"Neo-liberal" Barebone Capitalism or "Keynesian" Socially-Protected Capital Accumulation. A Comparative Evaluation for Closed KMG Economies

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April 30, 2015

Abstract

In this first paper on the subject of 'Socially-protected' capitalism we present the modules of a general, hierarchically structured continuous-time model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and considered in detail in Chiarella, Flaschel and Franke (2005). the model improves and modifies the preliminary 18D format used in Charpe, Chiarella, Flaschel and Semmler (2010, ch.7), here applied to the case of a closed economy. The model is sufficiently rich with respect to markets, sectors and agents and complete with respect to budget constraints or – as one now prefers to say – is stock-flow consistent.

We describe the model at the level of national accounts and then introduce against this background its extensive or structural form. Its laws of motion and supplementing static equations are derived next on the basis of which a balanced growth path can be obtained and investigated from a comparative dynamic perspective, in particular with respect to a rich set of taxation and transfer schemes between households, firms and the government. Stability is discussed in terms of various feedback channels which characterize the private sector of the economy and which in sum tend to destabilize it if fiscal and monetary policy remain passive.

Due to the size of the model, the set of fully interacting feedback channels can be studied only numerically, while isolated feedback chains can be investigated theoretically as in earlier work. The model allows for two contrasting limit cases, the Keynesian case of a 'socially-protected' form of capital accumulation as against a Neo-liberal 'barebone, form of capitalism where among certain conditions some credit support is given to firms in order to allow for an acceptable form of income distribution that provides worker households with the necessary income in a capitalist environment where public goods or transfers are completely absent.

Keywords: high order macro-dynamics, stability, fiscal and monetary policy, sustainable balanced growth, social protection schemes

JEL CLASSIFICATION SYSTEM: E32, E64, H11.

Contents

1 Introduction

In the following sections, we present in various ways the modules of a hierarchically structured continuous-time model of Keynesian monetary growth, of the variety introduced in Chiarella and Flaschel (2000) and considered in detail in Chiarella, Flaschel and Franke (2005). The model is sufficiently rich with respect to markets, sectors and agents and consistent with respect to budget constraints to capture the important details of actual macro-economies. We describe the model on the level of national accounts and then derive its extensive form dynamics. Our model allows for coherent stock-flow considerations, a compact intensive form for its theoretical and numerical investigation and a locally unique interior balanced growth positions which we use as starting and reference point for our simulations of its laws of motion.

These simulations provide a persuasive foundation for a basic understanding of the interaction of its various economic feedback channels some well-known known by partial reasoning from Keynesian economic theory, like the Harrod-Domar theory of the instability of balanced growth, the Goodwin-Rose distributive cycle mechanism, and the Blanchard analysis of bond and asset markets dynamics. The basic task here is to tame these generally destabilizing forces from a Keynesian perspective by way of suitably chosen fiscal and monetary policy rules, a quite demanding task in view of the many laws of motion of the considered macro-dynamics.

Of primary interest is however the question how the many tax-, transfer- and government-expenditure parameters of the full model can be used to improve the social protection of the sector of worker households, as compared to the barebone version of the macro-dynamics, without loosing the efficiency of a well-performing labor market (with its partial modelling of Friedmanian supply side forces), but also without neglecting the creation of a sufficient "infra-structure" for education, health care and care for the elderly, i.e., for the young people, the labor market participants and the retired. We therefore compare in this paper a capitalist system, where public goods and transfers to worker households are absent, with the situation where social protection is given to them, as workers as well as as retirees (and also for the rest of the society).

We can show that the case with social protection will indeed be advantages in basically all respects, as compared to the case of barebone capitalism, leading to more prosperity and a better income distribution on the basis of one important positive externality, which is the increase in the potential production of firms per unit of their capital following an increase in the public capital stock, the "infrastructure of the society". Such an infrastructure covers a lot of details, represented by a single public good on the macro-level, ranging from schools, hospitals, public transportation facilities to old-age homes and more. Of course, public services must supplement such public goods to make them productive ones.

Since residential issues are also a matter of great importance for the worker households, we finally add residential services to this households sector, services which are supplied from of a stock of houses created by the housing investment of the asset holders in our economy, the sole real asset these asset holders administer in this Keynesian model of monetary growth.

Concerning the topics just enumerated we will provide a range of answers showing the macro-advantages of an advanced type of "social protection" through public investments into the "infrastructure" of the economy, through various types of income transfers, and through Keynesian fiscal and monetary policy, but we will also find some obstacles which prevent the creation of what is called a "free lunch" by mainstream economics. These aspects are illustrated by some simulations of the laws of motion of our macro-dynamical system in the sections that follow the determination and study of its balanced growth path.

2 The real and the financial part of the economy

The following two tables provide a survey of the structure of the economy to be modelled in this paper. The aim of the paper is to establish an integrated continuous-time dynamical model, leading to an autonomous system of differential equations, where all sectors are fully specified with respect to their behaviour and their budget constraints from the viewpoint of complete theoretical macro-models of monetary growth. A bridge will thereby be provided between the Keynes-Metzler type monetary growth models of Chiarella and Flaschel (2000), Chiarella et al. (2005) and the applied Powell and Murphy (1997) approach for the Australian economy, where we however use only a closed economy perspective here. The open economy case will be considered in a companion paper to the present one.

2.1 The structure of the real part

Let us start with a presentation of the variables that comprise the real part of the economy. Table 1 provides the data for the temporary equilibrium position of the economy, based on given prices and expectations and it also shows the real stock variables of the model and their rates of growth.

	Labor	Goods	Dwellings
Workers	$L^w = \alpha^w L$	C_w	C_h^d
Asset holders	–	C_c	C_h^s, I_h
Firms	L_f^d, L_f^w	$Y^p, Y, I, \mathcal{I}$	–
Government	$L_g^d = L_g^w$	G	–
Taxes	$\tau_w, \tau_{wp}, \tau_c$	$\tau_v.\tau_{fp}$	–
Wages, Prices	w, w^n, w^r, w^u	$p = (1+\tau_v)p_y$	p_h
Expectations	π^c	π^c	π^c
Stocks	L	K, N	K_h
Growth	$\hat{L} = \bar{\gamma} - m$	$\hat{K} = I/K - \delta_f$	$\hat{K}_h = I_h/K_h - \delta_h$
	$\dot{N} = Y - Y^d$	–	–

Table 1: *The real part of the economy.*

We use a superscript d for demand and (sometimes) s for supply. The symbol α_w denotes the participation rate of the labor force L, the employment of which in the sector of firms is given by L_f^w. The symbol L_f^d denotes the employment of the employed workforce in hours and w the hourly gross wage. The symbols used for net wages, unemployment benefits and pension payments should self-explaining.

Payroll taxes are shared between workers and firms. We denote by p the price level that includes value added taxes at the rate τ_v. The expression π^c will be used to describe the expected medium-run inflation climate of the economy. The stock of inventories of firms is denoted by N. Finally, labor force growth is determined from outside the household sector through the trend rate in investment minus the growth rate of labor productivity m, towards which the growth rate of the labor force and of the population is adjusting. This assumption will be reconsidered later on.

2.2 The structure of the financial part

Let us next consider the financial part of the economy which we will keep less advanced as compared to the real part of this macro-economy.

	Money	Short-term Bonds	Long-term Bonds
Workers	–	$\dot{B}_w$	–
Asset holders	$\dot{M}$	$\dot{B}_c$	$\dot{B}_c^l$
Firms	–	–	$\dot{B}_f^l$
Government	–	$\dot{B}_g, \dot{B}_b$	$\dot{B}_g^l$
Prices	1	1 $[r]$	$p_l = 1/r_l$
Expectations	–	–	$\pi_l = \hat{p}_l^e$
Stocks	–	$B_g = B_w + B_c + B_b$	$B_g^l + B_f^l = B_c^l$
Growth	–	$\hat{B}_g$	$\hat{B}_g^l$

Table 2: *The financial part of the economy.*

We use the subscripts g, c, w for government, pure asset holders and workers respectively and assume as usual in a continuous-time stock-flow model (where stocks and flows have different economic dimensions) the existence of flow consistency by assuming the inflow of new stocks are just accepted by asset holders (here c and w) (while the central bank can change the money supply through the purchase of short-term bonds solely ($\dot{M} = \dot{B}_b$).

This trivial Walras' law of flows is to be supplemented by a dynamic Walras' law of stocks within which dynamic reallocations of the stocks held by pure asset holders take place, in particular the enforced inflow of new assets from the government and the central bank. We will do this in a very simple way here in order to allow the interest rate policy r of the CB operating on the interest rate of fixed-price short-term bonds B and to have in this way an indirect impact effect on the long-term rate of interest $r_l = 1/p_l$ of the perpetuities B^l, we use here as the only risky type of asset on the financial markets.

3 The structure of the economy from the viewpoint of national accounting

We consider in this section the production accounts, income accounts, accumulation accounts and financial accounts of the four internal agents in our economy. These accounts provide notation and basic information on what is assumed for these four sectors as well as which of their activities are excluded from the present theoretical framework. These accounts also serve the purpose of checking that ex post results of the economy are consistent with each other.

3.1 The four sectors of the economy

We start with the accounts of the sector of firms (shown in Table 5.3) which organise production Y, employment L_f^d of their workforce L_f^w and gross business fixed investment I and which use (in the present formulation of the model) only corporate bomds B_f^l as financing instrument (no debt in the form of bank loans nor equities issued by firms). There are value added taxes τ_v on consumption goods and payroll taxes τ_{fp} with respect to hours worked L_f^d, but no further taxation in the sector of firms and there are no subsidies (up to an exceptional numerical example).

Firms build and sell dwellings, which are of the same aggregate type as all other domestic production, and sell them to the asset holders (as investors) and thus have no own investment in the housing sector. They sell consumption goods to workers, asset holders and the government, organise fixed gross investments with respect to their capital stock K (as well as voluntary inventory

Production Account of Firms:

Uses	Sources
–	Consumption $p_v C_w$
Depreciation $p_y \delta_f K$	Consumption $p_v C_c$
Indirect Taxes $\tau_v p_y (C_w + C_c + G)$	Consumption $p_v G$
Wages (including payroll taxes) $\tau_{wp} w L_f^d$	
	Gross Investment $p_y I$
	Durables (Dwellings) $p_y I_h$
Profits $\Pi = \rho^{eg} p_y K + p_y \mathcal{I} = \rho p_y K + p_y \dot{N}$	Inventory Investment $p_y \dot{N}$

Income Account of Firms:

Uses	Sources
Profit payments (including interest paid by firms) $\rho^{eg} p_y K$	Profits Π
Savings $S_f^n = p_y \mathcal{I}$	

Accumulation Account of Firms:

Uses	Sources
Gross Investment $p_y I$	Depreciation $p_y \delta_k K$
Inventory Investment $p_y \dot{N}$	Savings S_f^n
	Financial Deficit FD

Financial Account of Firms:

Uses	Sources
Financial Deficit FD	Bond Financing $p_l \dot{B}_f^l$

Table 3: *The production, income, accumulation and financial accounts of firms.*

changes $\mathcal{I}$ with respect to finished goods) and experience involuntary inventory changes $Y - Y^d$ due to the deviation of aggregate demand Y^d from output Y (which is based on expected sales Y^e and planned inventories $\mathcal{I}$).

Firms have replacement costs with respect to their capital stock and pay wages including payroll taxes. Their accounting gross profit (including interest payments B_f^l on their perpetuity issue) is equal to expected profits (based on sales expectations. As is obvious from the narrow income account of firms, firms thus only save an amount equal to their intended inventory changes. The accumulation account is self-explanatory as is the financial account.

Note that all investment is traded without value added taxes and thus at producer prices p_y in place of the domestic consumer prices $p = (1 + \tau_v) p_y$, which include indirect taxes (value added taxes). Such taxes thus only apply to consumption activities, not to gross investment, and also not on housing investments and the inventory investment of firms. All expected profits are distributed to asset holders (including the interest payments of firms to them). Note however that the wages w paid by firms are augmented by payroll taxes $\tau_{wp} w$ (for unemployment benefits, medicare and other social insurance, as well as pensions) and that wage income w of workers is taxed at the rate τ_w. Note finally that the accumulation account of firms is based on realised magnitudes and thus does not refer explicitly to their intended inventory changes.

Production Account of Households (Asset Owners/Housing Investment):

Uses	Sources
Depreciation $p_y \delta_h K_h$	Rent $p_h C_h^d$
Earnings Π_h	

Income Account of Households (Asset Owners):

Uses	Sources
Tax payment $\tau_c r B_c$	Interest payment $r B_c$
Tax payment $\tau_c B_c^l$	Interest payment B_g^l
Taxes $\tau_c (p_h C_h^d - p_y \delta_h K_h)$	–
Tax payment $\tau_c \rho^{eg} p_y K$	Profits and interest from firms $\rho^{eg} p_y K$
Consumption $p_v C_c$	Earnings Π_h
Savings S_c^n	

Accumulation Account of Households (Asset Owners):

Uses	Sources
Gross Investment $p_y I_h$	Depreciation $p_y \delta_h K_h$
Financial Surplus FS	Savings S_c^n

Financial Account of Households (Asset Owners):

Uses	Sources
Short-term bonds $\dot{B}_c$	Financial Surplus FS
Long-term government bonds $p_l \dot{B}_c^l$	
Corporate bonds B_f^l	–

Table 4: *The production, income, accumulation and financial accounts of asset owners.*

Consider next the sector of asset-holders (see Table 4). Investment in housing as well as the supply of housing services has been exclusively allocated to this sector. The production account thus shows the actual sale (not the potential sale) of housing services (equal to the demand for housing services by assumption) which is divided into replacement costs and actual earnings or profits on the uses side of the production account.

The income of asset holders comes from various sources: interest payments on short- and long-term bonds, interest payments of firms (as part of their expected profit) and profits from housing services. All domestic profit income is subject to tax payments at the rate τ_c. After tax income by definition is subdivided into the consumption of domestic commodities (including houses, but not housing services) and the nominal savings of asset owners.

The accumulation account shows the sources for gross investment of asset-holders in the housing sector, namely depreciation and savings, the excess of which (over housing investment) is then invested in financial assets as shown in the financial account. Note here that short-term bonds are fixed price bonds $p_b = 1$ (which are perfectly liquid), while long-term bonds have the variable price $p_b = 1/r_l$ (with fixed nominal interest payments of one unit of money per period and bond), so-called consols or perpetuities

There is no taxation of financial wealth (held or transferred) in the household sector. Furthermore, though asset holders will consider expected gross rates of return on financial markets in their investment decision, there is no taxation of capital gains on these markets.

