Beyond the Accounting: Macroeconomic Limits on Deficit Spending in an Continuous-time SFC-PK Model

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Abstract

Modern Monetary Theory (MMT) holds that governments with sovereign currencies and flexible exchange rate regimes can run deficits without worrying about possible default, maintaining a desired interest rate set by the monetary authorities. We sympathize with calls for tests of such a system within the context of a full macro model. We first adopt a neo-Kaleckian growth model with sovereign currency and government production. Using analytical methods, we investigate potential and putative instabilities and dynamical properties related to stock-flow inconsistency, distributional drift, poor policy settings, and others. We also lay out a larger stock-flow consistent post-Keynesian (SFC-PK) model from a recent working paper [Hannsgen and Young-Taft 2015]. This model requires a historical-time approach that relies on numerical solution techniques and goes beyond the steady-state fiscal sustainability formulas of Domar, allowing for path dependency, etc. The model keeps track of simulated amounts of accumulated greenhouse-gas emissions, along with inflation, the growth rate, the wage-share, the profit rate, etc., in the model economy. This model also makes use of a Poisson model of financial crises/shocks, which generates crises/shocks whose exact timing depends on chance.

1. INTRODUCTION

Modern Monetary Theory (MMT) holds that governments with sovereign currencies and flexible exchange rates can run deficits without worrying about possible default, maintaining a desired interest rate set by the monetary authorities. In a rejoinder in a recent exchange on modern monetary theory (MMT), the name for neochartalism in current public debate, Tom Palley (2015a, b) calls for an actual MMT model that can go beyond what he regards as uninformative and unoriginal statements about the role of money in public finance in the world of modern macro models. That is, as adherents to MMT, in a model with a separate central bank we have Keynesian budget identities implied by MMT such as:

$$df \equiv tY - R - W - G = H_D + B_D$$

where D is the deficit, tY is tax revenues, R is interest and principal payments to the private sector, W is transfer payments, G is government spending, H_D is the portion of the deficit accounted for by issuance of high-powered government liabilities, and B_D is the portion covered by bond sales. In the interpretation of such an equation offered by MMT, debt costs are predetermined, except to the extent they can be rolled over at the current interest rate. The interest rate on government debt is exogenous in the sense that it is determined by policy. Total private-sector net accumulation of assets, shown on the right hand side is a residual determined by the other variables, though of course, H_D and B_D or their accumulated amounts can enter into a policymaking function such as, for example

$$G = f(H_D, B_D)$$

Then, asset generation is not a residual, though it is still (formally) determined by policy choices about the policy instruments t, , W, G, etc., on the right hand side of the equation. Adding a system of demand functions determines how the total on the right hand side is divided between the two summands H_D and B_D. Palley points out that in a model, such a mathematical identity does not lead to specific conclusions about the impact of deficits or how to determine appropriate fiscal policies in a quantitative sense. It depends on the rest of one's model:

> "The critical economic policy question is what does the power to money finance deficit spending mean for the government's ability to promote full employment and price stability? This question can only be answered by placing that power within a theoretical model and exploring its implications."

While we feel Palley's argument is perhaps a bit unfair, we subscribe to his argument that MMT's account of government money creation leaves much up in the air. We however feel that an appropriate model can show the benefits of many of the types of aggregate demand policies that have been advocated by MMT economists while not generating results so perfect as to cast doubt on the reliability of the MMT view. Moreover, the model's features will not cut against the grain of common sense as suggested to some by the mere statement of the ability of the government to pay for any fiscal policy without failing to pay back its debts. That is, it is quite reasonable to say that stock-flow consistent asset and debt effects, putative real balance effects, effects on asset returns, etc., are unlikely to generate unacceptable consequences—as long as available and known policies are used in the way called for by functional finance and other Keynesian theories and are not taken too far or implemented in a corrupt or inept manner. The implication of MMT is then good: proper macro policies can bring full employment with reasonable levels of inflation over the medium term, leaving aside of course the numerous kinds of disasters and foibles that always impair efforts to ensure a happy outcome. Yet inflation and various other clearly undesirable outcomes can get out of hand if we push the model to its limits. MMT, within the context of our model, will at least be vindicated from charges of implausibility once we show that something can go wrong in a world modeled after MMT.

Given our large highly nonlinear full model of 20 or more differential equations and a model of stochastic shocks, we will eventually rely on computational results. We hope to generate compelling examples by taking care to use reasonable parameter values.

2. THE LITERATURE

James Galbraith (2011) defines the issue of fiscal sustainability as follows:

> We can say that a path that leads to uncontrolled and explosive increases in the ratio of debt to GDP is 'unsustainable'—in the precise sense that the path will have to be changed to prevent the explosion from occurring. We can say this without having to specify what the bad consequences actually are, as these may vary according to institutional context, from hyperinflation to debt default. By the same definition, anything that can be reproduced year over year has to be considered sustainable. Any path that eventually stabilizes is sustainable, even if the debt-to-GDP ratio that finally results seems high to us. Again, we can say this without being forced to specify the economic conditions that would pertain. All that matters, for the question of sustainability, is whether a path stabilizes, or not."

