphi_{Bh}, \tau_W, \tau_B, \tau_\pi, \tau_V, \beta_h, \beta_c, \theta_h, \theta_c, \gamma_0, \gamma_1, \gamma_2, \delta, r, B.X)$$
(16)

Table 3: Shocks in exogenous variables in output and growth

	Y	g
ω	+	+
ψ_W	+/-	+/-
ψ_L	+	+
ψ_{Bh}	+	+
φ_B	+/-	+/-
φ_{Bh}	+	+
$ au_W$	+	+
$ au_{\pi}$	+	+
$ au_B$	+	+
$ au_V$	+	+
β_h	+	+
β_c	+	+
$ heta_h$	+	+
θ_c	+	+
γ_0	+	+
γ_1	+	+
γ_2	+	+
δ	+	+
r	+	+

The short-run output and growth have positive dependence of propensities to consume out of disposable income, β_h and β_c , by propensities to consume out of wealth, θ_h and θ_c , then it holds the paradox of thrift. The distribution of income of wages, ψ_L , and of financial, ψ_{Bh} through to the poorer class has a positive impact on the output and growth, since these classes have a higher propensity to consume. In the same way, an increase in distribution of wealth, ϖ_{B2} , also increases output and growth. Regarding to the taxes, both effects are positive, because the decrease in the total consume is smaller than the increase in government expenditure.

However, the impact of changes on the function distribution of income and in the proportion of the financial wealth in the total wealth is ambiguous on output and growth. The increase in proportion of wages in total income rises consumption while drops investment. If the first is higher than the second, the impact on the output is positive. On the other hand, if the fall in investment is higher than the surge in consumption, the income will decrease. The impact on growth also depends on size of effects on consumption and investment. If the impact on the output is negative, the growth will decrease. Even if the impact on output is positive, the growth could rise or fall. This results in compatibles with Badhuri e Marglin (1990).

The increase of financial wealth in total wealth, φ_B , will rise the consumption and government expenditure. However, this effect will drop the investment, since the accumulation is a positive function of capital stock, K. Then, the result in output will be positive if the first were higher than the second. In the same way that changes in function distribution of income on growth have an ambiguous result, if the impact on output is positive, the impact on growth could be positive or negative. Although, if the impact on output is negative, the growth will fall.

The impact of the rise in retained profits is negative because it drops the consumption of capitalists. Finally, the surge in the interest rate increases the output and growth, as a result of a rise in disposable income of high skilled workers and of capitalists.

6 Long-run solution

$$Y_{Dl} = C_l (17)$$

$$Y_{Dh} = C_h (18)$$

$$Y_{Dc} = C_c \tag{19}$$

$$I - \xi K = 0 \tag{20}$$

$$G + r(B_h + B_c) = T \tag{21}$$

The long-run solution is solving the output, Y, supposing savings zero, equations (17), (18) and (19); net investment zero, equation (20); and zero public bonds issue, (21). The first occurs because, in transition, the wealth rises. This rises the consumption, decreasing savings. Since, the wealth is the sum of accumulated wealth in the previous period with current saving, when the savings become zero, the wealth becomes constant. The second supposes that an constant increase in capital stock, K, will rise the depreciation, then the investment needs to be equal to the depreciation, i.e, the growth of capital stock is null. Finally, the third is a condition on equilibrium of the debt in the long run. Since the output do not grow, if the government issues more bonds, the relation between debt and income will arise continuously, which is not possible.

Besides that, when the long-run is solved, it is possible to turn exogenously the distributional variables: the proportion of high skilled worker's financial income in total financial income, ψ_{Bh} , and the distribution of financial wealth, φ_{Bh} .

6.1 Distributional parameters

6.1.1 Proportion of public bonds owned by high skilled workers in the total bonds

$$\psi_{Bh} = \psi(\lambda_0, \lambda_1, \lambda_2, r, \beta_h, \theta_h, \psi_W, \psi_L, \tau_W, \tau_B, \tau_V, \omega) \tag{22}$$

The proportion of public bonds owned by high skilled workers in the total bonds depends on the disposable income of this class and the parameters of the equation (7). The table 4 summarizes the effects of shocks in the exogenous variables on ψ_{Bh} .