The next set of accounts, the ones of worker households in Table 5, are fairly simple and easy to

Production Account of Households (Workers):

Uses	Sources
–	–

Income Account of Households (Workers):

Uses	Sources
Taxes $(\tau_w + \tau_{wp})wL^d + \tau_c rB_w$	Wages wL^d
Consumption $p_v C_w + p_h C_h^d$	Unemployment benefits $w^u(\alpha_w L - L^w)$
–	Pensions $w^r \alpha_r L$
Savings S_w^n	rB_w Interest payments

Accumulation Account of Households (Workers):

Uses	Sources
Financial Surplus FS	Savings S_w^n

Financial Account of Households (Workers):

Uses	Sources
Short-term bond accumulation $\dot{B}_w$	Financial Surplus FS

Table 5: *The production, income, accumulation and financial accounts of worker households.*

explain. First, there is no production account in this sector. Income of the members of the workforce, which may be employed, unemployed or retired, thus derives from wages, unemployment benefits or pension payments where $\alpha_w L$ denotes the total number of persons in the currently registered workforce (L^w the part that is employed) and $\alpha_r L$ the number of retirees who get pension income (α_w=const. the participation rate of the potential workforce L). To this we have to add the interest income on saving deposits (short-term bonds) which is taxed at the general rate used for financial asset income. All and only wage income is subject to taxation at the rate τ_w and total wage related income is again by definition subdivided into nominal consumption (consumption goods and housing services) and savings. Note here that the employment L^d of the employed L^w can differ from their normal employment which is just measured by L^w, the number of persons who are employed. Note also that wages w are net of payroll taxes (used to finance unemployment benefits, social insurance and pensions in particular).

We assume in the following that workers have a positive savings rate and that they hold their savings in the form of short-term bonds solely, which is mirrored here in the accumulation and financial account in a straightforward way.

There are finally the accounts of the fiscal and monetary authority (see Table 6), which due to the many taxation rules and transfer payments that are assumed are more voluminous than the preceding accounts – at least with respect to the income account. There is first however a fictitious production account where the supply of public goods is valued at production costs which consist of government expenditures for goods and labor.

The sources of government income consist of taxes on workers' income (taxed at a uniform rate plus their payroll tax contribution), of taxes on the various forms of profit, interest and rental income (again taxed at a uniform rate), payroll taxes from firms and value added taxes. Uses of the tax income of the government are interest payments, transfers to the unemployed and retirees, and the costs of the aforementioned government 'production'. In general all these uses of the tax income of the government will exceed its income so that there will result a negative amount of nominal

"Production Account" of Fiscal and Monetary Authorities:

Uses	Sources
–	Costless Provision
Government expenditure for services wL_g^d	of public goods

Income Account of Fiscal and Monetary Authorities:

Uses	Sources
Interest payment rB	Wage income taxation $(\tau_w + \tau_{wp})wL^d$
Interest payment B^l	Profit/interest taxation $\tau_c[\rho^{eg}p_yK + rB_g + B_g^l]$
Pensions $w^r\alpha_rL$	Rent income taxation $\tau_c(p_hC_h^d - p_y\delta_hK_h)$
Unemployment benefits $w^u(\alpha_wL - L^w)$	Payroll taxes from firms $\tau_{fp}wL^d$
Government consumption p_vG	Value added tax $\tau_vp_y(C_w + C_c + G)$
Salaries wL_g^d	
Savings S_g^n	

Accumulation Account of the Fiscal Authority:

Uses	Sources
	Savings S_g^n
	Financial Deficit FD

Financial Account of Fiscal and Monetary Authorities:

Uses	Sources
Financial deficit FD	change in short-term debt $\dot{B}$
	change in long-term debt $p_l\dot{B}^l$

Table 6: *The production, income, accumulation and financial accounts of the monetary and fiscal authorities.*

savings S_g^n which balances the income account of the government.

There is however accumulation of real assets in the government sector, called "infrastructure" for briefness, which is part of their total expenditure on the one good of the economy. This means that we only have to look into the financial account of the government to see how the excess of government outlays over government revenue is financed through short- or long-term debt. Note that there is central bank money in the economy used in the background of their interest rate policy rule.

4 The model

In this core section 4 of the paper we develop the extensive form equations of the general model, based on the accounting structure we presented in sections 2 and 3. We significantly reformulate the equations of CCFS (2010), but not the "philosophy" of their chapter 7 model, there derived from the Powell and Murphy (1997) model for the Australian economy, by making it a continuous-time macro-dynamical theoretical model of monetary growth, where all discrete lag structures of the originally quarterly model are depressed.

Our interest is not to fully mirror the dynamical structure and implications of the "Murphy" model, but to make use of such qualitative understanding of applied Keynesian theory to formulate and to investigate, in a first approximation of this 100 equations approach to macroeconometric model building, a set of prominent feedback structures of macro-dynamic theory and their role for economic stability analysis, before fiscal, monetary and social policy will enter the scene.

We thereby attempt to build a bridge between empirically motivated work on structural macro-dynamic model building (where there generally is no analysis of the mechanisms that are hidden in the formulated structure) and theoretical investigations of a reasonably large representation of such economies, where the interest is to see how the balanced growth path will look like, in particular with respect to the share of wages, and its dependence on various forms of taxation, social protection and fiscal and monetary policy.

4.1 Basic definitions

Let us start with some notation to be used in the structural equations of our approach to Keynesian monetary growth.

1. Definitions (Rates of Return, Wages and Prices)

$$\rho^e = \frac{p_y Y^e - B_f^l - (1+\tau_{fp}) w l_f Y - p\delta_f K}{pK} \tag{4.1}$$

$$\rho^{eg} = \frac{p_y Y^e - (1+\tau_{fp}) w l_y Y - p\delta_f K}{pK} \tag{4.2}$$

$$\rho_h = \frac{p_h C_{wh}^d - p\delta_h K_h}{pK_h} \tag{4.3}$$

$$w^n = (1-\tau_w-\tau_{wp})w = q^n w \tag{4.4}$$

$$w^u = q^u w^n, q^u < 1 \tag{4.5}$$

$$w^r = q^r w^n, q^r < 1 \tag{4.6}$$

$$p_l = 1/r_l \tag{4.7}$$

$$p = (1+\tau_v)p_y, \quad p(0) = p_{lo} \quad \text{as initial condition} \tag{4.8}$$

$$p_h = q_h p \tag{4.9}$$

Module 1 of the model provides definitions of expected rates of return ρ^e, ρ_h, based on expected sales in manufacturing (with and without the interest payments B_f^l of firms on corporate bonds, given by perpetuities), and for residential services, notation for hourly wages , w, including income taxation, and augmented by payroll taxes, prices p, p_h, p_l for goods, residential services and for perpetuities, the first including value added taxation, of untaxed pension payments to the retired worker of the workforce, w^r, and untaxed unemployment benefits per unemployed worker (of the workforce), w^u.

Module 2 concerns the household sector where two types of households are distinguished, workers and pure asset holders. Of course, these two types of households are only polar cases in the actual distribution of households types. Nevertheless we believe that it is useful to start from such polar household types before intermediate cases are introduced and formalized.

4.2 Households

We consider the behavioral equations of worker households first:

$$
\begin{aligned}
& 2a. \quad \underline{\text{Households (Workforce)}} \\
Y_w^{Dn} &= w^n L^d + w^u(\alpha^w L - L^w) + w^r \tilde{\alpha}^r L + (1-\tau_c) r B_w \\
&= Y_{ww}^{Dn} + (1-\tau_c) r B_w && (4.10) \\
L^w &= L_f^w + L_g^w && (4.11) \\
L^d &= L_f^d + L_g^d = L_f^d + L_g^w && (4.12) \\
pC_{wc}^g &= c_{wc}(q_h) Y_{ww}^{Dn}, \quad c_{wc}(q_h) = c_{wc}^o + c_1(q_h - q_{ho}), && (4.13) \\
q_h = p_h/p & \quad \text{residential services, price ratio} \\
pC_{wh}^g &= c_{wh}(q_h) Y_{ww}^{Dn}, \quad c_{wh}(q_h) = c_{wh}^o - c_1(q_h - q_{ho}), && (4.14) \\
& \quad \text{demand for residential services in terms of goods}, p_h C_{wh}^d : \text{ in terms of rent prices} \\
S_w^n &= Y_w^{Dn} - pC_{wc}^g - q_h C_{wh}^g = \dot{B}_w && (4.15) \\
\hat{L} &= \hat{L}_r = \hat{L}_{k(ids)} = \bar{\gamma} - m > 0, \quad (L(0), L_r(0) = \alpha^r L(0), L_{k(ids)}(0) \quad \text{given}) && (4.16)
\end{aligned}
$$

We start the description of workers' consumption and savings decision by distinguishing between labor income, unemployment benefits and pensions payments as income items behind workers' consumption plans (all based on the participation rate α^w), retired persons being given by $L^r = \alpha^r L$, with $\tilde{\alpha}^r L, \tilde{\alpha}^r = \alpha^w \alpha^r$ being the number entitled for receiving pension payments, plus interest income on their saving deposits (which however is simply saved again by them).

In the first equation of this module we provide the definition of the disposable income of workers, wages, unemployment benefits and pension payments, the first after taxes, and of their interest rate income after capital taxation. Next, the total employment of the workforce by firms and the government is defined in terms of the number of employed people. By contrast, the third equation defines hours worked within firms and the government sector, assuming that there is no overtime or undertime work in the government sector. The consumption function of workers, the fourth equation, is based on their perception of disposable work-related income in the usual linear fashion. Workers' savings is the difference between their total income and their actual consumption.

The final two equations define the here still very simple demographic structure of our model with respect to worker households. We assume that all age groups or generations (children and juveniles, potential workforce, retired persons) grow with the same rate (in this paper determined by trend investment and its implicit workforce plus family recruitment activities) and that only the proportion α_w of the potential workforce L belongs to the actual workforce L and thus gets pension payments when retired (i.e., $\tilde{\alpha}^r = \alpha_w \alpha^r$ gives the basis for the payments of pensions). Initial conditions with respect to these three cohorts of worker households are considered as given and determine, together with the assumed uniform rates of population growth, the time profile of the portions of the people not in the potential workforce, since already too old. This sector is more advanced than traditional presentations of differentiated households and saving habits, since we consider unemployment benefits and pensions explicitly (and for example the provision of health care implicitly as part of the goods and services provided by the government sector), but needs further elaboration later on.

Next, we consider the other type of household of our model, the (pure) asset owners who desire to consume C_c (goods and houses as supplied by firms through domestic production Y) at an amount that is growing exogenously at the rate $\bar{\gamma}$ and which is thus in particular independent of their current nominal disposable income Y_c^{Dn}. The consumption decision is thus not an important decision for pure asset holders. Their nominal income diminished by the nominal value of their consumption pC_c is then spent on the purchase of financial assets (two types of bonds and money) as well as on investment in housing supply (for worker households). Note here that the one good view of the production of the domestic good entails consumption goods proper and houses (both at commodity prices p) so that asset holders buy houses for their consumption as well as for investment purposes. Investment in the supply of residential services (and that of firms) is not subject to value added taxation.

2b. Households (Asset-Holders, flow-consistency assumed):

$$Y_c^{Dn} = (1-\tau_c)[\rho^e pK + B_f + rB_c + B_c^{lg} + \rho_h pK_h], \quad B_c^l = B_f^l + B_c^{lg} \tag{4.17}$$

$$\hat{C}_c = \bar{\gamma} \tag{4.18}$$

$$S_c^n = Y_c^{Dn} - pC_c \tag{4.19}$$

$$= \dot{M}_c + \dot{B}_c + p_l(\dot{B}_c^{lg} + \dot{B}_f^l) + pI_h, \quad \dot{B}_c = \dot{B} - \dot{B}_w - \dot{M}_c,\ \dot{B}_c^l = \dot{B}_f^l + \dot{B}_g^l$$

$$C_h^s = \alpha_h K_h \quad [C_h^d = ... \text{ see module 2a}] \tag{4.20}$$

$$g_h = (\frac{I_h}{K_h})^d = \alpha_{\rho h}(\rho_h - (r_l - \pi^c)) - \alpha_{rh}(r^l - r^*) + \alpha_{uh}(\frac{C_{wh}^d}{C_h^s} - \bar{u}_h) + \bar{\gamma} + \delta_h \tag{4.21}$$

$$\hat{K}_h = g_h - \delta_h \tag{4.22}$$

4.3 Firms

In module 3 of the model we describe the sector of firms, whose planned investment demand is assumed to be always served, just as all other consumption and investment plans. We thus assume for the short-run of the model that it is of a Keynesian nature since aggregate demand is never rationed, due to the existence of excess capacities, inventories, overtime work and other buffers that exist in real market economies. There is thus only one regime possible, the Keynesian one, for the short-run of the model, while supply side forces come to the surface only in the medium and the long run of the model. Up to certain extreme episodes in history this may be the appropriate modeling strategy for the macro-level of a market economy.

3. Firms (Technology, Production, Employment and Investment)

$$Y^p = y^p(k_g)K, \quad y^p(k_g) = y_o^p + y_1(k_g - k_{go}), k_g = K_g/K \tag{4.23}$$

$$\hat{l}_y = -m, \quad l_y = L_f^d/Y, m > 0 \tag{4.24}$$

$$u = Y/Y^p \in (0,1) \tag{4.25}$$

$$\hat{L}_f^w = \beta_{lf}(L_f^d/L_f^w - \bar{u}_f^w) + \bar{\gamma} - m, \quad \bar{u}_f^w \in (0,1) \tag{4.26}$$

$$g_k = I/K = \alpha_\rho(\rho^{eg} - (r_l - \pi^c)) + \alpha_u(u - \bar{u}) + \bar{\gamma} + \delta_f \tag{4.27}$$

$$Y_f^e = Y - Y^e = \mathcal{I} \tag{4.28}$$

$$S_f^{ne} = p_y Y_f^e \tag{4.29}$$

$$p_l \dot{B}_f^l = pI - p\delta_f K + p_y(\dot{N} - \mathcal{I}) \tag{4.30}$$

$$I^a = I + \dot{N} \tag{4.31}$$

$$\hat{K} = g_k - \delta_f \tag{4.32}$$

We assume for reasons of simplicity a fixed proportions technology[1] with output-employment ratio $1/l_y$ and potential output-capital ratio y^p. Labor productivity $z = 1/l_y$ is growing at the constant rate m, which together with the given potential-output capital ratio suggests that technological change in this model is exogenous and of neutral Harrod type.

Note however that Kaldor's stylized fact of a steady output-capital ratio in our view is based on statistics which neglects product innovation, i.e., that for example "hardisks" have become smaller and smaller in size and weight, but larger and larger in their capacity. Implicitly, the above assumption on potential output therefore contains the possibility that this ratio is rising significantly over time if quality changes were measured in such a quantity in an appropriate way. The model therefore can cover process as well as product innovation in its empirical applications.

[1] See Chiarella and Flaschel (2000) for the treatment of neoclassical smooth factor substitution in place of such a fixed proportions technology.

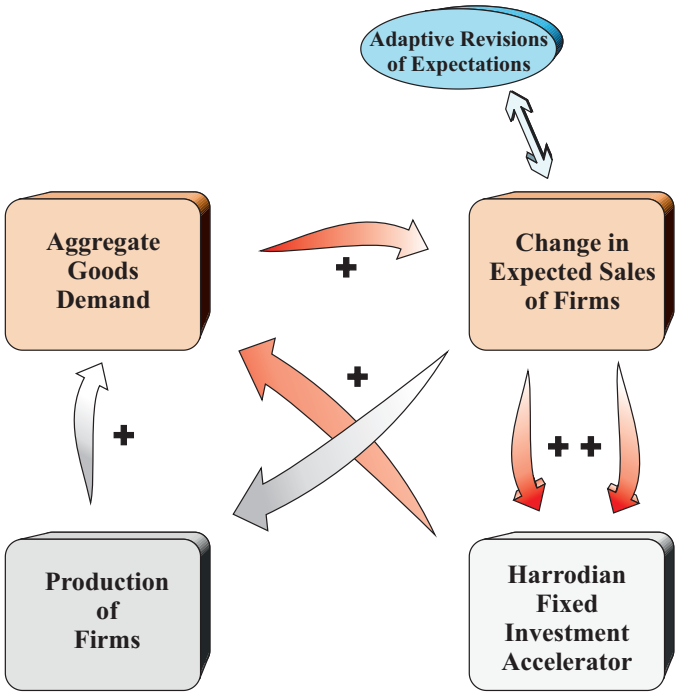


Figure 1: Harrod's Investment Accelerator: Multiplier-accelerator instability.