Often, one uses a formula such as the following, which we draw from Galbraith (2011)

$\Delta d = -s + d * [(r - g)/(1 + g)]$

which posits constant values for the real interest rate r, the growth rate g, the primary-surplus-to-GDP ratio s, and the initial debt-to-GDP ratio d. This approach determines whether a given primary surplus "can be reproduced year over year" which an unchanging debt ratio. This latter definition is asymptotic: It evaluates the stability of the debt ratio, given a deficit or surplus ratio that is sustained forever. It does not model the determination of GDP as an endogenous variable, though it measures stability as a ratio to GDP, which potentially grows at a different rate. It is potentially misleading in the sense that it does not allow for situations in which fiscal stimulus can be used to avoid self-defeating austerity, or fiscal-policy traps, which can arise because the terms in the formula above can in fact change.

What might be called the fiscal sustainability literature has seen a resurgence in the years since governments began to increase fiscal stimulus during the global financial crisis. In the face of demands for sustainability, many observers made the point that GDP growth is endogenous.[1] In the midst of an austerity debate, Aspromourgos (2014), Skott (2015b), and Watts and Sharpe (2014) extended the Keynesian tradition of modeling nominal GDP growth as endogenous in model-based discussions. Watts and Sharpe emphasize the distinction between state-money economies and economies in currency unions or blocs, such as the Eurozone, a point many seemed to be missing. These three papers mostly attempt to solve a sustainability problem posed in terms of finding a steady growth equilibrium in an economy with a P-sector, though Aspromourgos in particular carefully notes the problems posed by attempting to move from steady-growth solution to another, which might upset the state of expectation and other underlying model parameters that underlie the state of liquidity preference.

We extend this approach to a larger model, with nonlinearities and stochastic shocks, and consider solution pathways through time for given initial conditions x_0, a vector of reasonable parameter values

[1] An early post-Keynesian contribution with an endogenous GDP was Cunningham (1986–7).

θ, and a realization s of our probabilistic model of financial shocks (Hannsgen and Young-Taft 2015; Hannsgen 2012)

$$x_t = x(x_0, t, s; \theta), \qquad 0 \le t \le t_E, s \in S$$

$$s = \{(\tau_1, s_1), (\tau_2, s_2), (\tau_3, s_3), \dots, (\tau_N, s_N)\}$$

where t_E is the endpoint of a simulation, the τ_i are shock dates, and N is the total number of shocks in the simulation interval.[2] See the next section for a detailed description of the model and the appendix for a list of the technical details**. Hence, we take a historical approach and one in which many key variables, including the markup and labor productivity, are allowed to change. We take dynamical possibilities such as limit cycles and other nonlinear phenomena seriously. We will compare our results and the characteristics of our model at times in the analysis below. Such a model allows for cases in which, for example, timely policy interventions can reduce the risk of large financial shocks that change the solution x_t for all future t. In such growth models, history and aggregate demand matter. Moreover, we will attempt through repeated simulation to generalize somewhat to expected frequencies within a bounded subset of our state space P(x; x_0) for $x \in \mathbb{R}^K$.

Leão (2015) provides a fine account of the chartalist position on the question, "Is Very High Public Debt a Problem?" He even offers one numerical example which adopts a finite-time but long-run perspective, taking into account the possibility of changes in nominal GDP and government spending (no imposition of eternal balanced growth), but does not use a formal model. Tauheed and Wray (2006) use a welcome holistic system-dynamics approach and adhere to a chartalist account of monetary and fiscal institutions. To date, chartalist models of fiscal policy have used exclusively nominal variables, and do not consider changes in the value of the unit of account (i.e. inflation or deflation) or their economic effects in their numerical scenarios. This approach reflects a highly agnostic approach to the Phillips curve and similar relationships.

Some other recent models of fiscal policy elide this Keynes-Domar sustainability problem, often despite great rigor. Often, in contrast to the chartalist approach, their accounts do not attempt to show how all financial assets are voluntarily held (e.g., Palacio-Vera 2012), completely omit government money or interest-bearing liabilities, or leave accumulated assets or debts out of behavioral functions (e.g., Asada 2011; Hannsgen 2014). These common assumptions in the literature are relevant to the problem at hand, given that some policy commentators and MMT critics fret about situations in which investors' might not be willing to hold government debt at a reasonable interest rate (e.g., Aspromourgos 2014).

The stock-flow consistent method developed in Godley and Lavoie (2012) is related to the chartalist story and has a "medium run" concept of sustainability that is somewhat historical, making use of particular solutions; moreover, it is characteristically concerned with accounting for all stocks and flows implied by a pathway for the economy. Godley and Lavoie ([2007] 2012) developed a functional

[2] Harris (2005) makes the case that in a Robinsonian world with irreversible time, one knows only this much. That is, there is no behavior that does not depend on historical accident, including initial conditions, chance events that occur along the way, and so on. We add the random matrix s to the formulation in Harris's article.

finance type rule within this modelling approach. One post-Keynesian SFC study that generates endogenous financial instability using shocks to psychological factors, a feature in common with our model, is Le Heron (2011).

Dutt (2013) considers fiscal sustainability with endogenous productivity growth. Skott (2015a) derives some results showing ambiguous stability properties as a function of policy parameters and Skott (2015b) discusses policymaking from the theorist's perspective, using a small model. A number of papers in the Sraffian tradition written by Salvatore, Panico, Pinto, Sushko, and others emphasize endogenous technological change as a function of government policy in discrete nonlinear models. Our model of technological change differs somewhat from the ones used by Dutt (2013) and these latter authors.