Table 4: Shocks in exogenous variables on the proportion of high skilled workers' bonds

	ψ_{Bh}
ω	+
ψ_W	+
ψ_L	-
$ au_W$	-
$ au_B$	-
$ au_V$	-
β_h	-
θ_h	-
r	+

As a result of high skilled workers' public bonds being a positive function of the disposable income, the increase on the proportion of wages on total income will rise the high skilled workers' ownership of public bonds. In the same way, any increase of the proportion of low wages in total wages, of wage tax, of financial income tax and of financial wealth tax, will drop the ψ_{Bh} . The rise of propensities to consume and of the λ_2 will drop this proportion too, since this changes will affect (7). However, rises in λ_1 and λ_0 , will increase this proportion. Finally, the rise in interest rate impacts positively both the disposable income and (7). Then, the proportion will rise.

In face of the government budget in equilibrium, without issued of bonds, the target of government deficit in long run must be negative. In other words, the government must tax more than expenditure in order to pay the interests. Then, an increase in ω indicates smaller surplus. Since the size of surplus indicates the quantity of bonds government has already issued, the rise of ω means less bonds in the market. Therefore, the proportion of high skilled workers' public bonds will surge.

6.1.2 Distribution of financial wealth

$$\varphi_{Bh} = \varphi(\lambda_0, \lambda_1, \lambda_2, r, \beta_h, \theta_h, \psi_W, \psi_L, \tau_W, \tau_B, \tau_V, \omega, \beta_c, \theta_c, \tau_{\pi}, \delta)$$
(23)

Differently from public bonds, the capitalist's wealth, V_c is not a function of high skilled workers' wealth, V_h . In view of the fact that wealth is an cumulative process savings, its long-run value is a negative function of the propensities to consume, β_h and θ_h to V_h , and β_c and θ_c to V_c . Moreover, likewise the public bonds, the wealth is a positive function of disposable income. The table 5 summarizes the effects of shocks in the exogenous variables on φ_{Bh} .

The results of shocks on wealth distribution must be correlated with the effects on financial income distribution, since the last impacts directly the disposable income. For this reason, an increase in the tax of wages, in the functional distribution of income, in the proportion of low wages on total wages and in the government deficit will effect in the same direction as income disposable. Nevertheless, an increase in the tax of financial income, in the tax of financial wealth, and in interest rate could have an ambiguous effect.

Table 5: Shocks in exogenous variables on the distribution of financial wealth

	φ_{Bh}
ω	-
ψ_W	+
ψ_L	-
$ au_W$	-
$ au_B$	+/-
$ au_V$	+/-
$ au_{\pi}$	+
β_h	-
θ_h	-
β_c	+
θ_c	+
r	+/-

On the one hand, the non ambiguous results occur thanks to the fact that the effect on high skilled workers' financial wealth is the opposite of the effect on capitalists' financial wealth, i.e, one will rise and the other will fall. On the other hand, the ambiguous exists thanks to the fact that the effect could be in the same direction. For example, in accordance with the last subsection, a rise in the interest rate will rise the high skilled workers' disposable income more proportional than the capitalists' disposable income¹¹. However, the result on the wealth of both classes is a negative function of the propensities of consume. As long as the propensities to consume of capitalists is sufficiently lower than the propensities to consume of high skilled workers, the wealth of the firsts will rise more than the wealth of the second, deepening the distribution of wealth. The same result occurs for the tax of financial wealth and for the tax of financial income, but in this case, drooping the wealth.

The increase of capitalists' propensities to consume out of income, β_c , and out of wealth, θ_c , will rise the distribution of financial wealth, even though it happens because the wealth of capitalists drops. Besides that, the increase in high skilled workers' propensities to consume out of income, β_h , and out of wealth, θ_h will reduce the distribution of financial wealth, not only because of the drop in high skilled workers' wealth, but also because the wealth of capitalists will surge. Finally, since the tax of profit reduces the capitalists disposable income, if it rises, the wealth distribution will rise too.