The output level Y actually produced by firms will be provided in a later module by a Metzlerian output-inventory adjustment mechanism. Depending on this output level we define the rate of capacity utilization u and the employment L_f^d of the workforce employed by firms, which in the short-run is assumed to supply any amount demanded by firms through over- or under-time work.

For the adjustment of the workforce of firms we assume as given a normal rate of employment $\bar{u}_w$ of the workforce, which in principle could be set to unity, representing the normal hours worked by the workforce L_f^w currently employed by firms. Due to absenteeism however, the hours supplied by the employed under normal conditions will be less than the norm L_f^w, and is here represented explicitly by the benchmark level $\bar{u}_w \in (0, 1]$, separating over-time from under-time work caused by fluctuating aggregate demand, expected sales and adjusting output levels. The number of workers L_f^w employed by firms is adjusted by some sort of Okun's Law with speed β_l according to the over- or under-time work $L_f^d - \bar{u}_w L_f^w$ they experience, augmented by a term that accounts for trend growth (always set equal to the trend growth rate $\bar{\gamma}$ in firms' investment decision – to which all other trend growth terms adjust with infinite speed here). The intended rate of investment, finally I^d/K is driven by two forces in this module of the model, Goodwinian profitability and Harrodian capacity utilization. We assume that potential output depends positively on the stock of "infrastructure" k_g.

4.4 The government

In the next module 4 we describe the public sector of the economy in a way that allows for government debt in the steady state and for a monetary policy that adjusts the rate of interest on short-term bonds in view of the level of the long-term rate of interest, the domestic rate of inflation as compared to a target level and the excess activity of firms.

4. Government (Fiscal and Monetary Authority):

$$
\begin{aligned}
T^n &= (\tau_w + \tau_{wp}) w L^d + \tau_{fp} w l_y Y + \tau_v p_y (C^d_{wc} + C_c + G) \\
&+ \tau_c [\rho^{eg} pK + \rho_h pK_h + rB + B^l] - w^u(\alpha^w L - L^w) - w^r \tilde{\alpha}^r L - w L^d_g && (4.33) \\
G &= g_y \bar{u} Y^p(k_g) - g_p(Y^e - Y^e_o) = G_i - G_{bc}, && (4.34) \\
\text{where } & G_i = g_y \bar{u} Y^p = g_i K,\ g_y = g_{y0} - g_{y1}(d - d_o),\ d = \frac{B_g + p_l B^l_g}{Y^e} && (4.35) \\
L^w_g &= l_y g_y \bar{u} Y^p && (4.36) \\
\dot{r} &= \beta_{r_l r}(r_l - r^*) + \beta_{pr}(\hat{p} - \pi^*) + \beta_{ur}(u - \bar{u}), \quad \dot{B}_b = \dot{M}_c && (4.37) \\
\dot{\tau}_{fp} &= \beta_{d\tau_{fp}}\left(\frac{(B + p_l B^l)}{Y^e} - d_o\right) - \beta_{\dot{d}\tau_{fp}}(\tau_{fp} - \tau_{fpo}) && (4.38) \\
S^n_g &= T^n - pG - (1 + \tau_c)(rB + B^l),\ \tau_c(rB + B^l) = \text{"public aid" expenditure} && (4.39) \\
\dot{B} &= -\alpha_{fp} S^n_g && (4.40) \\
p_l \dot{B}^l &= -(1 - \alpha_{fp}) S^n_g && (4.41) \\
\dot{K}_g &= \alpha_g g_y \bar{u} Y^p - \delta_g K_g && (4.42)
\end{aligned}
$$

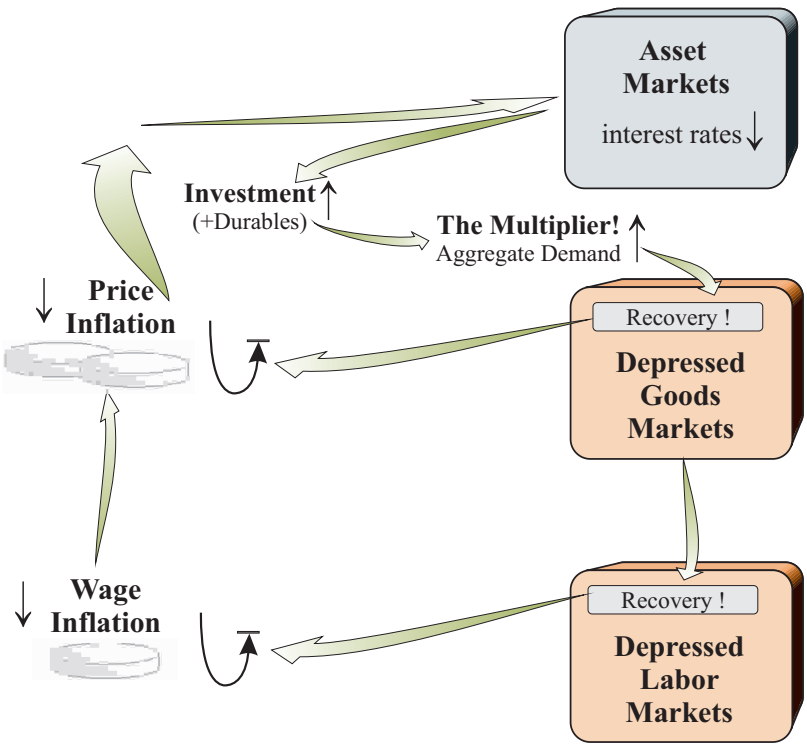


Figure 2: Interest rate policy: The current form of the usage of the stabilizing Keynes-effect.

In the government sector, wage income taxes are raised with rate τ_w on wages w (as well as a payroll taxes $\tau_{wp} w L^d$) and there are untaxed unemployment benefits and pension payments. Unemployment benefits w^u and retirement payments w^r are in fixed proportion to net wages w^n. The capital income tax rate τ_c is applied to the interest income of workers as well as to profit and interest income of asset holders. Finally, the untaxed interest income of the central bank is not – as is often done – assumed to be transferred back into the government sector. With respect to pension payments we have assumed that only the proportion $alpha^w \alpha^r L$ of all people above retirement age receive such payments.

Government expenditures for goods and services are both assumed to be constant fractions normal output, the former however augmented by an anti-cyclical fiscal policy rule. With respect

to the provision of services we in addition assume – in contrast to the sector of firms – that there is no overtime work in the government sector. From the expenditures for goods and services, the interest payments and the transfers made by the government we obtain the savings of the government sector by deducting the sum of these items from T^n, the sum of the received tax payments. These savings will in general be negative in our investigations and thus imply debt financing. Government allocates its debt financing needs in nominal terms in constant proportions to short- and long-term debt and can always realize its intended debt financing due to the flow consistency requirements of such macro-dynamical models.

The contribution of firms to payroll taxation is adjusted towards the objective of a given debt target, and this if needed in a different way in the case of a rising or falling ratio d. The current debt to (expected) sales ratio d determines the government expenditure ratio g_y in a negative way. By contrast, we have already assumed that the stock of public capital k_g, where the rate of change is given by the last equation in this module, exhibits a positive influence on the potential output capital ratio y^p.

Keynesian business cycle policy is characterized by the minus sign in front of the parameter β_{fp}. They (wisely enough) reduce government expenditures in good states of the economy and increases them in bad states (in order to neutralize the pro-cyclical behavior in the private sector), where it may depend on the model builder whether stress will be put on the labor market or on the performance of firms as measured by their rate of capacity utilization. The big question is of course whether undamped private sector business cycle fluctuations can be damped by the intervention of the assumed fiscal and monetary policy rules.

4.5 Quantity and price adjustment processes on the firm level

We now come to the description of the dynamics of quantities (module 5a) and prices (module 5b). Module 5a of the model basically describes a Metzlerian inventory adjustment process for the goods produced by firms (an advanced form of a Keynesian multiplier dynamics). Module 5b then describes the nominal price adjustments in the goods and in the labor market, as well as the adjustment of long-term inflationary expectations π^c, understood as measuring the inflationary climate in which the economy is operating.

$$
\begin{aligned}
& 5a. \quad \underline{\text{Quantity Adjustments in the Production of the Domestic Good}} \\
Y^e &\neq Y^d = C^d_{wc} + C_c + I_h + I + G && (4.43)\\
I^{na} &= pI + pI_h + p_y \dot{N} && (4.44)\\
N^d &= \alpha_{n^d} Y^e && (4.45)\\
\mathcal{I} &= \beta_n (N^d - N) + \bar{\gamma} N^d && (4.46)\\
Y &= Y^e + \mathcal{I} && (4.47)\\
\hat{Y}^e &= \beta_{y^e} (Y^d / Y^e - 1) + \bar{\gamma} && (4.48)\\
\dot{N} &= Y - Y^d && (4.49)
\end{aligned}
$$

In this simple Metzlerian approach to goods market disequilibrium we assume that the output decisions Y of firms are based on expected sales Y^e and intended inventory changes $\mathcal{I}$. The intended inventory changes in turn are based on the desired inventory level N^d of firms assumed to be proportional to their expected sales. Inventories are then adjusted according to the discrepancy $N^d - N$ between desired and actual inventories with speed β_n, the inventory accelerator mechanism, again augmented by a term that accounts for trend growth. Actual inventory changes are given by output minus aggregate demand (which in this Keynesian approach is always served). We ignore here the possibility that inventories may become exhausted, which would provide a situation of rationing with respect to goods demand. The last equation of this module of the model provides the adjustment mechanism for sales expectations Y^e which are assumed to follow observed domestic

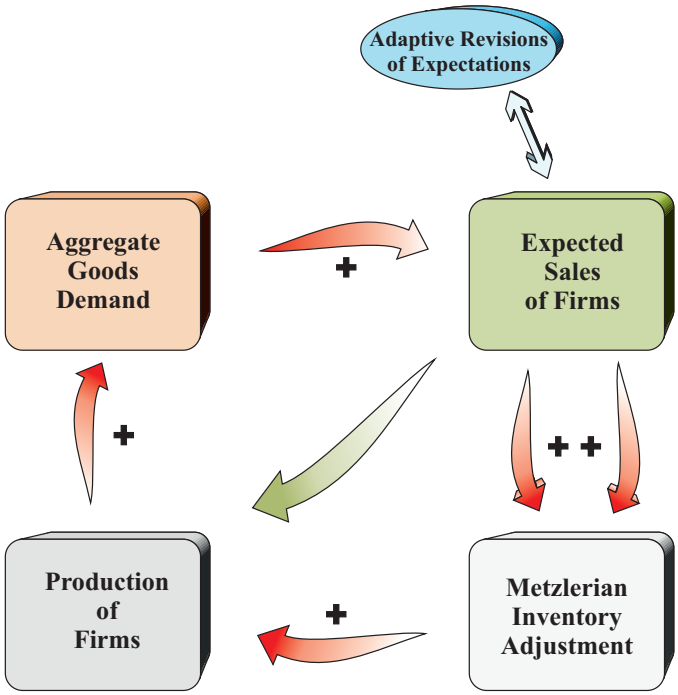


Figure 3: Metzler's accelerator mechanism: inventory-investment driven multiplier-accelerator instability

aggregate demand in an adaptive fashion, again augmented by a term that accounts for trend growth.

This module of the model basically represents a refined dynamic multiplier story to the extent that output adjustment towards aggregate demand is not represented by only one – dynamic - equation, but augmented by inventory adjustments that such a process entails and by the assumption that aggregate demand is not perfectly foreseen. We know that the dynamic multiplier is unstable when the marginal propensity to spend is larger than one, as in the famous Kaldor (1940) trade cycle model, a condition which is here slightly more difficult to establish due to the distinction between output, expected demand and aggregate demand. In addition we may now also have instability due to the Metzlerian inventory adjustment process, which – if sufficiently fast – also establishes a positive feedback chain between output, expected demand and aggregate demand.

These are the basic pure quantity adjustment processes of our Keynesian macrodynamics. A further and final one, the Harrodian mechanism of unstable warranted growth – is shown in figure 1 of the firm sector and is working through the investment function of our model.

Next we consider the wage-price spiral of the model. This type of dynamics represents an important module of the present stage of modeling of an integrated treatment of short-, medium- and long-run behavior.