Hannsgen (2014), which develops a chartalist model of growth and distribution, lacked a substantive behavioral role for assets and debts denominated in units of currency. It assumed fixed technological parameters and nominal wages. Hannsgen and Young-Taft (2015), which included greater financial and monetary detail (in fact, the model has at least 20 differential equations along with a conditional model of rare shocks) did not contain much discussion of fiscal policy implications. Moreover, given time and space constraints we could not report many relevant simulations. We use this paper as a vehicle to report and discuss fiscal policy results. We made the case for our model in more detail in Hannsgen and Young-Taft (2015).

Recent papers in macrodynamics that take a functional finance view of fiscal policy and that include accounts of the monetary and financial systems similar to the one in our model include Asada (2011), Asada et al. (2010), Costa Lima et al. (2014), Dafermos (2015), Hannsgen and Young-Taft (2015), Keen (2013), Le Heron (2011), Nikolaidi (2014), Ryoo and Skott (2015), and Skott (2015). This list is by no means all inclusive. Few formal papers in nonlinear macroeconomic dynamics have taken an avowedly MMT or chartalist approach. Hannsgen (2014) is an exception and Asada (2010) would appear to be entirely consistent with MMT in the way the present model is, without referring to MMT or chartalism. We follow Taylor, Rezai, and Foley (2015) in incorporating atmospheric greenhouse gases in a model of this type. Tauheed and Wray (2006) take a system dynamics approach that brings uses a computational approach to deal with a complex macrodynamic system. As we explain in the following section, we lack simulation results at the time of the writing of this paper for the full model of Hannsgen and Young-Taft (2015), which is nonlinear and contains at least 20 differential equations along with our model of financial crises/shocks. We nonetheless intend to have simulation results done on time for the conference. In this paper, we develop some formal results using elaborations on the simpler 3D model in Hannsgen (2014) and some elaborations of the latter model developed in more detail in Hannsgen (forthcoming).

3. Characteristics of the Model

Hannsgen (2014), among other things, examines the dynamical behavior of a model of growth and distribution with endogenous fiscal policy in the form of a countercyclical government expenditure rule and a system of unemployment compensation benefits. A key part of the original paper analyzed the nonlinear dynamics of a system of three variables—public production p, capacity utilization u, and the markup m—leaving out the dynamics of the labor force.

$$\dot{p} = -\alpha_p \left(\alpha_{pu}(u - u_T) + \alpha_{pp}(p - p_T) \right)$$

$$\dot{u} = \alpha_u \left(u^d(p, u, m) - u \right)$$

$$\dot{m} = f_2 \left(u - u_{EQ} \right)$$

In more detail, the aggregate demand function in the 3D model above is

$$u^d(p, u, m) = (1 - \tau)\left(\frac{1}{1+m}\right)u + (1 - \tau)p + \chi\left\{(1 - \tau)\left[\left(\frac{m}{1+m}\right)u - \delta + \bar{\imath}b\right]\right\} + f_1\left(u - u_{EQ}, m\right),$$

where we have of course used overdots to indicate differentiation by the time variable t. Moreover, $b > 0$ signifies K-sector holdings of treasury bills divided by the stock of capital goods, $\bar{\imath} > 0$ represents the real interest rate, $\delta \in (0, 1)$ is the rate of depreciation of the capital stock, and $\tau \in (0, 1)$ is the tax rate on all income. The variable b is assumed to be positive and is a constant for reasons to be explained later. $\chi > 0$ is a positive propensity to consume out of K-sector disposable income, which includes net profits and interest from government securities. The parameter u_{EQ} is a desired level of capacity utilization that we set equal to u^T in the policy function for simplicity. The other target must be set at a value that is consistent with this desired/targeted value of u.

Our restrictions on the shapes of the functions f_i in this system were as follows.

investment function: $f_1(u - u_{EQ}, m) = auton + f_1(u - u_{EQ}) + \kappa(1 - \tau)(m/(1+m))u + \delta$

$$\kappa > 0;\ 0 < \delta < 1; f_1' > 0;\ f_1''(x) > 0\ for\ all\ x\ <\ 0;\ f_1''(x) < 0\ \ for\ all\ x\ >\ 0.$$

(Commendatore, Panico, and Pinto 2011; see also Kaldor (1940))

m: $df_2/du \leq 0$; $d^2f_2/dm^2 > 0$; for $u < u_{EQ}$, and $d^2f_2/dm^2 < 0$ for $u > u_{EQ}$; and $f_2(0)=0$. Note that these conditions assure (1) very slow markup adjustment near the equilibrium; (2) a rapidly falling markup with very high output; and (3) a rapidly rising markup with very low output. See Branston, Cowling, and Tomlinson (2014) for evidence bearing on the sign of the first-order effect from recent data.

In addition, we make the following assumptions (see Hannsgen 2014 and the references therein):

Assumption 1 (unstable output-adjustment process) $\frac{\partial u^d(p,u,m)}{\partial u} - 1 > 0$

Assumption 2 (policy instrumentalism). The values of the fiscal policy function parameters α_p, α_{up}, and α_{pp} in equation are such that the matrix formed by the first two entries in the first two rows has positive determinant.

Assumption 3 (wage-led AD): $\partial u^d(p, u, m)/\partial m < 0$

I also briefly discuss below a two-equilibrium case below in which assumption 3 does not hold at one of the fixed points.