6.2 Long-run Output

$$Y^{**} = Y_{Dl} + Y_{Dh} + Y_{Dc} + I + G + X$$
(24)

$$Y^{**} = y(\xi, \gamma_0, \gamma_1, \gamma_2, \psi_W, X) \tag{25}$$

According to the beginning of this section, all classes consume their disposable income. In doing so, the wealth does not affect the output, due to the compensated effect that increasing taxes have on the lower

¹¹Actually, the disposable income of capitalists could fall. On this condition, the distribution of financial wealth must increase.

consumption of high skilled workers and capitalists is compensated by increasing taxes, rising the government expenditure. For the same reason, the propensities to consume also do not impact the output. In addition, the public bonds and the interest rate also do not influence the output. It happens because the consume of high skilled workers and capitalist out of financial income, plus financial income taxed is equal to the government surplus. In other words, the quantity of income that the government must withdraw from his expenditure is the same as the total expenditure out of financial income, private or public.

The taxes also do not impact the output, since the withdraw of private income is compensated by the increase of government expenditure. Finally, since there is not a different propensity to consume out of income, the proportion of low wages in total wages, ψ_L , does not impact the output too. The table 6 summarizes the effects of shocks in the others exogenous variables on the output.

Table 6: Shocks in exogenous variables on output

	Y^{**}
ψ_W	+
ξ	-
γ_0	+
γ_1	+
γ_2	+
δ	-

The increase in the depreciation drops the stock of capital in the long run. For this reason, the gross fixed capital formation declines, decreasing the output. The retained profits reduces the disposable income, then the rise of this variable will decline the output. Finally, the rise of wage share impacts positively the output, i.e, in the long-run the demand is wage-led. It happens because of the stability of the model, the surge in wage share must impact the consumption more than the investment.

7 Conclusion

The neo-Kaleckians theorists have developed growth models studying the functional and personal distribution of income. However, these models usually forget to treat an important variable: the wealth. When we suppose that a given class saves a proportion of its income, we should also model where these reserves area accumulating. In addition to this, it is necessary to think about the effects these savings have on the choices in this class makes - like in consumption.

This paper tried to work out answers for these questions. In the short-run, the wealth, wealth distribution and its allocation affects the output and the growth. This happens because classes have different propensities to consume out of income and out of wealth.

In the long-run the wealth distribution does not impact the output and growth. Nevertheless, it is the increasing of wealth in the transition that turns the consumption equal to disposable income. For this reason, the impact of functional income distribution in output must be positive. Then, the wealth has a main importance in this model.

Even though the wealth and its allocation do not affects output in the long-run, both are very important for the extent of how much each class earns and consumes. According to section 4, a higher wage share means

more income for the workers and a rise in the output. In the same section, we demonstrated that higher interest rate brings a more equal distribution of finance income, but can cause a more unequal distribution of wealth. After all, the size of classes' wealth reduces their own income and consume. In doing so, the surge of interest rate can increase the total consume of workers. This will happen without any consequence to output.

In addition, higher taxes on income of income of bonds will redistribute wealth to capitalists. Besides that, this will decrease high skilled workers' income. However, since the others taxes do not changes the output, it could be compensated by a reduction of the others taxes. In the face of these reductions, high skilled workers' financial income could be increased. Then, at the best, the wealth and income distribution could increase without any change in the output.

In spite of the four models presented in section 2 have defined the variables in different meanings, it is possible to compare their results with our results. It is important to remember, apart from Godley and Lavoie (2007) and our model, that all the models are evaluated in capacity utilization. This difference occurs because their models have different closures. However, since the utilization has the same meaning as the demand, the connections still hold. The table 7 summarizes it 1213.