5*b*. Wage-Price Adjustment Equations, Expectations:

$$\hat{w} = \beta_{ew}(k_g)(\frac{L^w}{\alpha^w L} - \bar{e}(k_g)) + \beta_{uw}(k_g)(\frac{L^d_f}{L^w_f} - \bar{u}^w_f) + \beta_{hw}(\frac{q_h}{q_o} - 1) - \beta_v(v - v_o) + \kappa_w \hat{p} + (1 - \kappa_w)\pi^c + m$$

$$\bar{e}'(k_g) > 0, \beta'_{ew}(k_g) < 0, \beta'_{uw}(k_g) < 0 \tag{4.50}$$

$$\hat{p} = \beta_p(u - \bar{u}) + \kappa_p(\hat{w} - m) + (1 - \kappa_p)\pi^c, \quad p(0) \quad \text{as initial condition, see } d_0 \tag{4.51}$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p} - \pi^c) + (1 - \alpha_{\pi^c})(\pi^* - \pi^c)) \tag{4.52}$$

$$\hat{p}_h = \beta_{hp}(\frac{C^d_{wh}}{C^s_h} - \bar{u}_h) + \pi^c \tag{4.53}$$

With respect to gross nominal wages w (which include income, but not yet payroll taxes) we assume that their rate of growth $\hat{w}$ depends positively on demand pressure on the external labor

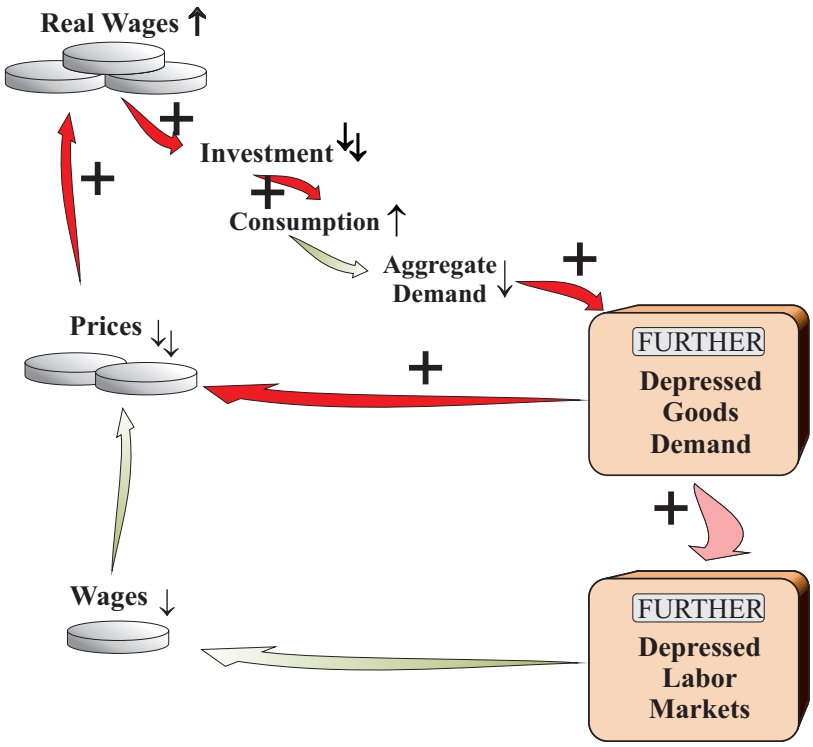


Figure 4: An adverse Rose real-wage effect when investment dominates consumption.

market, measured by the deviation of the rate of employment from the NAIRU rate of employment $\bar{e}$, and on labor demand pressure within the firms, measured by the degree of over- or undertime work compared to the normal worktime of the employed. Cost pressure for wage earners is measured by two related expression. Firstly, and on the one hand, we assume – in order to show that myopic perfect foresight is not at all a problem for Keynesian macroeconomics – that workers have perfect knowledge of the short-term evolution of price inflation, but use in addition, and on the other hand, on the basis of this knowledge, an inflation rate expression, π^c, representing the inflationary climate in which the current inflation regime is operating. The inflationary climate variable π^c is thus a magnitude that is related to the medium-run and is assumed to be updated in the adaptive fashion shown in the last equation in this module. Cost pressure for workers is then measured as a weighted average of these two expressions for price inflation $\hat{p}$ and π^c, implying that workers and their unions look beyond the short-run (for $\kappa_w < 1$) and thus take into account also the climate in which current inflation is evolving. This guarantees that the dynamics of the model is not heavily dependent on whether short-term expectations are perfect or not. We here simply save, by the assumption of myopic perfect foresight, another dynamic law that would describe the evolution of short-term expectations, without much change in the implied dynamics if these expectations are revised sufficiently fast. We finally state that the assumption of Harrod neutral technical change of rate m requires that this term has to be added to the right-hand side of the money wage dynamics in order to allow for a steady state solution later on. Wage claims thus include the observed change in labor productivity in a one to one fashion, called a complete productivity pass-through in the literature (not fully confirmed by empirical estimates in this very strict way). The wage bargaining process is mitigated in an economy with a higher level of infrastructure k_g, since the role of corporatism is increased thereby.

Turning next to price inflation we assume (analogously to wage inflation) that it is also based on demand pressure here measured by the rate of capacity utilization u in its deviation from what firms conceive as normal capacity utilization $\bar{u}$. Regarding cost pressure we assume again myopic perfect foresight, now of firms with respect to wage inflation, and form again a weighted average with the inflationary climate π^c also characterizing the medium-run expectations of firms. Note that the growth rate of labor productivity has been subtracted from the cost pressure term $\hat{w}$ on the right hand side of the price level dynamics in order to allow for a steady state solution later on.

Figure 6 shows in this context an example of adverse or destabilizing Rose-effects, the other adverse Rose (1967) effect being given by the situation where all arguments in this figure are reversed. Below we summarize all four possibilities by way of their partial feedback chains. Note that we do not yet consider asymmetric Phillips curves, which implies that Rose adjustment patterns can be explained in terms of inflation as well as deflation.

The case of a destabilizing Rose or real wage effect shown in Figure 6 is based on the situation that wages are less flexible than prices (the wage-price is then called labor-market led) and that investment is more responsive to real wage changes than consumption (the well-known profit led case). In deflationary periods, we therefore have that real wages will indeed rise, instead of falling as is generally assumed, which implies a decline in aggregate demand and thus further depressed situation with further increases in the real wage, further reduction in economic activity and so on, i.e., a deflationary spiral will be established in this way.

Two empirical observations are here needed in order to prevent such a spiral, the first being that the typical situation may rather be one where wages are more flexible with respect to demand pressure than prices which implies an "normal" real wage adjustment, see Flaschel and Krolzig (2003) for such empirical estimates of wage and price flexibilities for the US-economy. The second observation is that this may apply only to cases of high economic activity, while the hierarchy of price and wage flexibilities is reversed in depressions (though an asymmetry in the money wage Phillips curve), leading to stable real wage adjustments in such situations, see Hoogenveen and Kuipers (2000) for an empirical investigation of this type.

The four partial Rose wage adjustment mechanisms in sum are:

$$w/p \uparrow \begin{cases} \Longrightarrow C \uparrow \quad Y^d, Y^e, Y \uparrow \quad w \uparrow \quad w/p \uparrow \\ \Longrightarrow C \uparrow \quad Y^d, Y^e, Y \uparrow \quad w \uparrow \quad w/p \downarrow \\ \Longrightarrow I \downarrow \quad Y^d, Y^e, Y \downarrow \quad w \downarrow \quad w/p \downarrow \\ \Longrightarrow I \downarrow \quad Y^d, Y^e, Y \downarrow \quad w \downarrow \quad w/p \uparrow \end{cases}$$

<u>Normal Rose Effects:</u> Rose effect

1a) Real wage increases (decreases) will be reversed in the case where they reduce (increase) economic activity when nominal wages respond stronger than the price level to the decrease (increase) in economic activity.

1b) Real wage increases (decreases) will be reversed in the case where they increase (reduce) economic activity when the wage level responds weaker than the price level to the increase (decrease) in economic activity.

<u>Adverse Rose Effect:</u>

2a) Real wage increases (decreases) will be further increased in the case where they reduce (increase) economic activity when the wage level responds weaker than the price level to the decrease (increase) in economic activity.

2b) Real wage increases (decreases) will be further increased in the case where they increase (reduce) economic activity when the wage level responds stronger than the price level to the increase (decrease) in economic activity.

4.6 The dynamics of asset market prices and expectations

The sixth module lists the dynamic adjustment equations we assume to hold for the single flexible asset price of our model: long-term bonds, p_l.

6. Financial Markets and Expectations: Implications of portfolio re-allocations

$$\hat{p}_l = \beta_{p_l}(r_l + \pi_l - r), \quad dB_c + p_l dB_c^l = 0 \tag{4.54}$$

$$\dot{\pi}_{ls} = \beta_{\pi_{ls}}(\hat{p}_l - \pi_{ls}) \tag{4.55}$$

$$\dot{\pi}_{lr} = \beta_{\pi_{lr}}(\hat{p}_{lo} - \pi_{lr}) \tag{4.56}$$

$$\pi_l = \alpha_s \pi_{ls} + (1 - \alpha_s)\pi_{lr} \tag{4.57}$$

Since we allow for only one risky financial asset, we simplify the dynamic portfolio approach on the basis of Walras' law of stocks to just one adjustment equation for the price of the considered perpetuities. Moreover we use the expectations formation approach of CFPS (2013)[2] without the opinion dynamics investigated there in detail. We thus make use here of a fixed number of chartists and fundamentalist only, which are characterized by adaptive and regressive expectation formation, respectively. This approach is used here to indicate that the interest rate policy of the CB does not operated directly on the real part of the economy (as it is often assumed), but must channel itself through the financial markets first.

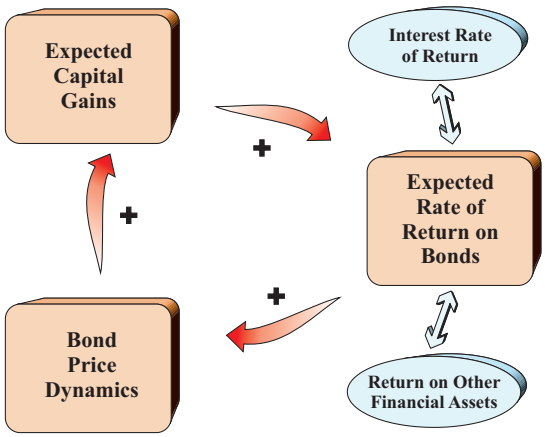


Figure 5: Blanchard-type bond-market instability [Boundedness by interacting "naive" and "rational" endogenous opinion formation

In the framework of an open economy, a Dornbusch-type Exchange Rate Dynamics can be formulated in a similar fashion by means of a delayed form of Uncovered Interest Rate adjustment process, leading from increasing expected depreciation of a currency to increasing expected returns of the foreign currency to increasing actual depreciation and from there again to further increases in expected depreciation of the domestic currency.

5 Collecting the Growth Laws of Motion $\hat{\cdot}$

In order to study the dynamics of our disequilibrium growth model analytically and numerically it is necessary to reduce the equations of the model to intensive or per (value) unit of capital form. To simplify subsequent presentations of the dynamics of the model and also its steady state solution we assume in the remainder of this paper for the consumption of asset owners $\boxed{C_c(0) = 0}$. Moreover we set $\boxed{\delta_f = \delta_h = \delta_g.}$ These two assumptions do not restrict the dynamical behavior of the system in an important way. We will also use the abbreviations $\boxed{\tilde{q}^r w = q^r(1 - \tau_w - \tau_{wp})w,\ \tilde{q}^u w = q^u(1 - \tau_w - \tau_{wp})w}$ in the following intensive form of the model.

Note also that the model is still based on a complete productivity pass-through into the wage share and the inflation rate, i.e., the rate of labor productivity growth does not yet appear as a parameter in the dynamics considered below.

[2]prepared as working paper for the ILO in a preceding project.

5.1 The laws of motion

The Quantity Dynamics of Firms:

$$\hat{y}^e = \beta_{y^e}(y^d/y^e - 1) + \bar{\gamma} - (g_k - \delta) \quad [\text{ Metzlerian Sales Expecations}] \tag{5.1}$$

$$\dot{\nu} = y - y^d - (g_k - \delta)\nu \quad [\text{ Metzlerian Inventory Adjustment}] \tag{5.2}$$

$$\hat{l}_f^{we} = \beta_{lf}(u_f^w - \bar{u}_f^w) + \bar{\gamma} - (g_k - \delta), \quad l_f^{we} = L_f^w/l_y, u_f^w = l_f^{de}/l_f^{we} \quad [\text{ Okun's Law}] \tag{5.3}$$

The Growth Law of Industrial Wage Share $v_f = \frac{wL_f^d}{pY}$, Prices $p, q_h = \frac{p_h}{p}$ and Inflation Climate π^c:

$$\hat{v}_f = \frac{(1-\kappa_p)[\beta_{ew}(\frac{l^{we}}{\alpha^w l^e} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1) - \beta_v(v - v_o)] - (1-\kappa_w)[\beta_p(u - \bar{u})]}{1 - \kappa_p\kappa_w} \tag{5.4}$$

$$\hat{p} = \frac{\beta_p(u - \bar{u}) + \kappa_p[\beta_{ew}(\frac{l^{we}}{\alpha^w l^e} - \bar{e}) + \beta_{uw}(u_f^w - \bar{u}_f^w) + \beta_{hw}(\frac{q_h}{q_o} - 1) - \beta_v(v - v_o)]}{1 - \kappa_p\kappa_w} + \pi^c$$

$$\dot{\pi}^c = \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p} - \pi^c) + (1 - \alpha_{\pi^c})(\pi^* - \pi^c)) \tag{5.5}$$

$$\hat{p}_h = \beta_h(\frac{c_{wh}^d}{\alpha_h k_h} - \bar{u}_h) + \pi^c, \quad i.e.$$

$$\hat{q}_h = \beta_h(\frac{c_{wh}^d}{\alpha_h k_h} - \bar{u}_h) + \pi^c - \hat{p} \tag{5.6}$$

Asset Prices and Medium-run Expectations:

$$\hat{p}_l = \beta_{p_l}[r_l + \pi_l - r] = -\hat{r}_l \quad [\text{ Interest-spread driven Bond-price Dynamics}] \tag{5.7}$$

$$\dot{\pi}_{ls} = \beta_{\pi_{ls}}(\hat{p}_l - \pi_{ls}) \quad [\text{ "Naive" Expectations}] \tag{5.8}$$

$$\dot{\pi}_{lr} = \beta_{\pi_{lr}}(\hat{p}_{lo} - \pi_{lr}) \quad [\text{ "Rational" Expectations}]$$

$$\pi_l = \alpha_s\pi_{ls} + (1 - \alpha_s)\pi_{lr} \quad [\text{ Average Expectations}] \tag{5.9}$$

Growth Dynamics:

$$\hat{l}^e = \bar{\gamma} - (g_k - \delta), \quad l^e = \frac{L/l_y}{K} = \frac{L/L^d}{K/Y} \quad [\text{ Labor Intensity}] \tag{5.10}$$

$$\hat{k}_h = g_h - \delta - (g_k - \delta) \quad [\text{ Housing Stock}] \tag{5.11}$$

$$\dot{k}_g = \alpha_g g_y \bar{u} y^p - (g_k - \delta)k_g \quad [\text{ Public Capital Stock}] \tag{5.12}$$

Monetary and Fiscal Policy Rules:

$$\dot{r} = \beta_{r_l r}(r_l - r^*) + \beta_{qr}(\frac{q_h}{q_o} - 1) + \beta_{pr}(\hat{p} - \pi^*) + \beta_{ur}(u - \bar{u}) \quad [\text{ Interest Rate Policy}] \tag{5.13}$$

$$\dot{\tau}_{fp} = \beta_{d:\tau_{fp}}(\frac{b + p_l b^l}{y^e} - d_o) - \beta_{\dot{d}:\tau_{fp}}(\tau_{fp} - \tau_{fpo})) \quad [\text{ Payroll Tax Policy}] \tag{5.14}$$

$$s_g^n = t - g - (rb + b^l), \quad s_{go}^n = t_o - g_o - (r_o b_o + b_o^l) \tag{5.15}$$

$$t = (1 - q^n)v_f y + \tau_{fp}v_f y + \tau_v(c_{wc}^d + c_c + g) + \tau_c[\rho^{eg} + \rho_h k_h]$$
$$- [\tilde{q}^u v_f(\alpha^w l^e - l_f^{we} - g_y\bar{u}y^p) + \tilde{q}^r v_f \tilde{\alpha}^r l^e + q^n v_f g_y \bar{u} y^p]$$

$$g = g_y\bar{u}y^p - g_p(y^e - y_o^e), \; g_y = g_{y0} - g_{y1}(d - d_o)$$

Government Debt Accumulation: $d = (b + p_l b^l)/y^e, b = \frac{B}{pK}, b_l = \frac{B^l}{pK}$

$$\dot{b} = -\alpha_{fp} s_g^n - (g_k + \hat{p} - \delta) b \quad [\text{ Short-term Debt } b] \tag{5.16}$$

$$\dot{b}^l = -\frac{(1-\alpha_{fp}) s_g^n}{p_l} - (g_k + \hat{p} - \delta) b^l \quad [\text{ Long-term Debt } b^l] \tag{5.17}$$

The above laws of motion do not yet form an autonomous system of differential equations, but must be augmented by certain identities and algebraic equations about macro-economic behavior in order to become a complete Keynesian model of the working of the macro-economy.