The public sector issues two liabilities—real money $\mathcal{M}$ and treasury bills b—as a consequence of its stock-flow accounting identity

$$\dot{\ell} \equiv \dot{b} + \dot{\mathcal{M}} = p + \bar{\imath}b - \tau u$$

The financial stock variables b and $\mathcal{M}$ are reported as ratios to the nominal capital stock $p_U k$, where p_U is the retail price of goods. This identity can be used as a rough guide to extreme problems with stock-flow consistency, but our models of consumption and investment here do not include the financial assets b and $\mathcal{M}$. However, K-sector disposable income includes interest payments $\bar{\imath}$b. We assume that K-sector bill demand b^d is constant for constant i, implying that $\dot{\ell} \equiv \dot{\mathcal{M}}$. More generally, we could just as easily make the b^d a function of other variables, with open market operations continuing to assure i = $\bar{\imath}$ (See Section 6).

This sector also produces a service using Leontief technology

$$L_p = \ell_P p$$

where the variable L_P is the number of hours of labor employed by the government and ℓ_P>0 which it provides at no cost. We also consider the effects of using an alternative rule for p with a deficit target. Some lessons about stability and sustainability from this smaller model are mentioned below in Section 6.

As mentioned above, we now summarize the more extensive model to be used in our simulations, which contains either 20 or 21 differential equations, along with a probabilistic model used to generate financial crises/shocks. The essentials of government finance remain the same as in the 3D model above from Hannsgen (2014). We take care to provide a relatively high level of detail about the model of crises/shocks in this section. A full list of equations for the entire model, along with separate lists of parameters and initial conditions, appears in a separate section below.

In a working paper by the present authors (Hannsgen and Young-Taft 2015), we add a net revenues variable nr to the net investment function above. All variables in this function now enter nonlinearly, as the term $u - u_{EQ}$ did before. Now all stock and output variables are nominal values normalized by the capital stock times the price of goods. We add a term to the K-sector consumption function that depends on the deviation of assets divided by capital from a constant stock-flow norm. We also make the growth rate of labor productivity ℓ_U an increasing function of the capital accumulation rate g = $\hat{k}$ and P-sector expenditures on research and development p_{ℓ_U}. We add an identity from physics making the rate of increase of atmospheric greenhouse gas concentrations a function of physical output u.k. We make the wage unit variable, either by having it rise at an exogenous rate set by the P-sector (option 1) or by using a nonlinear wage-Phillips curve (option 2). We allow the P-sector and K-sector wages to differ by a factor of proportionality c that we call the wage contour. As in the 5D model in Hannsgen (2014, section), we allow for an endogenous labor force, making its growth rate a function of the unemployment rate un. We change our main fiscal policy function to

$$\dot{p} = -\left[\alpha_{p,p}(p - p^T) + \alpha_{p,un}(un - un^T)\right], \qquad p^T > 0, \quad 0 \le un^T \le 1$$

where we have replaced the capacity utilization target u^T with an unemployment rate target un^T. An alternative "sound finance" approach can be implemented using the policy function:

$$\dot{p} = -\alpha_{df}(df)$$

We add a stock market with endogenous determination of prices. We add a banking system, which provides consumer loans d_W to the W-sector and margin loans (loans to buy stock) ml to the K-sector. The degree of leverage used by the K-sector to hold stock enters into the desired price-earnings ratio *pe_ratio,* which acts as an center of gravity for the dynamics of the stock price pr_S. Wealthy households hold deposits dep that are liabilities of this sector, along with a portion $\mathcal{M}_h$ of the government money stock and equity issued by the K-sector. Its disposable income includes dividends paid by the banks, capital gains on all assets, and interest on bills. The banking system sets interest rates by marking the Treasury bill rate up or down. An equation for the this markup pulls m toward a desired level that depends on the markup in the goods-producing K-sector and a cash flow variable. It is required to hold government money in the form of reserves $\mathcal{M}_F$ in direct proportion to deposits. Monetary assets (defined as state money) were accounted for by the identity $\mathcal{M} \equiv \mathcal{M}_h + \mathcal{M}_F$. Building on work in Hannsgen (2012), we add a model of financial crises in which the rate of occurrence of a shock/crisis (say, the number every 100 years) depends on an endogenous financial fragility variable

$$\lambda = f_\lambda\left(\overbrace{\tilde{u}}^{-}, \overbrace{FF}^{+}\right) \ (rareshock\ frequency)$$

where the fragility variable FF depended on the ratio of disposable income to debt for both household sectors and the ratio of government liabilities to the capital stock.

$$FF = f_{FF}\left(\overbrace{(y_W - i_w d_W)/d_W}^{-}, \overbrace{(y_K - i_{ml}ml)/_{ml}}^{-}, \overbrace{\tilde{\ell}}^{-}\right)$$

where i_W is the rate of interest on consumer loans, d_W is the ratio of these loans to the capital stock, i_{ml} and ml are the corresponding variables for margin loans.

Behaviorally, the proclivity of the two household sectors to take on debt and the willingness of the banking system to lend to them depends upon a psychological variable P

$$P = f_P\left(\overbrace{\tilde{T}}^{-}, \overbrace{p_F}^{+}, \overbrace{\tilde{u}}^{-}\right), \qquad T \equiv \int_{\tau_{-1}}^{t} 1\, dt$$

where τ_{-1} is the date of the most recent crisis and $p_F < p - p_{\ell_U}$ is government expenditures on the regulation and supervision of the banks. The inclusion of the variable T in the function above instantiates the Minskyan hypothesis of the "instability of stability": the mere passage of time leads to falling margins of safety, which relentlessly increase financial fragility. For the specifics of the loan demand and supply functions in which P appears, see the full model in Hannsgen and Young-Taft (2015) or as summarized below.[3] The size of individual shocks s was drawn from a power distribution $F(s) = c_1 s^{-\alpha}\ for\ s \geq 0$. The crises/shocks were meant to generate discrete (finite) movements in financial variables such as stock prices, loan markups, holdings of uninsured large bank deposits, and the like, in a model in which the pathways of the state variables were otherwise continuously differentiable. The effects of the shocks are meant to be accounted for a stock-flow consistent manner. An extensive list of references are provided and more detail are provided in Hannsgen and Young-Taft (2015) and a version of that paper now being prepared for journal submission.