Table 7: Summarizes of derivatives on long -run Output

	Palley (2012)	Palley (2016)	Dutt (2015)	Godley and Lavoie (2007)	Our Model
r	X	X	+/-	+	0
β_h	0	+	-	-	0
β_c	+	+	X	+	0
θ_h	0	X	-	-	0
θ_c	+	X	X	+	0
ψ_L	X	+/-	X	X	0
ψ_W	+/-	+/-	+/-	X	+
$ au_W$	X	X	X	-	0
$ au_B $	X	X	X	X	0
$ au_{\pi}$	X	X	X	X	0
$ au_V $	X	X	X	X	0
δ	X	X	+/-	X	-

Therefore, our model's result is different from that observed in the literature. First, differently of Palley (2012,2016), in the long-run the paradox of thrift does not hold, but the propensities to consume do not decreases demand. Second, the first three models presented in that section 2 still have the profit-led demand way, while our model just remains the wage-led demand.

However, it is important to remember the constrained conditions made in the long run. The net investment equal to zero imposes a zero growth in long run. If it is relaxed, the investment could be higher. Besides that, the other conditions are able to be relaxed. If the government can issue more bonds in the long run, public bonds that have been already issued would affect the output. Changing the third condition (zero savings) would turn effective the changes in the propensities to consume. But, even if all these conditions remain

 $^{^{12}}$ The X in the table indicates that variable there is not in the model

 $^{^{13}}$ Except for our model, all models has two or one class. Then, in order to connections, we will consider the subscript h for workers and the subscript c for capitalists. Regarding to Godley and Lavoie (2007), the household propensity to consume is in the subscript h. Besides that, the taxes on Godley and Lavoie (2007) are in τ_W

with a little alteration in the function of government expenditure, the output will be modified and turned to a function of other variables.

Then, many extensions could be done to improve the results of the model. As in Van Treek (2009), the banks lend to the richest classes on bases on expected financial wealth. Then, if the model has two classes that own wealth, the wealth distribution may impact aggregate demand. Besides that, as in Dutt (2015) and Piketty (2016), the richest ones have access to profitable investments. Therefore, a model with multiple assets could capture this effect.

Finally, the table 8 summarizes the effects of shocks in the other exogenous variables on short run output, Y^* , on short-run growth, g^* , on distribution of financial income, ψ_{Bh} , on distribution of financial wealth, φ_{Bh} , and on long-run output, Y^{**}

Table 8: Resumo das derivadas								
	Y^*	g^*	ψ_{Bh}	φ_{Bh}	Y^{**}			
r	+	+	+	+/-	0			
β_h	+	+	-	-	0			
β_c	+	+	+	+	0			
θ_h	+	+	_	_	0			
θ_c	+	+	+	+	0			
ψ_L	+	+	_	_	0			
ψ_W	+/-	+/-	+	+	+			
τ_W	+	+	_	_	0			
τ_B	+	+	_	+/-	0			
τ_{π}	+	+	X	+	0			
τ_V	+	+	_	+/-	0			
ω	+	+	+	+	0			
δ	-	-	X	+	_			

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9 Appendix

9.1 Distributional Variables - Short-Run

9.1.1 Wages and Profits

$$Y_L = W_l L_l + W_h L h (9.1)$$

$$Y_{\pi} = \pi Y \tag{9.2}$$

$$\psi_L = \frac{W_l L_l}{Y_L} \tag{9.3}$$

$$\psi_W = \frac{Y_L}{Y} \tag{9.4}$$

Then:

$$W_l L_l = \psi_W \psi_L Y \tag{9.5}$$

$$W_h L_h - \psi_W (1 - \psi_L) Y \tag{9.6}$$

$$\pi Y = (1 - \psi_W)Y\tag{9.7}$$

9.1.2 Public Bonds

$$B = B_h + B_c (9.8)$$

$$\psi_{Bh} = \frac{B_h}{B} \tag{9.9}$$

Then:

$$B_h = \psi_{Bh}B \tag{9.10}$$

$$B_c = (1 - \psi BhB \tag{9.11}$$

9.1.3 Wealth

$$V = V_h + V_l + K \tag{9.12}$$

$$V_F = V_h + V_c \tag{9.13}$$

$$\varphi_{Bh} = \frac{V_h}{V_F} \tag{9.14}$$

$$\varphi_B = \frac{V_h + V_c}{V} \tag{9.15}$$

Then:

$$V_h = \varphi_B \varphi_{Bh} V \tag{9.16}$$

$$V_c = \varphi_B(1 - \varphi_{Bh})V \tag{9.17}$$

$$K = (1 - \varphi_B)V \tag{9.18}$$

9.2 Short-run solutions

$$Y^* = \frac{\Omega_1}{\Delta_1} \tag{9.19}$$

So that,

$$\Omega_{1} = rB(\psi_{Bh}(\beta_{h}(1 - \tau_{B}) + \tau_{B}) + (1 - \psi Bh)(\beta_{c}(1 - \tau_{B}) + \tau_{B})) + V((1 - \varphi_{B})\gamma_{0} + \varphi_{B}(\varphi_{Bh}(\theta_{h}(1 - \tau_{V}) + \tau_{V}) + (1 - \varphi_{Bh})(\theta_{c}(1 - \tau_{V}) + \tau_{V}))$$

$$\Delta_1 = 1 - (\omega + \gamma_2 + \gamma_1(1 - \psi_W) + \psi_W(\psi_L + (1 - \psi_L)(\beta_h(1 - \tau_W) + \tau_W))) + (1 - \psi_W)(1 - \delta)(\beta_c(1 - \tau_\pi) + \tau_\pi)$$

$$g^* = \gamma_0 + (\gamma_1(1 - \psi_W) + \gamma_2) \frac{Y^*}{(1 - \varphi_B)V}$$
(9.20)

9.3 Long-Run Solutions

9.3.1 Public bonds issued and the surplus

From (12) and (21):

$$B - \frac{-\omega}{r}Y\tag{9.21}$$

9.4 Disposable income, public bonds and wealth

9.5 High Skilled Workers

$$S_h = Y_h - C_h = Y_h - \beta_h Y_h - \theta_h V_h \tag{9.22}$$

So that, S_h is the high skilled workers' savings.

Then, from (18)

$$V_h = \frac{(1 - \beta_h)}{\theta_h} Y_{Dh} \tag{9.23}$$

Besides that, from (7) and (9.23)

$$B_h = aY_{Dh} (9.24)$$

So that, $a = ((\lambda_0 + r\lambda_1)((1 - \beta_h)/\theta_h) - \lambda_2)$

Then, from (8)

$$Y_{Dh} = \frac{\theta_h(1-)\psi_L(1-\tau_W)}{\theta_h(1-a(1-\tau_B)r) + (1-\beta_h)\tau_V}Y$$
(9.25)

$$B_h = \frac{a\theta_h(1-)\psi_L(1-\tau_W)}{\theta_h(1-a(1-\tau_B)r) + (1-\beta_h)\tau_V}Y$$
(9.26)

$$V_h = \frac{(1 - \beta_h)\theta_h(1 -)\psi_L(1 - \tau_W)}{\theta_h(1 - a(1 - \tau_B)r) + (1 - \beta_h)\tau_V}Y$$
(9.27)

9.5.1 Capitalists

From (10), (9.21) and (9.26)

$$B_c = -\frac{\omega}{r} - \frac{a\theta_h(1-)\psi_L(1-\tau_W)}{\theta_h(1-a(1-\tau_B)r) + (1-\beta_h)\tau_V}Y$$
(9.28)

Besides that, solving the wealth as (9.23)

$$V_c = \frac{(1 - \beta_c)}{\theta_c} Y_{Dc} \tag{9.29}$$

Then:

$$Y_c = \frac{\theta_c Y}{\theta_c + (1 - \beta_c)\tau_V} ((1 - \delta)(1 - \tau_\pi)(1 - \Psi_W) + r(1 - \tau_B)(-\frac{\omega}{r} - \frac{a\theta_h(1 -)\psi_L(1 - \tau_W)}{\theta_h(1 - a(1 - \tau_B)r) + (1 - \beta_h)\tau_V}))$$
(9.30)

$$V_c = \frac{(1 - \beta_c)Y}{\theta_c + (1 - \beta_c)\tau_V} ((1 - \delta)(1 - \tau_\pi)(1 - \Psi_W) + r(1 - \tau_B)(-\frac{\omega}{r} - \frac{a\theta_h(1 -)\psi_L(1 - \tau_W)}{\theta_h(1 - a(1 - \tau_B)r) + (1 - \beta_h)\tau_V}))$$
(9.31)