Supplementing Static Relationships for the Laws of Motion of the Baseline Model:

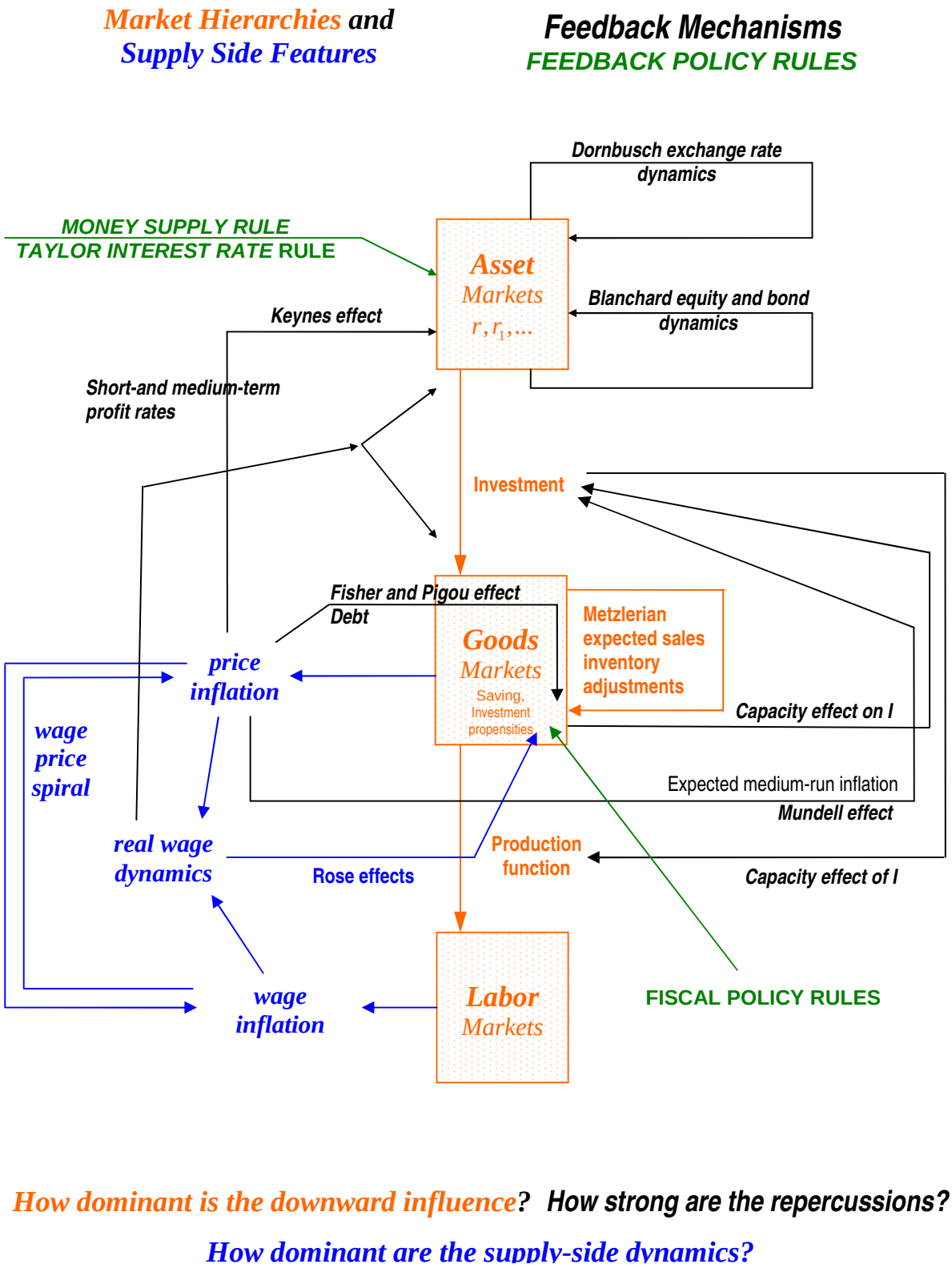
$$\begin{aligned}
y^d &= c_{wc}^g + c_c(0) + g_k + g_h k_h + g_y \bar{u} y^p - g_p(y^e - y_o^e) \\
y_{ww}^D &= q^n v_f l^{we} + \tilde{q}^u v_f (\alpha^w l^e - l^{we}) + \tilde{q}^r v_f \tilde{\alpha}^r l^e \\
&= ((q^n - \tilde{q}^u) l^{we} + (\tilde{q}^u \alpha^w + \tilde{q}^r \tilde{\alpha}^r) l^e) v_f, \quad l_w^e = l_{wf}^e + \quad g_y \bar{u} y^p, \; v_f = \frac{w L_f^d}{pY} \\
y &= y^e + \beta_n(\alpha_{n^d} y^e - \nu) + \bar{\gamma} \alpha_{n^d} y^e \\
c_{wc}^g &= c_{wc}(q_h) y_{ww}^D \\
c_{wh}^g &= c_{wh}(q_h) y_{ww}^D, \quad \text{in terms of goods, just as supply} \quad \alpha_h K_h \\
\rho^{eg} &= y^e/(1+\tau_v) - (1+\tau_{fp}) v_f y - \delta \\
\rho_h &= q_h c_{wh}^g / k_h - \delta \\
g_k &= \alpha_\rho(\rho^{eg} - (r_l - \pi^c)) - \alpha_r(r^l - r^*) + \alpha_u(u - \bar{u}) + \bar{\gamma} + \delta \\
g_h &= \alpha_{\rho h}(\rho_h - (r_l - \pi^c)) - \alpha_{rh}(r^l - r^*) + \alpha_{uh}(\frac{c_h^d}{\alpha_h k_h} - \bar{u}_h) + \bar{\gamma} + \delta \\
d &:= \frac{b + p_l b^l}{y^e}, \quad d_0(t) = d(t)/p(0)!
\end{aligned}$$

The logic of Keynes' approach to macro-statics (and macro-dynamics in the chapter "Notes on the Trade Cycle" of his "General Theory") is summarized in the following diagram which shows the assumed market hierarchy of his theory, the repercussions this theory allows for and the impact of fiscal and monetary policy, nowadays often formulated as fiscal and monetary policy rules.

The diagram in particular shows that the argument that nominal wage decreases immediately imply price level decreases must be considered as rather naive from his perspective, to say the least. Note in this respect that the contributions by Goodwin (1967) and Rose (1967) have significantly enlarged the perspective of Keynesian macro-dynamics due to the complex working of the wage-price spiral their approaches have led us to.

Note also that the contribution by Metzler, his completion of the dynamic multiplier story, is a compelling one, since the dynamic multiplier simply ignores what happens to inventories when the goods market is not in equilibrium. By contrast, assuming that it is always in IS-equilibrium in our view represents an assumption which is hard to swallow.

Traditional Keynesian Theory: Summary



How dominant is the downward influence? ***How strong are the repercussions?***

How dominant are the supply-side dynamics?

Figure 6: Keynes' causal nexus and his repercussion analysis

6 The " Balanced Growth Reference Path" of the model

In this section of the paper we show that there is a uniquely determined, economically meaningful balanced growth path or steady state solution of our model which provides us with a useful reference path for the dynamical evolutions over time the model implies, which may or may not converge to this steady state reference solution.

The first set of steady state conditions presented below concerns the growth rates of our economy:

$$\begin{aligned} g_{ko} &= g^d_{ko} = \bar{\gamma} + \delta \\ g_{ho} &= g^d_{ho} = \bar{\gamma} + \delta \end{aligned}$$

These equations state that capital (and also output) will grow with the external rate $\bar{\gamma}$, to which also the natural rate of growth of the working population will adjust (in the present model through instantaneously fast "migration" processes, giving rise to their growth rate $\bar{\gamma}(k_{go}) - m$).

The next set of steady state conditions concerns inflation and expected inflation – for all prices and the capital gains that exist in our model which are all equalized , except for wage rates. Note that only the state variables of the model are numbered in the following set of steady state equations:

$$\pi^* = \pi^c_o = \hat{p}_o = \hat{p}_{yo} = \hat{p}_{ho} \tag{6.1}$$

$$r^* = r_{lo} = 1/p_{lo} = r_o \tag{6.2}$$

$$\pi_{ls} = 0 \tag{6.3}$$

$$\pi_{lr} = 0 \tag{6.4}$$

$$r^* - \pi^* = \rho^{eg}_o = \rho_{ho}$$

The next block concerns the steady state determination of various quantities of the model. If one wants also y^p_o to depend on the relative size of the infrastructure k_g by way of a function $f(\cdot)$, one has to solve the first of the following equations in a different way, by inverting the function $f(k_g)/k_g$ and applying it to the argument $\frac{1}{\alpha_g g_y \bar{u}}$ thereafter.

$$k_{go} = \text{from} \quad \alpha_g g_y \bar{u} y^p_o = \bar{\gamma} k_{go}, \ i.e.: \tag{6.5}$$

$$\begin{aligned} k_{go} &= \frac{\alpha_g g_{y0} \bar{u} y^p_o}{\bar{\gamma}} \\ y_o &= \bar{u} y^p_o \\ l^{de}_{fo} &= l^{we}_{fo} = y_o \end{aligned} \tag{6.6}$$

$$\begin{aligned} l^{we}_{go} &= g_{y0} y_o \\ l^{we}_o &= l^{we}_{fo} + l^{we}_{go} = (1 + g_{y0}) y_o \\ l^e_o &= l^{we}_o / (\alpha^w e_o) \quad [e_o = \bar{e}(k_{go})]] \end{aligned} \tag{6.7}$$

We clearly see how government can influence the size of its infrastructure relative to the capital stock of firms. Moreover it can create in this way a more tranquil scenario for wage negotiations, in particular concerning the level of the so-called NAIRU.

Further steady state relationships on the side of quantities are:

$$y^e_o = \frac{y_o}{1 + \bar{\gamma} \alpha_{n^d}} \tag{6.8}$$

$$\nu_o = \alpha_{n^d} y^e_o \tag{6.9}$$

$$\begin{aligned} y^d_o &= c^g_{wco} y^D_{wwo} + \bar{\gamma} + \delta + (\bar{\gamma} + \delta) \frac{c^g_{who} y^D_{wwo}}{\bar{u}_h \alpha_h} + g_{y0} y_o \\ &= (c^g_{wco} + (\bar{\gamma} + \delta) \frac{c^g_{who}}{\bar{u}_h \alpha_h}) y^D_{wwo} + \bar{\gamma} + \delta + g_{y0} y_o = y^e_o \rightarrow \\ y^D_{wwo} &= \frac{y^e_o - (\bar{\gamma} + \delta + g_{y0} y_o)}{c^g_{wco} + (\bar{\gamma} + \delta) \frac{c^g_{who}}{\bar{u}_h \alpha_h}} > 0 \end{aligned} \tag{6.10}$$

We see that a variety of parameters influence the relative level of the disposable wage-related income of worker households which is determined in the market for goods and there in fact from the side of uses, not resources. We in particular see that increasing long-run government expenditure does not have a positive influence on this income. There is thus from this angle no "free lunch" for the creation of public infrastructure, which however may increase y^e via y^p, if the linear function assumed in module 3 is suitably modified.

Similar effects hold for the wage share in GDP of workers in the private sector of the economy, which is dependent on all the sources of worker households's incomes. The minus sign in front of $\tilde{q}^u$ is dominated by the first appearance of $\tilde{q}^u$ in the denominator below and thus not giving rise to a positive impact of this parameter on v_{fo}. The negative impact of unemployment benefits on the share of wages may also be small.

Increasing the tax rates on workers' income influences the gross wage share of the workforce of firms in a positive way, while it has a negative effect on their net wage income $q^n v_{fo}$. And a higher level of the ratio k_g can improve their wage share if it really has the effect of making the bargaining process more moderate.

$$
\begin{aligned}
y^D_{wwo} &= [(\tilde{q}^u\alpha^w + \tilde{q}^r\tilde{\alpha}^r)l^e_o + (q^n - \tilde{q}^u)l^{we}_o]v_{fo} \rightarrow \\
v_{fo} &= \frac{y^D_{wwo}}{[(\tilde{q}^u\alpha^w + \tilde{q}^r\tilde{\alpha}^r)/(\alpha^w e_o(k_g)) + (q^n - \tilde{q}^u)](1+g_{y0})y_o} \in (0,1) \qquad (6.11) \\
\rho^{eg}_o &= \frac{y^e_o}{1+\tau_v} - (1+\tau_{fp})v_{fo}y_o - \delta = r^* - \pi^* \rightarrow \\
\tau_{fpo} &= \frac{y^e_o(k_g)/(1+\tau_v) - v_{fo}(k_g)y_o - \delta - (r^* - \pi^*)}{v_{fo}(k_g)y_o} > 0 \qquad (6.12) \\
k_{ho} &= c^g_{who}y^D_{wwo}/(\bar{u}_h\alpha_h) \qquad (6.13)
\end{aligned}
$$

We see that an increase in the value added tax rate and the long-run interest rate decreases the payroll tax burden for firms. The determination of the relative size of the housing stock is as expected.

Note again that the fraction $\frac{y^D_{ww}}{y}$ is the share of work-related incomes in gross output, while v_{fo} is the share of wages paid by firms per output unit and is thus also measuring the unit wage costs of firms. One has to take note of the fact that the logic of steady states is not mirroring the logic of the Keynesian business cycles of the model, since decreases in the consumption coefficients imply increases of the above income fraction. Note however that the goods consumption of workers plus gross investment in housing per unit of capital remain unchanged in such cases.

An isolate decrease of c^g_{wco} must therefore result in a decrease of $c^g_{wco}y^D_{wwo}$ and an increase of $c^g_{who}y^D_{wwo}$ and thus a shift from goods consumption to "flat let for rent" consumption in the long-run, with no definite conclusion concerning the total consumption effect. An increasing portion of pensioners α^r works on the balanced growth path via v_{fo} and t_o, see below. It affects v_{fo} negatively and increases public debt without much consequences as long as the economy remains stable.

And for the aggregate of government bonds we finally get (per unit of pK):

$$(\bar{\gamma}+\pi^*)(b_o+b_o^l/r^*) = -s_{go}^n = g_o - t_o + r^*(b_o+b_o^l/r^*), \quad i.e.$$

$$b_o + b_o^l/r^* = \frac{g_o - t_o}{\bar{\gamma}+\pi^*-r^*}, \quad \text{and thus have as result of debt financing:} \tag{6.14}$$

$$d_o = \frac{g_o - t_o}{(\bar{\gamma}+\pi^*-r^*)y_o^e}, \text{ and in the "actual world":} \tag{6.15}$$

$$\begin{aligned}
d_0 &= \frac{g_o - t_o}{(\bar{\gamma}+\pi^*-r^*)p(0)y_o^e}] \\
t_o &= (1-q^n)v_{fo}(k_g)y_o + \tau_{fpo}v_{fo}(k_g)y_o + \tau_v(c_{wc}(q_{ho})y_{wwo}^D + g_o) + \tau_c[(r^*-\pi^*)(1+k_{ho})] \\
&- [\tilde{q}^u v_{fo}(k_g)(\alpha^w l_o^e - l_o^{we}) + \tilde{q}^r v_{fo}(k_g)\tilde{\alpha}^r l_o^e + q^n v_{fo}(k_g)g_{y0}y_o] \\
&= [(1-q^n)y_o + \tau_{fpo}y_o - [\tilde{q}^u((1+g_{y0}y_o)\frac{1-e_o}{e_o}) + \tilde{q}^r\tilde{\alpha}^r\frac{(1+g_{y0})y_o}{e_o} + q^n g_{y0}y_o]]v_{fo}(k_g) \\
&+ \tau_v(c_{wc}(q_{ho})y_{wwo}^D + g_o) + \tau_c[(r^*-\pi^*)(1+k_{ho})] \\
g_o &= g_{y0}\bar{u}y_o^p
\end{aligned}$$

from which the individual distribution of bonds (between workers and pure asset holders) can be derived if this is desired.