The full system can likewise be stated as a stochastic differential equation (SDE). In particular for $X_0 = x_0$ the initial value of a process, W_t a standard Brownian motion, and Z_t a compound Poisson process,[4] consider a multidimensional model, solution to

$$dX_t = a(t, X_t, \theta)dt + b(t, X_t, \theta)dW_t + c(t, X_t, \theta)dZ_t,$$

or more generally

$$dX_t = a(t, X_t, \theta)dt + b(t, X_t, \theta)dW_t + \int_{|z|>1} c(X_{t-}, z)\mu(dt, dz)$$
$$+ \int_{0<|z|\leq 1} c(X_{t-}, z)\{\mu(dt, dz) - v(dz)dt\},$$

for μ a random measure defined on the jumps of X,

$$\mu(dt, dz) = \sum_{s>0} 1_{\{\Delta Z_s \neq 0\}} \boldsymbol{\delta}_{(s,\Delta Z_s)}(\boldsymbol{dt}, \boldsymbol{dz}),$$

for δ the Dirac measure, Z the driving pure-jump Lévy process

$$Z_t = \int_0^t \int_{|z|\leq 1} z\,\{\mu(ds, dz) - v(dz)ds\} + \int_0^t \int_{|z|>1} z\,\mu(ds, dz),$$

and v (the Lévy measure) any measure such that $v(0) = 0$ and $\int (1 \wedge |z|^2) v(dz) < \infty$ (Brouste *et*

[3] In earlier versions of this model (Hannsgen 2012), T entered directly into the formula for λ, the frequency of shocks. In Hannsgen and Young-Taft (2015), we take a less ad hoc route and provide a more complete account of how fragility increases.

[4] That is, jumps that are drawn from some distribution that occur a times drawn from a Poisson distribution.

al. 2014, Iacus 2011). As our model does not incorporate any Brownian motion and its drift components are entirely specified, we can omit the parameter script and write

$$dX_t = a(t, X_t)dt + \int_{|z|>1} b(X_{t-}, z)\mu(dt, dz)$$
$$+ \int_{0<|z|\le 1} b(X_{t-}, z)\{\mu(dt, dz) - v(dz)dt\},$$

Then the function can be thought of as a system of ordinary differential equations (ODEs) constituting the drift component in an additive relationship with one equation that consists solely of a jump process.

Technically, in relation to the 3D model described at the beginning of this section, we divide the balance sheets in the K-sector so that we now have a separate financial sector ("F-sector"), K-sector household sector ("KH-sector"), and nonfinancial business sector ("K-sector"). Consumption goods c and investment goods g now have separate adjustment functions that move actual normalized quantities produced and sold toward the amounts desired, g^d and c^d. In the full model, based on standard existence theorems for nonautonomous differential equations, we know that smooth solution pathways exist for a chosen compact region in state space and a time interval [0, t_{END}] that we specify, except for a finite set of jump discontinuities where crises occur (Hannsgen and Young-Taft 2015, 56–58). Given our assumption that the function for the number of crises period time unit is bounded above zero, the full system has no equilibrium or balanced growth path (Hannsgen and Young-Taft 2015, pp. 55–56). All one can solve for in this larger system of 20+ equations is a solution pathway that does not converge on an equilibrium point and may be chaotic. We can simulate pathways using the Euler-Maruyama scheme, and Monte-Carlo methods in conjunction with such simulation or asymptotic expansion to solve for expected value and evolution of density (Brouste *et al.* 2014). The pathway will vary depending upon the draws from the stochastic process used to generate crises/shocks. Also, given our Kaldorian assumptions about the determinants of technological progress, early shocks, recessions, or policy choices can change the "real" growth path altogether, setting off irreversible demand-led processes. And the entire process generates greenhouse gas emissions, which relentlessly lead toward a bound for total atmospheric carbon accumulations that is not very flexible—another sense in which the process generated by our model is irreversible. We discuss various aspects of stability and sustainability that are more amenable to policy intervention in a section below.

4. DEFINING SUSTAINABILITY IN AN MMT MODEL: RULING OUT POLICY COORDINATION PROBLEMS

One characteristic of MMT is that it argues that there is no affordability constraint on governments. In other words, there is no government "solvency constraint," etc. This characteristic seems to apply to canonical SFC models (e.g., Godley and Lavoie [2007] 2012, 2012). On the other hand, such models do generally have some "medium-term" role for stocks of assets and liabilities in their behavioral equations. It is sometimes questioned whether the MMT approach leads to an implausible situation in which deficits and debts could reach any levels whatsoever with no negative impacts on the economy. This is of particular interest in a model in which various effects are added to the basic mix, including greenhouse gas concentrations, financial crises whose exact timing is stochastic, and various dynamical effects, which arise from the richness of the detail in the model, relative to, say, Godley and Lavoie (2007).