9.6 Capital and Investment

From (2) and (20)

$$K^{**} = \frac{(1 - \psi_W)\gamma_1 + \gamma_2}{(\xi - \gamma_0)}Y \tag{9.32}$$

$$I^{**} = \frac{\xi((1 - \psi_W)\gamma_1 + \gamma_2}{(\xi - \gamma_0))}Y \tag{9.33}$$

So, $\xi > \gamma_0$

9.7 Distributional Variables - Long-Run

From (9.10), (9.26) and (9.21)

$$\psi_{Bh} = -\frac{ar\theta_h(1-)\psi_L(1-\tau_W)}{\omega(\theta_h(1-a(1-\tau_B)r) + (1-\beta_h)\tau_V)}$$
(9.34)

From (9.14), (9.23) and (9.31)

$$\varphi_{Bh} = \frac{(1 - \beta_h)(1 - \tau_W)(1 - \psi_L)\psi_W}{(\theta_h(1 - a(1 - \tau_B)r) + (1 - \beta_h)\tau_V)\Delta_2}$$
(9.35)

So that,
$$\Delta_2 = \frac{(1-\beta_c}{\theta_c + (1-\beta_c)\tau_V}((1-\delta)(1-\tau_m)(1-\Psi_W) + r(1-\tau_B)(-\frac{\omega}{r} - \frac{a\theta_h(1-)\psi_L(1-\tau_W)}{\theta_h(1-a(1-\tau_B)r) + (1-\beta_h)\tau_V})) + \frac{(1-\beta_h)(1-\tau_W)(1-\psi_L)\psi_W}{(\theta_h(1-a(1-\tau_B)r) + (1-\beta_h)\tau_V)\Delta_2}$$

9.8 Output in Long - Run

9.8.1 Households consume plus government expenditure

From (17), (18) and (19)

$$C + G = Y_{Dl} + Y_{Dh} + Y_{Dc} + G (9.36)$$

Besides that,

From (8)

$$Y_{Dh} = \psi_W (1 - \psi_L)(1 - \tau_W)Y + rB_h(1 - \tau_B) - \tau_V V_h$$
(9.37)

From (8)

$$Y_{Dc} = (1 - \psi_W)(1 - \delta)(1 - \tau_\pi)Y + rB_c(1 - \tau_B) - \tau_V V_c$$
(9.38)

Then, from (13) and (4)

$$C + G = (\psi_W + (1 - \delta)(1 - \psi_W))Y \tag{9.39}$$

9.8.2 Output

Then, from (24), (9.33) and (9.39)

$$Y^{**} = -\frac{(\xi - \gamma_0)X}{(\xi - \gamma_0)(-\delta(1 - \psi_W) + \xi((1 - \psi_W)\gamma_1 + \gamma_2)}$$
(9.40)

So $(\xi - \gamma_0)(\delta(1 - \psi_W) > \xi((1 - \psi_W)\gamma_1 + \gamma_2)$

9.8.3 Proof of Wage-Led

$$\frac{\partial Y^{**}}{\partial \psi_W} = \frac{X(\xi - \gamma_0)((\xi - \gamma_0)\delta - \xi\gamma_1)}{((\xi - \gamma_0)(-\delta(1 - \psi_W) + \xi((1 - \psi_W)\gamma_1 + \gamma_2))^2}$$
(9.41)

Note, from the Output $(\xi - \gamma_0)\delta(1 - \psi_W) > \xi(1 - \psi_W)\gamma_1$, since $\xi\gamma_2 > 0$. Then, $(\xi - \gamma_0)\delta > \xi\gamma_1$; Therefore, $\frac{\partial Y^{**}}{\partial \psi_W} > 0$