The tax to capital-stock ratio t_o is a fairly complicated expression, due to the encompassing tax and transfer system that characterizes the considered economy, where $\tau_w, \tau_{wp}, \tau_c, \tau_v$ are exogenously given and τ_{fp} endogenously (as well as export and import taxtion).

Summing up – and this conclusion holds due to the admitted neoclassical or Friedmanian supply side influences on the long-run output-capital ratio y_o and the long-run employment rate e_o – we have that the most effective way to increase the wage-related share of incomes y_{wwo}^D is to increase the steady state level of the stock ratio k_g, the size of the public capital stock relative to the industrial capital stock, because of its assumed impact on the potential output of firms (and also on taxes per unit of capital).

Such an assumption is a very natural one and also often assumed as a positive externality in the environment of neoclassical production functions when issues of for example endogenous growth are investigated from the perspective of policy making.

We conclude that an adequate distribution of income between capital and labor in capitalist economy that can be considered as an advanced one demands an advanced government sector with an advanced system of public investments in all sorts of things in order to create the frame within which capitalism can develop his innovative potential without endangering a social structure as we have experienced it in the property phase after World War II.

7 Numerical investigations

7.1 "Barebone" Capitalism: A core case, and also a case of "Government-funded" People's Capitalism

We here collect the equations[3] of the model which characterize a type of "barebone capitalism," lacking any supply of public goods and services, a concept comparable to a "barebone computer". The positive influence of the building-up of an infrastructure by the government remains totally untapped here. We here consider inflation-free steady states for simplicity only.

$$
\begin{aligned}
\hat{y}^e &= \beta_{y^e}(y^d/y^e - 1) + \bar{\gamma} - (g_k - \delta) \\
\dot{\nu} &= y - y^d - (g_k - \delta)\nu \\
\hat{l}_f^{we} &= \beta_{lf}(u_f^w - \bar{u}_f^w) + \bar{\gamma} - (g_k - \delta), \quad u_f^w = l_f^{de}/l_f^{we} \\
\hat{v}_f &= \frac{1}{1-\kappa_p\kappa_w}[(1-\kappa_p)[\beta_{ew}(k_g)(\frac{l^{we}}{\alpha^w l^e} - \bar{e}(k_g)) + \beta_{uw}(k_g)(u_f^w - \bar{u}_f^w) + \beta_{hw}(q_h/q_o - 1) - \beta_v(v - v_o)] \\
&- (1-\kappa_w)\beta_p(u - \bar{u})], \quad v_f = \frac{wL_f^d}{pY} \\
\hat{p} &= \frac{\beta_p(u-\bar{u}) + \kappa_p[\beta_{ew}(k_g)(\frac{l^{we}}{\alpha^w l^e} - \bar{e}(k_g)) + \beta_{uw}(k_g)(u_f^w - \bar{u}_f^w) + \beta_{hw}(q_h/q_o - 1) - \beta_v(v - v_o)]}{1-\kappa_p\kappa_w} + \pi^c \\
\dot{\pi}^c &= \beta_{\pi^c}(\alpha_{\pi^c}(\hat{p} - \pi^c) + (1-\alpha_{\pi^c})(0 - \pi^c)) \\
\hat{q}_h &= \beta_h(\frac{c_{wh}^g}{\alpha_h k_h} - \bar{u}_h) + \pi^c - \hat{p} \\
\hat{p}_l &= \beta_{p_l}[r_l + \pi_l - r^*] = -\hat{r}_l, \quad \pi_l = \alpha_s\pi_{ls} + (1-\alpha_s)\pi_{lr} \\
\dot{\pi}_{ls} &= \beta_{\pi_{ls}}(\hat{p}_l - \pi_{ls}) \\
\dot{\pi}_{lr} &= \beta_{\pi_{lr}}(\hat{p}_{lo} - \pi_{lr}) \\
\hat{l}^e &= \bar{\gamma} - (g_k - \delta) \\
\hat{k}_h &= g_h - \delta - (g_k - \delta) \\
s_g^n &= \tau_{fp} v_f l_f^d - (rb + b^l) \\
\dot{b} &= -\alpha_{fp} s_g^n - (g_k - \delta)b \\
\dot{b}^l &= -\frac{(1-\alpha_{fp})s_g^n}{p_l} - (g_k - \delta)b^l, \quad d_0 = (b_o + p_{lo}b_o^l)/p(0) \\
\dot{r} &= \beta_{r_l r}(r_l - r^*) + \beta_{qr}(q/q_o - 1) + \beta_{pr}\hat{p} + \beta_{ur}(u - \bar{u})
\end{aligned}
$$

[3] We make use the law of motion $\dot{\tau}_{fp} = \beta_{d:\tau_{fp}}(\frac{b+p_l b^l}{y^e} - d_o) - \beta_{d:\tau_{fp}}(\tau_{fp} - \tau_{fpo})$ in place of the algebraic equation for τ_{fp} and have simplified the investment functions in a secondary way.

Supplementing Static Relationships:

$$
\begin{aligned}
y &= y^e + \beta_n(\alpha_{n^d} y^e - \nu) + \bar{\gamma}\alpha_{n^d} y^e, \quad u = y/y^p(k_g) \\
y^d &= c^g_{wc} + g_k + g_h k_h \\
y^D_{ww} &= v_f l^{we}_f = v_f y \\
c^g_{wc} &= c_{wc}(q_h) y^D_{ww} \\
c^g_{wh} &= c_{wh}(q_h) y^D_{ww} \\
\rho^{eg} &= y^e - (1+\tau_{fp}) v_f y - \delta \\
\rho_h &= q_h c^g_{wh}/k_h - \delta \\
g_k &= \alpha_\rho(\rho^{eg} - r^*) - \alpha_r(r_l - r^*) + \alpha_u(u - \bar{u}) + \bar{\gamma} + \delta \\
g_h &= \alpha_{\rho h}(\rho_h - r^*) - \alpha_{rh}(r_l - r^*) + \alpha_{uh}(\frac{c^g_{wh}}{\alpha_h k_h} - \bar{u}_h) + \bar{\gamma} + \delta \\
\tau_{fp} &= \frac{y^e_o - v_{fo} y_o - \delta - \rho^{eg}}{v_{fo} y_o}
\end{aligned}
$$

```
                    PARAMETERS (yeshock=1.02, TimeHorizon=50)
betqw = 1;              betp =    1;                betpic    = 0.1;
betpl = 0.5;            betpils=  0.2;              betpilr = 0.2;
betye = 5;              betn =    1;                betlf   = 0.5;
beth =  0.5;

alpnd =  0.1;
alppic = 0.5;           alppils =0.5;
alpw=    1;             alpeh =       0.1;
alpuh=   0.3;           alprhoh =     0.2;  alprh=  0.2;
alpu =   0.5;           alprho =      0.5;
kapp =   0.5;           kapw =        0.5;
baru =   1;             baruw =       1.0;  baruh =  1;      alph=  0.15;
bargam = 0.06;          del =         0.1;  yp0    =  1;     yp1=   0;

betuw0 =      0.4;      betuw1= 0;
betew0 =      0.4;      betew1= 0;
barew0=       0.96;     barew1= 0;
cw0=          0.6;      cw1=    0;
ch0=          0.4;      ch1=    0;

betur=0;     betpr=  0; betrlr = 0; betqr  = 0;
betd= 0;     betdd=  0; gy0    = 0; gy1    = 0;  alpg=0;   gp=0;
tauv= 0;     tauwp=  0; tauw= 0; tauc  = 0;
rf =  0.05;  tilqu =  0;     tilqr=0;      tilalpr=0;  alpfp = 1;
```

```
Selected Steady State Values (c_w=cgwc+cgwh=ywwo):
   b_c              b_g            vf=v_f*y
-3.1663267       -3.1663267       0.81237252
    q_h             c_w             y^d=y^e
1.0000000       0.81237252        0.99403579
prime deficit(if -)                 b_g/20
0.031663267                      -0.15831634
   tau_fp           gov/y           k_g
0.038976290       0.00000000      0.00000000
```

We can see from the set of selected steady state values (all per unit of the capital stock) that the government runs a prime surplus (solely through collecting taxes from firms, at a fairly low rate) and buys and accumulates on this basis short-term bonds (like "gold", issued by the asset holders (workers do not save, due to the high level of residential prices, equal to the goods prices here). The government expenditure quota and the public stock are both zero. The simulation run, where sales expectations y^e are shocked by a factor 1.02 and show that this type of economy exhibits stable adjustment processes. Note that the wage share is high due to the assumed low level of the interest rate (and the lack of any risk premium here).

The economy is profit-led if there holds:

$$(1 + g_{y0})[c_{wc}(q_{ho})(q^n - \tilde{q}^u) + (\tilde{q}^u \alpha^w + \tilde{q}^r \tilde{\alpha}^r)/(\alpha_w e_o)] < \alpha_\rho(1 + \tau_{fpo}).$$

It is wage led if the opposite inequality holds. In the presently considered barebone case, this boils down to the simple condition:

$$0.6 = cw0 > \alpha_\rho(1 + \tau_{fpo}) = 0.5 * 1.038976290 \approx 0.519, \quad i.e.$$

the economy is wage-led in this case. This corresponds to the anti-clockwise orientation of the goods-demand augmented distributive cycle, see the following figure top-left.

The phase diagrams plot (top-left to bottom-right) are: Wage share vs. the employment rate, and vs. the government's stock of private bonds, the Metzlerian inventory adjustment process and finally the relative price q_h against the stock variable k_h.

All partial cycle mechanisms are counter-clockwise in their orientation, the first is therefore not an example for the Goodwin/Marx model of the distributive cycle. Overshooting is relatively weak in the first three cycles, while the stock of dwellings keeps on rising for a longer while, even after the rental price has passed its maximum value already.

We next consider an example of this special choice of our general model which at first sight appeared to be a miscalculation:

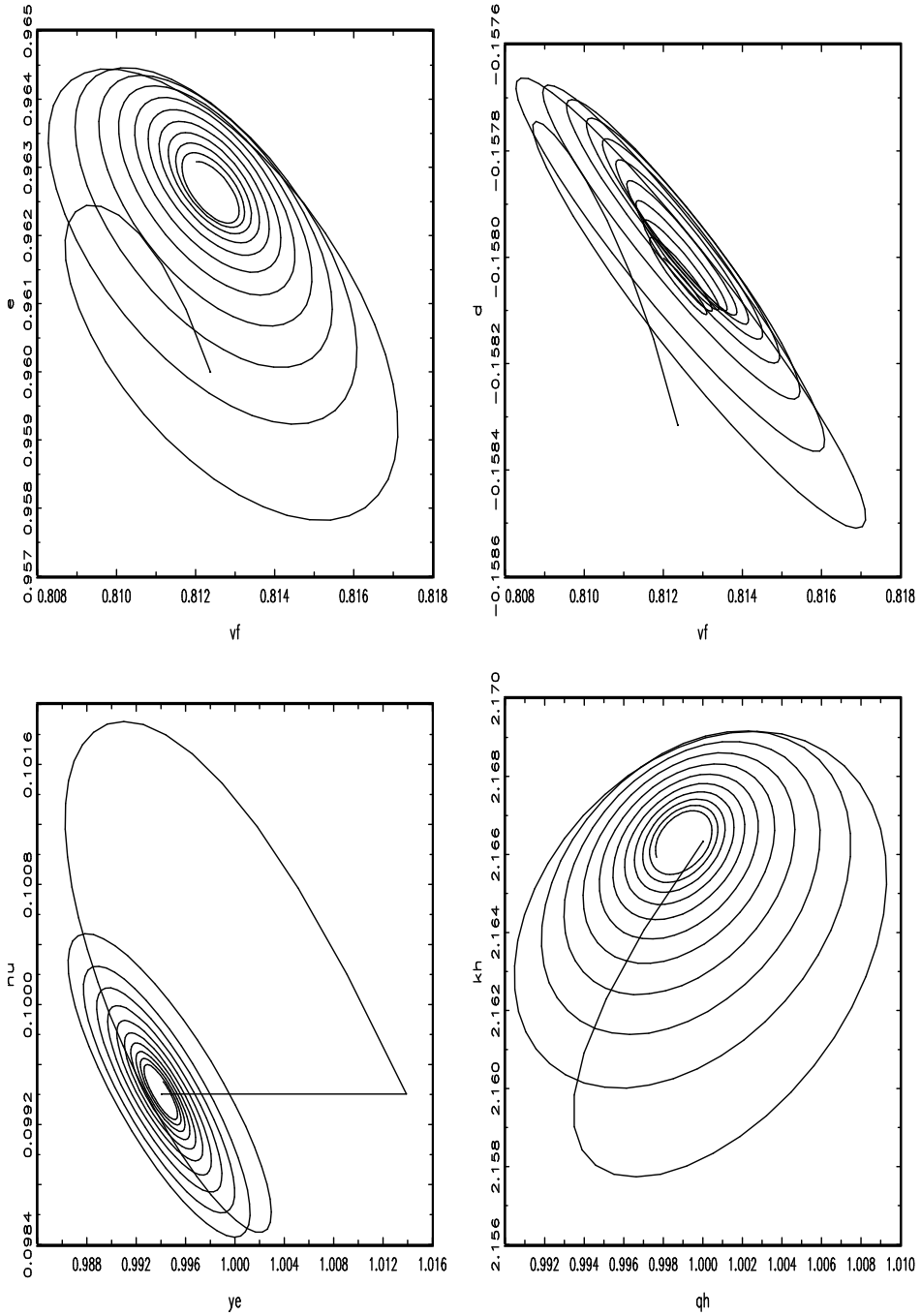


Figure 7: Simple barebone capitalism with a prime surplus of the government

```
                                PARAMETERS

betqw = 1;              betp =    1;              betpic =  0.1;
betpl = 0.5;            betpils=  0.2;              betpilr=0.2;
betye = 5;              betn =    1;                betlf =   0.5;
beth =  0.5;

alpnd =       0.1;
alppic = 0.5;           alppils =0.5;

alpw=         1;        alpeh =       0.1;
alpuh=   0.3;           alprhoh =     0.2; alprh=  0.2; alph=  1;
alpu =   0.5;           alprho =      0.5;

kapp =        0.5;
kapw =        0.5;
baru=         1;      baruw=  1.0;
baruh=        1;

bargam = 0.06; rhoego=0.05;

del =         0.1;
yp0 =         1;        yp1=    0;

betuw0 =      0.4;      betuw1= 0;
betew0 =      0.4;      betew1= 0;
barew0=       0.96;     barew1= 0;
cw0=          0.6;      cw1=    0;
ch0=          0.3;      ch1=    0;

taufpo=yeo-vfo*baru*yp0-del-rhoego/(vfo*baru*yp0);

TH=50         dtg        bbl       vf       taufp        gov/y      kg
49.900000 -0.44305648 2.2152824 1.2870923 -0.34423055 0.00000000 0.00000000
```

The steady value of the wage share is now approximately 128 percent of the output of firms. We notice however that firms are subsidized by the government by a high negative "payroll" tax rate. And the government is now running a prime deficit and has accumulated a high level of debt. Otherwise it is doing nothing for the society. The balanced growth path is again a stable one, i.e., it is surrounded by centripetal forces as the next figure shows.