There have been claims to the effect that the MMT approach ignores the possibility that the central bank fails to support the efforts of the budget-setting authorities in the government. Yet to simply answer a policy question with the statement that the government will not adopt a proposed policy often begs some interesting research question in macroeconomics, such as, determining the likely effects on growth and distribution of a more generous unemployment insurance scheme.

There are three approaches to take here: 1) one can assume the ability to set the fiscal policy stance and the monetary policy stance for the sake of argument in the interest of finding the best possible policies, rather than begging the question of what policies actually are in force; 2) one can take an approach of taking as given the forces that keep the policy stance at existing settings.; or 3) one can assume at least for the sake of argument that some such forces are given and that others fall are up for debate.

We assume here the full power of fiat for the following policies and policy stances (see Hannsgen and Young-Taft 2015 for a larger and similar model):

1) a fiscal policy function, which determines the rate of change of the P-sector's activity level p as a function of other macro variables
2) An given personal income tax rate (we use a flat rate purely for the sake of simplicity)
3) Proportions of spending devoted to 1) financial regulation and supervision; 2) Spending on "green" R&D and non-green R&D
4) A central bank interest rate, which is paid to the banking system, given the contemporaneous inflation rate
5) In the version of the model using option 1 for the wage-growth equation, the rate of inflation of the P-sector wage ω_P

In other words, we seek answers about how these choices about these policy instruments affect the dynamics of the economy with open eyes about how choices are often restricted or not carefully considered. These include the base interest rate, though we allow bank rates to vary endogenously with the F-sector markup, as explained above.

On the other hand, in all versions of this model, we take some policy parameters as fixed for the purposes of our analysis, namely, the values of the public production target p^T and the unemployment rate target un^T. These values represent central tendencies for policy that bias the system toward less-than-full employment in the world described by Kalecki and Steindl.

For example, a *central bank obstruction scenario* would be a relevant possibility if the central bank is considered to be a separate actor for the rest of the p-sector with interests of its own. We divide the P-sector balance sheet into two parts, CB and Gov, as follows, as in the actual model (Hannsgen and Young-Taft 2015,):

P (public)-sector	Assets	Liabilities
	None	M B

Here M is nominal high-powered money, and B is short term government security that pays interest continuously.

We continue to assume that the central bank is the bank of the government and that it pays for the government expenditures decided upon by the government. But government A may know that government B will raise interest rates if deficits become two high, perhaps because of different views regarding the appropriate amount of total stimulus or the level of the “non-accelerating inflation rate of unemployment (NAIRU). Disagreements between fiscal and monetary policymakers have arisen in many cases in US history, leading to mild jawboning of Congress in meetings with Congressional committees to encourage a tighter fiscal stance. For example, Paul Volcker made it quite clear in testimony before Congressional committees during the early 1980s that deficits would need to fall in order for him to raise the federal funds rate. Such as situation, in which the Fed chair, perhaps with the tacit approval of the rest of the open market committee, sets the central bank rate i_P as a function of the stock or flow of new interest-bearing government securities.

$i_P = f_{i_P}(b)$ or $i_P = f_{i_P}(\dot{b})$

and puts the Gov sector in the position of having to consider repercussions in interest rates if it fails to maintain fiscal targets. How it responds to the situation depends upon its objectives and concerns.

In this situation, unless there is institutional change, each of the two subsectors have a problem in negotiation and persuasion that we will not consider further. We put this problem aside in what remains, assuming that there is a given institutional structure that permits all possible policy settings to be considered.

5. RESULTS ON STABILITY IN A SMALLER VERSION OF THE MODEL

We first consider signs of unsustainability in the 3D model with which we started and then move to the full model. In the case of our full model, there is admittedly no steady state or balanced growth path of any kind. So we must redefine unsustainability in a way that does not make reference to values that can obtain forever. We are reduced to looking for pathways through the state space that avoid catastrophe, excessive unemployment, and the exhaustion of the posited carbon budget—Robinsonian histories of the type suggested by Harris (2005). Here, we have a problem with finding numerical solutions so far even for reduced versions of the system, as system stiffness[5] appeared to be a problem in early runs in 2014.

The initial 3D model posited at the beginning of the previous section is unstable, as the Jacobian matrix can be signed as follows

$$J = \begin{bmatrix} - & - & 0 \\ + & + & - \\ 0 & - & 0 \end{bmatrix} or \begin{bmatrix} - & - & 0 \\ + & + & - \\ 0 & 0 & 0 \end{bmatrix}$$

This is system lacks a stable equilibrium as argued in Hannsgen (2015), as the determinant of J is either zero or positive. There are two ways one might argue. First, given instability of the equilibrium, one can think about more global issues that are relevant with no gravitation toward a steady state. Boundedness of the consumption function yields a curved $\dot{u} = 0$ locus as a function along the m dimension. Moreover, the function also now depends on the capital stock k, which is normally eliminated from the system by an appropriate normalization. The results are to be issued in an imminent working paper (Hannsgen forthcoming), but to foreshadow the main picture in the system with a bounded capitalist consumption function, one finds that maintaining our earlier assumptions, one gets a system whose normal form is

$$\begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

which is indicative of a steady-state-Hopf bifurcation. We have used a specification for the markup equation

$$\dot{m} = -\alpha_m (u - u_{EQ})^5$$

for which obviously $\frac{\partial \dot{m}}{\partial u} = 0$ when $u = u_{EQ}$. The latter properties may hold approximately in any event. We have used an approximation using steps outlined in Appendix APPROX.