Notice that the stock of houses is supplying more residential area now (alph=1) which lets the steady state value of rental prices drop dramatically to $q_{ho} = 0.15$. Workers now only spend $c_w = cgwc + 0.15 * cgwh << ywwo$ and therefore can save a lot by buying short-term government debt which is used to finance firms. This is a situation comparable to Pasinetti's people capitalism of the capital debate of the 1960's. A crude form of capitalism may therefore look quite comfortable from a simple work-only perspective, but lacks public goods and services and thus any public infrastructure.

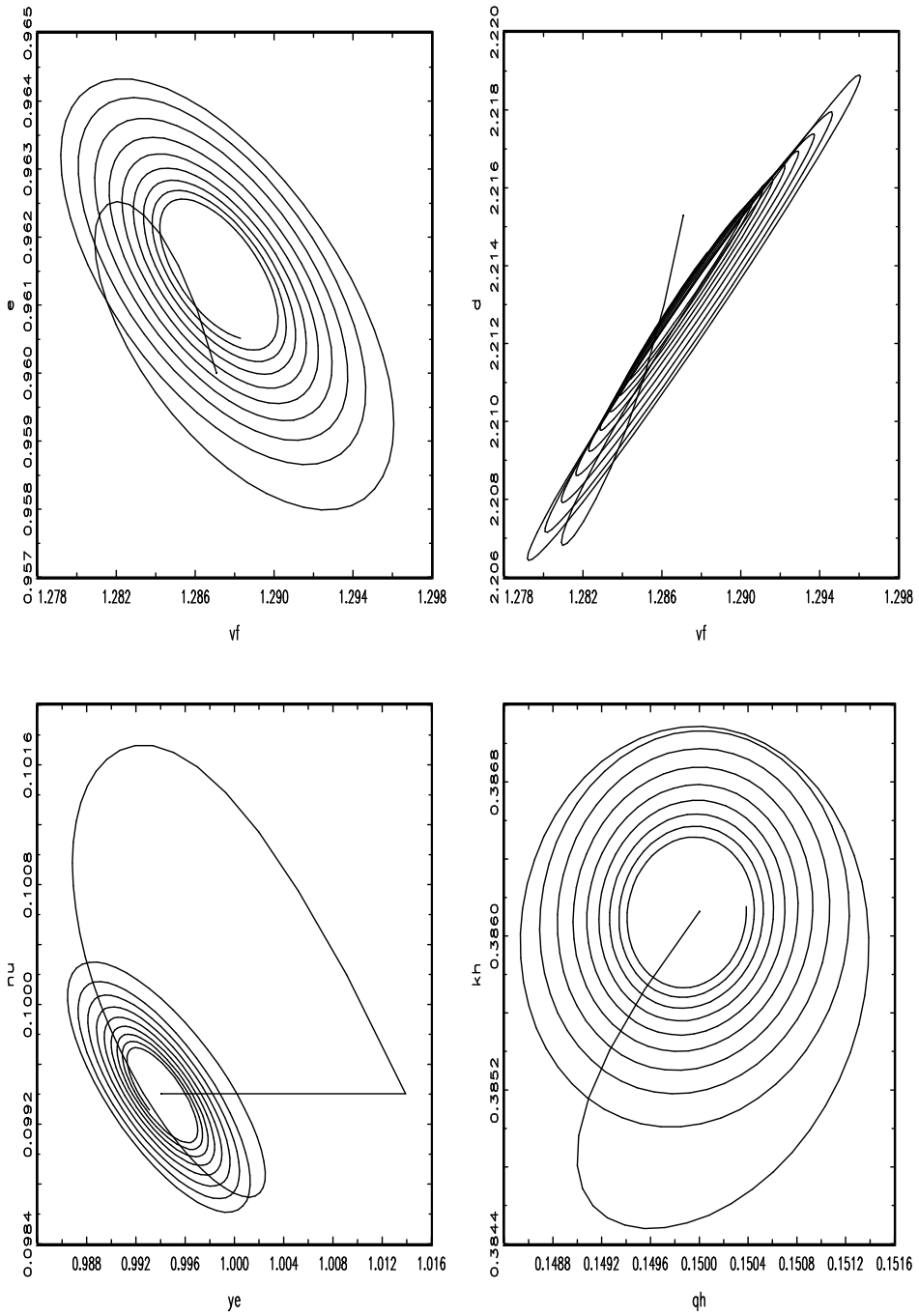


Figure 8: Credit-funded barebone capitalism: Convergence to an exceptional steady state

7.2 Socially-protected capitalism: Numerical investigation of an opposite "limit" case of the model

We now consider a case where the government intervenes heavily into the private sector, also by way of an anti-cyclical fiscal policy rule. Payroll taxes still remain fixed at their steady state value and the interest rate is pegged by the central bank to a low level.

```
                              PARAMETERS

betqw = 1;                betp =    1;                betpic =  0.1;
betpl = 0.5;              betpils=  0.2;              betpilr=0.2;
betye = 5;                betn =    1;                betlf =   0.5;
beth  = 0.5;

alpnd =  0.1;             alppic = 0.5;             alppils =0.5;

alpw =   1;               alpeh =       0.1;
alpuh=   0.3;             alprhoh =     0.2;        alprh=  0.2;   alph=  1;
alpu =   0.5;             alprho =      0.3;        alprg=  0;

kapp =   0.5;             kapw =        0.5;        baru =   1;    baruw=1;   baruh=1;

bargam = 0.06;            del =  0.1;  yp0 = 1;     yp1=    0;

betuw0 =      0.8;        betuw1= 0;
betew0 =      0.8;        betew1= 0;
barew0=       0.96;       barew1= 0;
cw0=          0.6;        cw1=    0;
ch0=          0.3;        ch1=    0;

betur=0; betpr=    0; betrlr =  0; betqr=  0;  betd=0;

gy0=   0.4; gy1 =   0.05; gp=.1;     alpg  = 0.5;
tauv = 0.2; tauwp=  0.1;  tauw= 0.2; tauc  = 0.5;
rf =   0.03; tilqu =  0.8;  tilqr=0.65; tilalpr=0.3;  alpfp =  1;

t      dtg          d          vf         taufp      gov/y        kg
50 -0.19277630 0.32123871 0.51094112 0.36693316  0.40292269 3.3384221
```

The government runs a prime deficit and accumulates punlic debt, still in short-term bonds solely. The wage share is at the lower end and taxed with 30 percent, but firms and asset holders are also strongly taxed. Moreover, the value added tax rate is high. Yet, the government expenditure quota is also high and the supply of public infrastructure more than three times as high compared to the private capital stock. Residential prices are very low ($q_{ho} = 0.13$) and workers consumption is $c_w = (0.6 + 0.13 * 0.3) * ywwo \approx 0.64 * ywwo$ that is they save at a very high rate (or change their consumption pattern significantly).

The dynamics shown in the next figure exhibits strongly damped oscillation (stochastic shocks as always neglected) with some monotonic adjustment to balanced growth at a later stage. The anti-clockwise orientation is as in the case of barebone capitalism. We have added now at the bottom of the figure the interaction of the prime deficit with government debt d and also the one between its expenditure quota and the stock of infrastructure, with the first cycle now clockwise in nature.

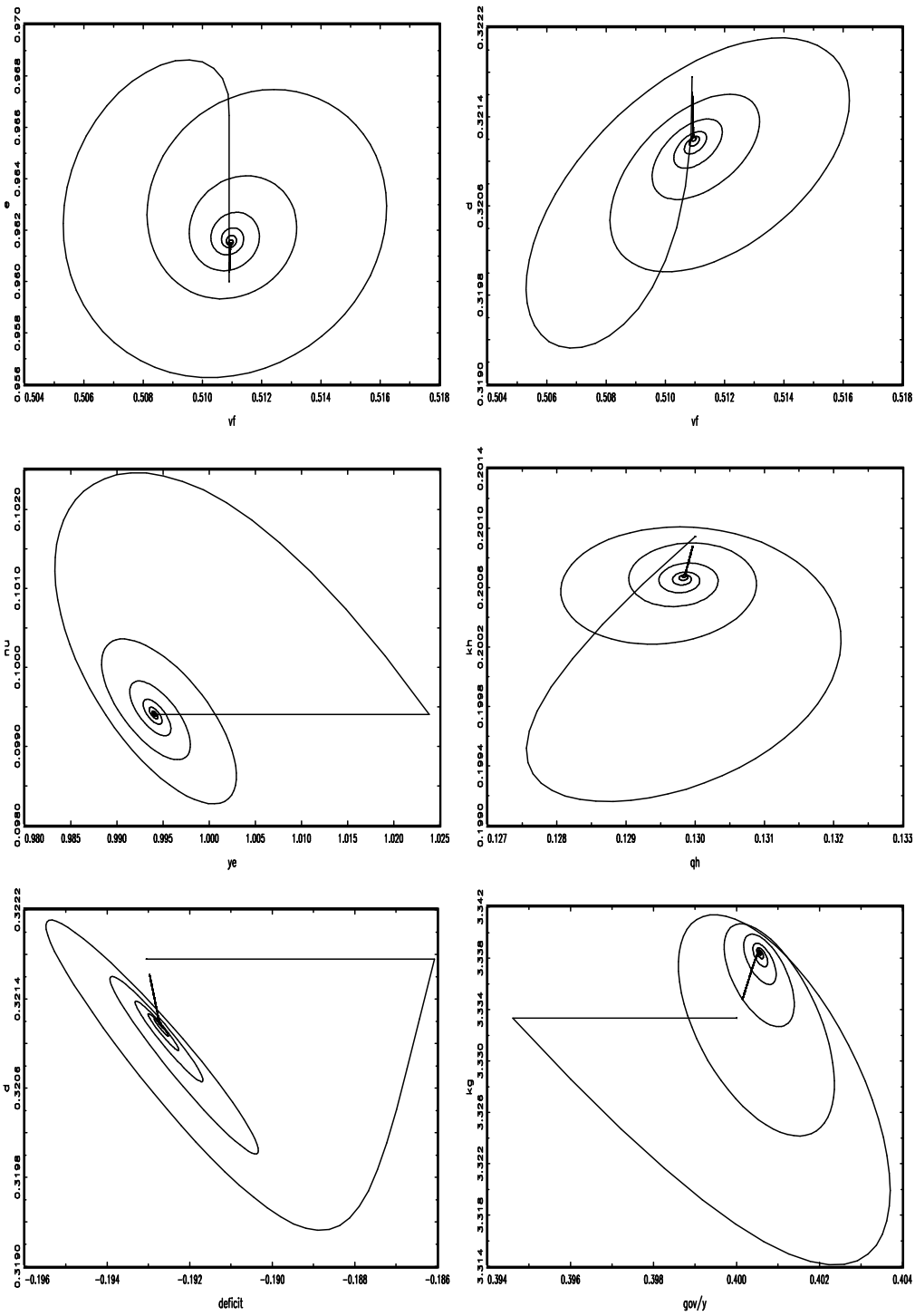


Figure 9: Socially-protected capitalism: A rapidly convergent baseline case

The positive shock in the sales expectations of firms lowers the deficit instantaneously, but it starts rising immediately after the shock, with government debt passing its steady state level, before strongly stabilizing forces take over and drive the economy towards a level that is lower than steady state debt and accompanied by a somewhat higher prime deficit. Thereafter, a monotonic adjustment back to the steady state values of these two state variables takes place. Something opposite happens in the partial phase plane shown bottom-right, due to the anti-cyclical policy of the government.

We consider next a case where business cycles become nearly undamped, associated with a higher share of wages however.

PARAMETERS

```
betqw = 1;              betp =    1;              betpic =  0.1;
betpl = 0.5;            betpils=  0.2;              betpilr=0.2;
betye = 5;              betn =    1;                betlf =   0.5;
beth =  0.5;

alpnd =       0.1;
alppic = 0.5;           alppils =0.5;

alpw=         1;        alpeh =       0.1;
alpuh=   0.3;           alprhoh =     0.2; alprh=  0.2; alph=  1;
alpu =   0.5;           alprho =      0.5;

kapp =        0.5;
kapw =        0.5;
baru=         1;      baruw=  1.0;
baruh=        1;

bargam = 0.06;
del =         0.1;
yp0 =         1;        yp1=    0;

betuw0 =      0.4;      betuw1= 0;
betew0 =      0.4;      betew1= 0;
barew0=       0.96;     barew1= 0;
cw0=          0.6;      cw1=    0;
ch0=          0.3;      ch1=    0;

betur=0; betpr=    0; betrlr =  0; betqr=  0; betd=0;

gy0=0.4; gy1=.062;    gp=0;
tauv = 0.2; tauwp=  0.1; tauw= 0.2; tauc  = 0.5; alpg=0.5;
rf =          0.03; tilqu =  0.8;  tilqr=0.65; tilalpr=0; alpfp =  1;
```

t	dtg	d	vf	taufp	gov/y	kg
50	-0.19305402	0.32175670	0.65241069	0.070434870	0.40240000	3.3333333