According to standard analysis of this case in 3D dynamics (e.g., the system bifurcates into two equilibria). The wage-led equilibrium, which has a higher value of m, remains unstable. The profit led equilibrium undergoes a Hopf-Andronov-Poincaré bifurcation (Wiggins 2003, section 20.7), yielding a limit cycle and interesting global dynamical properties to be described more fully in the forthcoming paper. Moreover, we will also deal there with the matter of the dependence of the equation for u on the capital stock, which produces long-run nonstability in the 3D model above and which we have elided here.

[5] See Hale and Koçak (1991, 236–37) who note that "the term stiff does not really have a precise mathematical definition; it usually means that the solutions are doing interesting things on a short, as well as on a long, time scale."

Another way of achieving stability is through some form of real wage norm, perhaps enforced by an applicable policy (Hannsgen 2012). Such policies might include incomes policies that stabilized the wage differential between the P- and K-sectors as well as the real wage. We then get an alternative equation for m:

$$\dot{m} = f_{mm}\overbrace{\left(m - \frac{1-\eta}{\eta}\right)}^{-} - \alpha_m(u - u_{EQ})^5$$

is the norm for the real wage in the K-sector $\omega_U = \left(\frac{1}{1+m}\right)$. For simplicity we use the same functional form for the term that depends on u.

The Jacobian for the system, evaluated at equilibrium, then becomes

$$J = \begin{bmatrix} - & - & 0 \\ + & + & - \\ 0 & 0 & - \end{bmatrix}$$

For simplicity, we deal with the second case. We recenter the m axis on the value used to compute our targets p^T and u^T, a shift that does not reduce our level of generality. The following proposition describes the effects of the added term.

Proposition (Hopf bifurcation in 3D model with wage norm): 1) Let J_i denote the ith principal minor, i.e. the determinant of the Jacobian matrix with row i and column i deleted. Assume

$$(det(J_1) + det(J_2) + det(J_3)) > 0$$

2) Assume further that $J_{22} > -J_{33}$

A limit cycle is born locally at a bifurcation value of α_P. If the equilibrium is asymptotically stable, i.e., trace(J) < 0, for calibrated values, then any limit cycle in observed state space will be unstable and vice-versa.

The implications are that the unique equilibrium or, in the two-equilibrium case, wage-led equilibrium is stabilized by this change, in the form of an asymptotically stable point or a stable limit cycle.

Proof: See appendix WAGENORMDYN

6. POLICY LESSONS: 3D INSTABILITIES

1) **Persistently Mistaken Expectations:** At the profit-led equilibrium, the equilibrium after-tax profit rate differs, and therefore, the equilibrium growth rate differs. Hence if the constant *auton* is derived in such a way that the expected growth rate g_{EXP} are met when the system is in equilibrium, as in the expression

$$g_{EXP} = g_{EQ} = \kappa s(1-\tau)u_{EQ} + auton$$

i.e., by setting

$$auton = g_{EQ} = \kappa s(1-\tau)u_{EQ} - g_{EXP}$$

then the profit-led equilibrium will obviously be such that

$$g_{EXP} \neq g_{EQ}$$

A recent treatment of this issue is provided by **. We then are prevented from using this interpretation of the parameter *auton.*

2) **Endlessly Rising K-sector Financial Wealth:** For our parameter values, we envision a spending rule that is countercyclical but results in budget deficits on average (see the examples in Hannsgen 2012). This is in keeping with arguments by Minsky, Steindl, and others. Our specification of K-sector preferences leads to constant b (bills over capital), but given systematic deficits, the stock of high-powered money $\mathcal{M}$ rises on average over decades. This fact seems consistent with conditions on bank balance sheets in recent years in the US, where the banking system has accumulated vast amounts of reserves as the government fought the financial crisis and recession. Also, population tends to rise in most countries.

We believe that the MMT/Chartalist logic holds: people in countries with sovereign currencies are more than willing to accept high-powered money as a form of payment for wages in the P-sector and interest on government securities. The financial system allows them to exchange these monies for interest-bearing assets. If they choose to purchase the liabilities of foreign countries, adjustment of the exchange rate permits these outflows to occur without jeopardizing the fiscal or monetary policy stance. In the larger model in Hannsgen and Young-Taft (2015), reserves are set at a fixed ratio rr to normalized deposits dep.

3) **Possible Inconsistencies with Asset Demand Curves** In our model, asset demand curves are independent of the forces that determine the amount of government liabilities in existence. In the 3D system's asset demand curves, $\mathcal{M}/b \rightarrow \infty$, so financial portfolios become increasingly imbalanced over time, assuming the systematic budget deficits mentioned above. This problem can be fixed easily by using a different, more plausible, set of demand curves, but then b becomes variable, allowing K-sector disposable income, which includes the term (1–τ).b.i, to change shifting the desired goods schedule u^d *upward,* and thereby upsetting equilibrium in u. Indeed, we assume a more realistic set of asset demand curves in the full model, with household demands for money and deposits in a non-fixed ratio and bank reserve demand in fixed proportion to deposits (Hannsgen and Young-Taft 2015). But in that larger model, additional variables can adjust, crises/shocks can occur, and other dynamical possibilities will exist that are characteristic of models of this type. In essence, this would lead to a bifurcation—a change in the qualitative properties of the dynamics—with the passage of time. We intend to investigate this phenomenon and the qualitative and quantitative dynamics to which it leads.

4) **Distributional Drift** As we have seen, **t**he wage-led, high-m equilibrium is unstable. Since this equilibrium has a higher u, it may be more desirable from a policy perspective. Yet, this equilibrium in not locally asymptotically or orbitally stable. (A change to balanced budget targeting rule that keeps everything else the same does not help matters (see Hannsgen forthcoming). Moreover, it will be the

only equilibrium in the 3D system with a linear consumption function. These problems do not appear in the 2D model, which possesses constant m (Hannsgen 2014). The case involving an upper bound on desired consumption entails highly complex behavior and is analyzed in Hannsgen (forthcoming). As we have shown above, a model with an additional term in the differential equation for m attains orbital stability. This additional term pulls real wages toward a norm and in the context of our model is suggestive of an incomes policy that coordinates wage setting across sectors in the manner of the successful Austrian model and similar policies in many other European countries.

7. Scenarios Involving Excessive Aggregate Demand Leading to Inflation

In a stock-flow consistent setting with a consolidated P-sector, as in our model, the initial effect of increased spending or reduced tax revenues is to inject money into the other sectors. Moreover, interest on the public debt does the same thing, so that, *ceteris paribus,* higher debt ratios have a stimulative effect, rather than a contractionary one. Specifically, rising interest payments might lead to higher AD via higher K-sector disposable income and thence to higher inflation. These would erode the real burden of debt for the rich and poor, initially reducing the probability of a financial crisis and leaving room on balance sheets for more new private-sector borrowing. Moreover, the wage share in our model is likely to rise, given our assumptions of wage-led AD (Assumption 3 above) and the form of our differential equation for m, especially in the absence of a wage norm, again producing self-reinforcing expansionary dynamics. Potential constraints within our full model include any finite carbon budget as well as the size of the labor force. It is all to likely that we will obtain simulation results that must be truncated before the intended time horizon $t = t_{END}$ has been reached. Because of these considerations, in our deliberate effort to look for downsides to excessive government debt within the context of our model, we expect to find scenarios with excessive inflation, to the extent our search is successful, rather than deflationary or low-growth, high-unemployment scenarios. In another irony, we find that even the long-run effects of high government debt promote growth—potentially in excessive amounts— within this framework. rising interest payments, leading to higher AD via higher K-sector disposable income and thence to higher inflation. Our simulations may shed light on this possibility.

Watts and Sharpe (2014, 78–79) themselves question the practical implications of this possibility, noting that “In practice, the likelihood that bondholder spending will impose major constraints on sovereign government net spending is remote, given, first, that current and prospective target (cash) rates are the main determinants of longer term bond rates, so that a low cash rate will keep the long-term rate low, and second, in the pursuit of its own political objectives, a sovereign government can vary tax rates in order to release real resources so that its economic and social programs can be implemented.”

8. TECHNICAL APPENDICES

Appendix APPROX

1) First, we shift the third axis so that when m=0, J_{23} =0.

2) Second, we guess that dynamical behavioral near equilibrium will not be qualitatively changed by the approximation α_{pp}= 0. Recall Assumption 2 (policy instrumentalism) which restricts the size of this

parameter to be small relative to the other policy function parameter α_{UP}. Ideally, we will check in various ways to ensure robustness numerically.

3) Third, using a reduction in ϕ, we set $J_{22} = 0$, on the grounds that the system is not too far from having the property of Keynesian stability, in spite of Assumption 1, which we have adopted from Kaldor (1940). Hence, local behavior in the neighborhood of the bifurcation will still be relevant.

4) Fourth, we set $\omega \equiv -J_{12} = J_{21} > 0$ by averaging the absolute value of the two relevant entries in J.

5) Fifth, using a change in the bifurcation parameter *auton,* we shift equation (2) in the system above so as to achieve tangency between the $\dot{u} = 0$ surface and the $\dot{p} = 0$ hyperplane.

The nonlinear part of the system can be dealt with in a straightforward way.

Appendix WAGENORMDYN

Proof of Hopf bifurcation for the 3D Model with a Wage Norm

The additional term in the equation for m means that the Jacobian becomes.

$$J = \begin{bmatrix} - & - & 0 \\ + & + & - \\ 0 & 0 & - \end{bmatrix}$$

The proof of a Hopf bifurcation involves showing that Routh-Hurwicz stability conditions switch from not holding to holding at a critical value of a parameter. These conditions are trace(J) < 0; det(J) < 0; $-trace(J)(det(J_1) + det(J_2) + det(J_3)) + \det(J) > 0$; $det(J_1) + det(J_2) + det(J_3) > 0$ (Gandolfo 1997, 479).

Our numbered points correspond to the conditions above, in order. (1) Given our assumptions, the sign of trace(J) is indefinite. (2) Det (J) < 0. (3) Furthermore, let J_i denote the ith principal minor, i.e. the determinant of the Jacobian matrix with row i and column i deleted. Then assuming we are still dealing with a unique wage-led equilibrium, the second-order principal minors of J can be signed as follows: $J_1 < 0$, $J_2 > 0$, and, remembering Assumption 2 (policy instrumentalism) of section 2.3, $J_3 > 0$. Hence, a restriction on our parameters can insure that locally$(det(J_1) + det(J_2) + det(J_3)) > 0$ Changes in α_P can then produce a change in the sign of trace(J), if we assume $J_{22} > -J_{33}$. This change occurs with nonzero speed. Hence, a limit cycle is born at a bifurcation value of α_P. If the equilibrium is asymptotically stable, i.e., trace(J) < 0, for calibrated values, then any limit cycle will be unstable and vice versa.

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