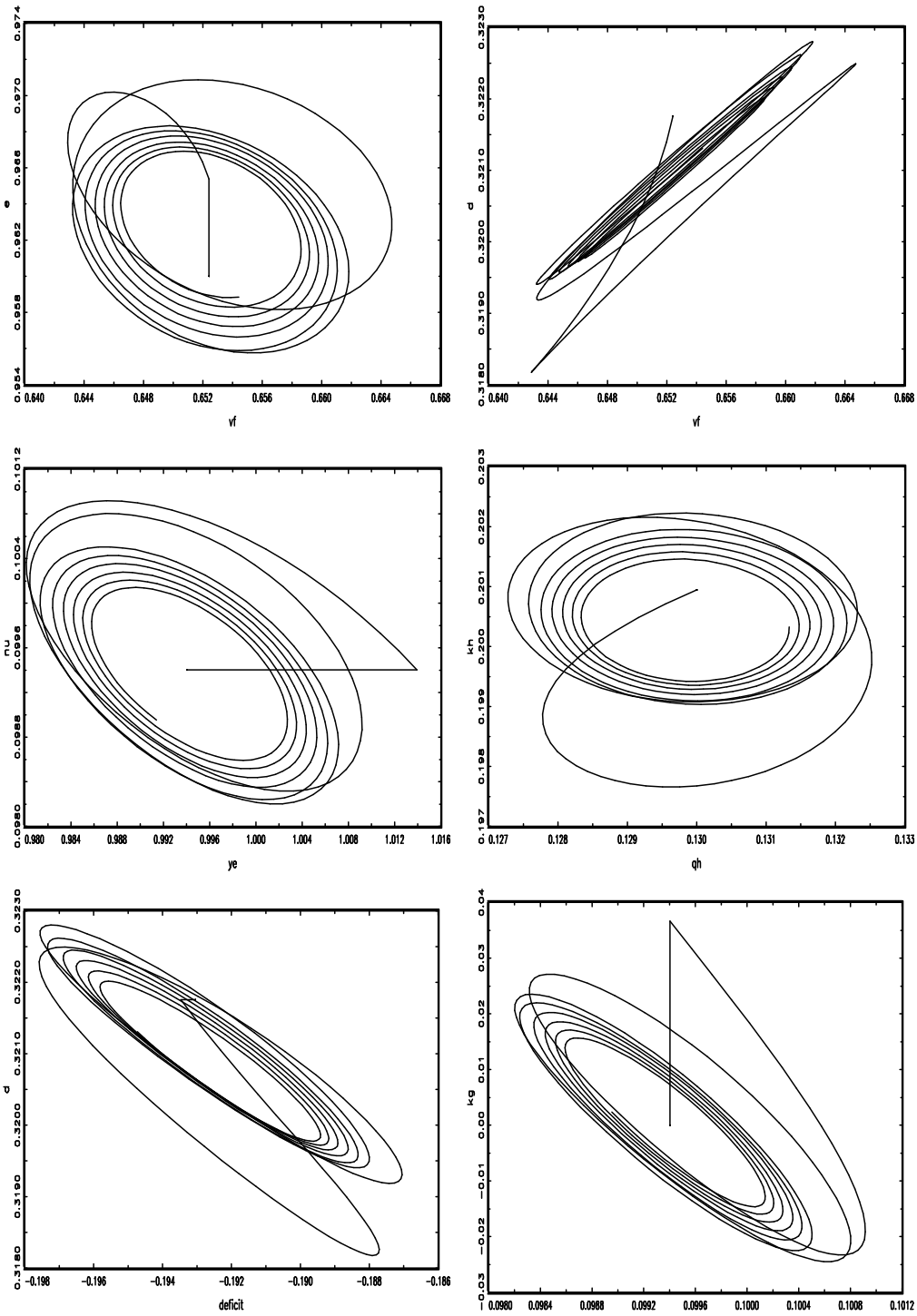


Figure 10: Socially-protected capitalism: Instability through debt-dependent government expenditures and inactive fiscal policy: $g_{y1} = 0.062, g_p = 0$.

PARAMETERS

```
betqw = 1;              betp =    1;                betpic =  0.1;
betpl = 0.5;            betpils=  0.2;              betpilr=0.2;
betye = 5;              betn =    1;                betlf =   0.5;
beth =  0.5;

alpnd =       0.1;
alppic = 0.5;           alppils =0.5;

alpw=         1;        alpeh =       0.1;
alpuh=   0.3;           alprhoh =     0.2; alprh=  0.2; alph=  1;
alpu =   0.5;           alprho =      0.5;

kapp =        0.5;
kapw =        0.5;
baru=         1;      baruw=  1.0;
baruh=        1;

bargam = 0.06;
del =         0.1;
yp0 =         1;        yp1=    0;

betuw0 =      0.4;      betuw1= 0;
betew0 =      0.4;      betew1= 0;
barew0=       0.96;     barew1= 0;
cw0=          0.6;      cw1=    0;
ch0=          0.3;      ch1=    0;

betur=0; betpr=    0; betrlr =  0; betqr=  0; betd=0.001;
gy0=0.4; gy1=.05;    gp=0;
tauv = 0.2; tauwp=  0.1; tauw= 0.2; tauc  = 0.5;
rf =          0.03; tilqu =  0.8;  tilqr=0.65; tilalpr=0;
alpfp =  1;  alpg=0.5;
```

t	dtg	d	vf	taufp	gov/y	kg
300	-0.19305402	0.32175670	0.65241069	0.070434870	0.40240000	3.3333333

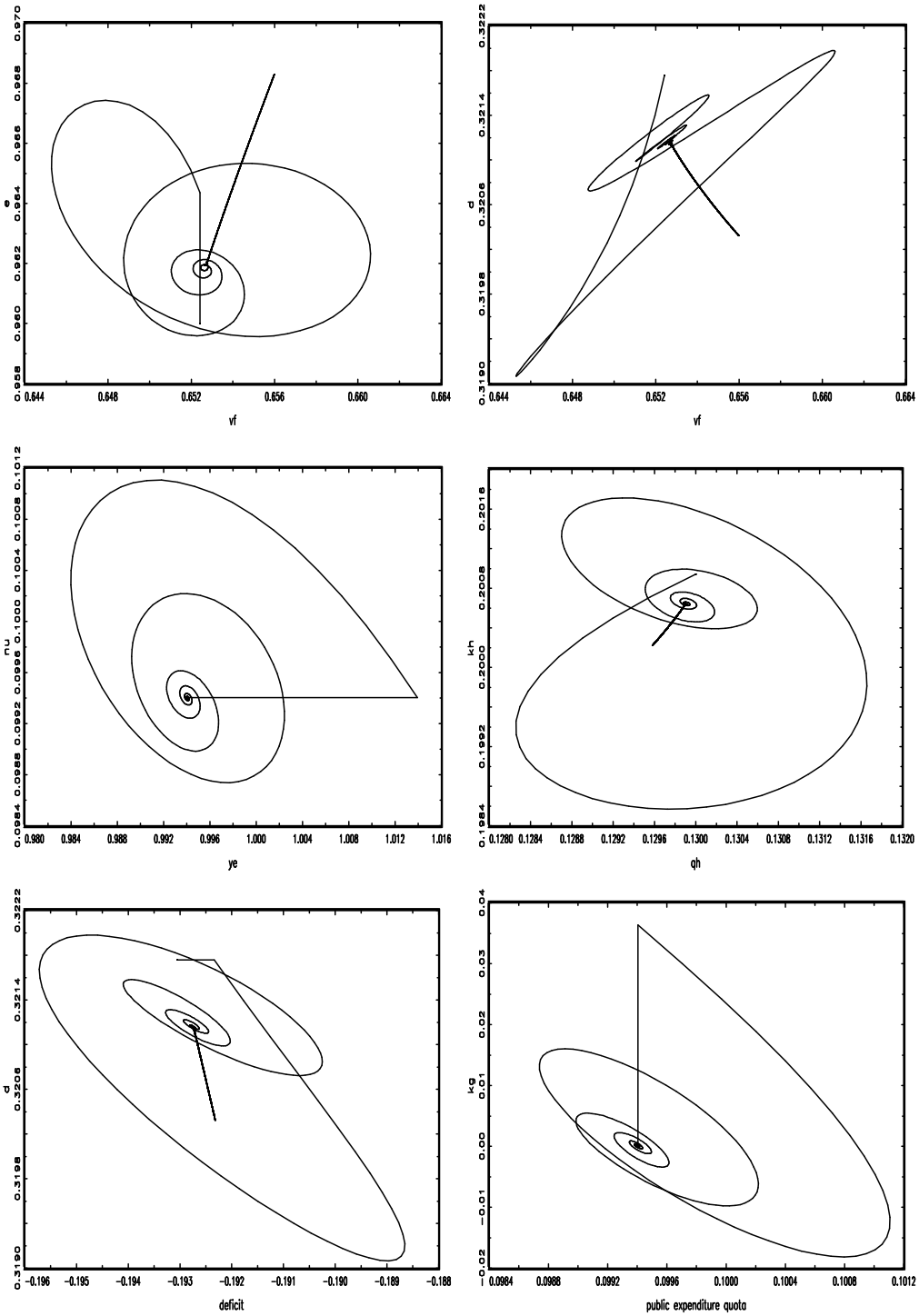


Figure 11: Convergence and after year "100" partial cumulative instability through a very weak adjusting, debt dependent payroll tax rule for firms: $\beta_d = 0.001$.

PARAMETERS

```
betqw = 1;              betp =    1;              betpic =  0.1;
betpl = 0.5;            betpils=  0.2;              betpilr=0.2;
betye = 5;              betn =    1;                betlf =   0.5;
beth =  0.5;

alpnd =       0.1;
alppic = 0.5;           alppils =0.5;

alpw=         1;        alpeh =       0.1;
alpuh=   0.3;           alprhoh =     0.2; alprh=  0.2; alph=  1;
alpu =   0.5;           alprho =      0.5;

kapp =        0.5;
kapw =        0.5;
baru=         1;      baruw=  1.0;
baruh=        1;

bargam = 0.06;
del =         0.1;
yp0 =         1;        yp1=    0;

betuw0 =      0.4;      betuw1= 0;
betew0 =      0.4;      betew1= 0;
barew0=       0.96;     barew1= 0;
cw0=          0.6;      cw1=    0;
ch0=          0.3;      ch1=    0;



betur=0; betpr=    0; betrlr =  0; betqr=  0; betd=-0.01;
gy0=0.4; gy1=.05;    gp=0;
tauv = 0.2; tauwp=  0.1; tauw= 0.2; tauc  = 0.5;
rf =          0.03; tilqu =  0.8;  tilqr=0.65; tilalpr=0;
alpfp =  1;  alpg=0.5;
```

t	dtg	d	vf	taufp	gov/y	kg
3000	-0.19305402	0.32175670	0.65241069	0.070434870	0.40240000	3.3333333

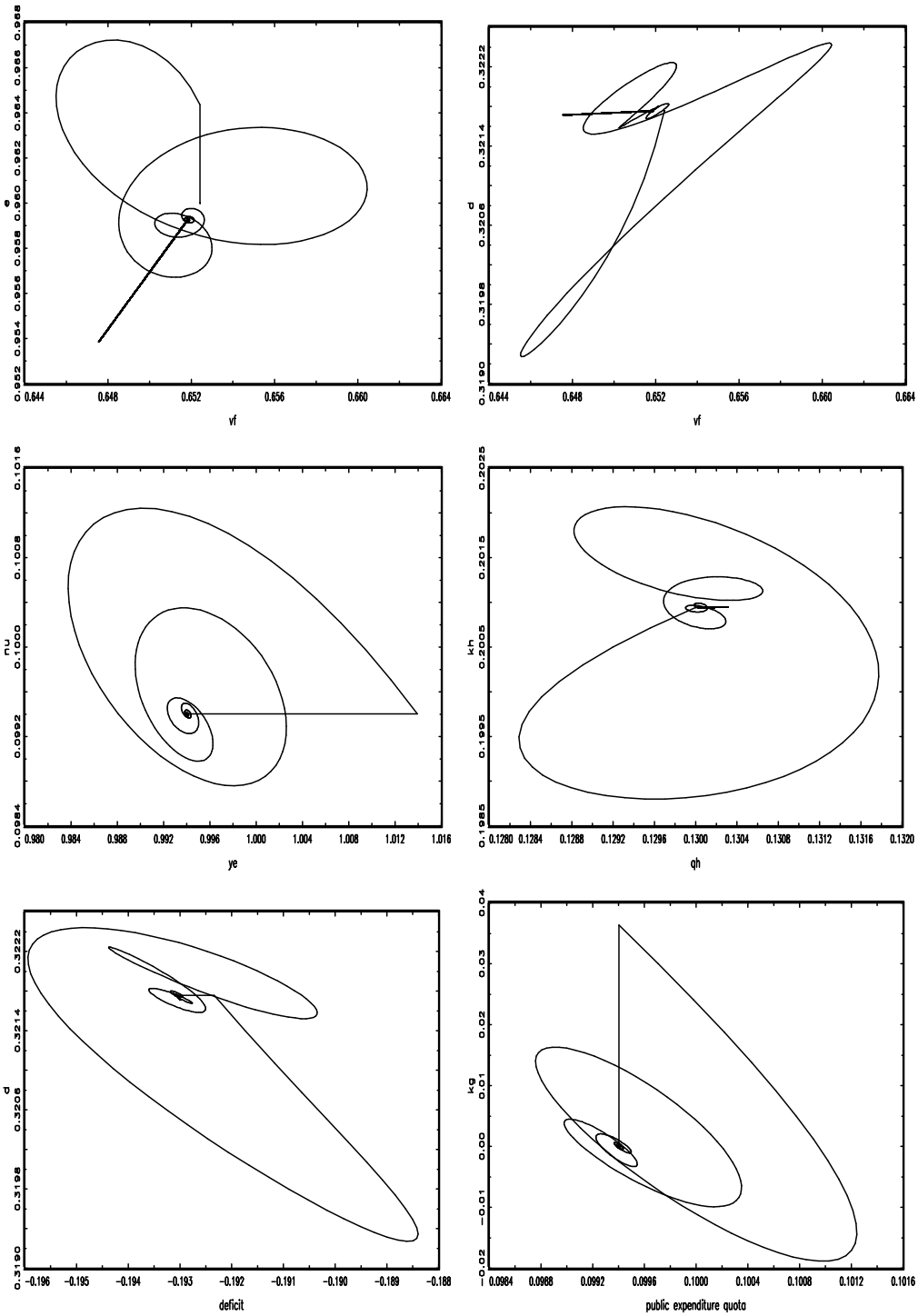


Figure 12: Convergence, but still partial cumulative instability through a weak, now negatively debt-dependent payroll tax of firms: $betd = -0.01$. Time Horizon 3000 years!

We finally consider the impact of financial markets and the conduct of monetary policy on the real part of the economy.

```
                              PARAMETERS

betqw = 1;              betp =    1;                betpic =  0.1;
betpl = 0.5;            betpils=  0.2;              betpilr=0.2;
betye = 5;              betn =    1;                betlf =   0.5;
beth =  0.5;

alpnd =       0.1;
alppic = 0.5;           alppils =0.5;

alpw=         1;        alpeh =       0.1;
alpuh=   0.3;           alprhoh =     0.2; alprh=  0.2; alph=  1;
alpu =   0.5;           alprho =      0.5; alprg=0.0001;

kapp =        0.5;
kapw =        0.5;
baru=         1;      baruw=  1.0;
baruh=        1;

bargam = 0.06;
del =         0.1;
yp0 =         1;        yp1=    0;

betuw0 =      0.4;      betuw1= 0;
betew0 =      0.4;      betew1= 0;
barew0=       0.96;     barew1= 0;
cw0=          0.6;      cw1=    0;
ch0=          0.3;      ch1=    0;

betur=0.0001; betpr=    0; betrlr =  0.0001; betqr=  0;
betd=-0.01; betdd=.5;
gy0=0.4; gy1=.05;    gp=0;
tauv = 0.2; tauwp=  0.1; tauw= 0.2; tauc  = 0.5;
rf =          0.03; tilqu =  0.8;  tilqr=0.65; tilalpr=0;
alpfp =  1;  alpg=0.5;

Steady        dtg          d         vf        taufp        gov/y         kg
State    -0.19305402  0.32175670 0.65241069 0.070434870  0.40240000    3.3333333
```

The outcome of various simulation runs of this extension is not very convincing yet. Monetary policy is – if really operate, see the parameters here chosen – destabilizing. We conclude that we should follow Keynes (1936) suggestion which recommended that monetary policy should operate directly on the long end of the financial markets (as the ECB is doing it now).

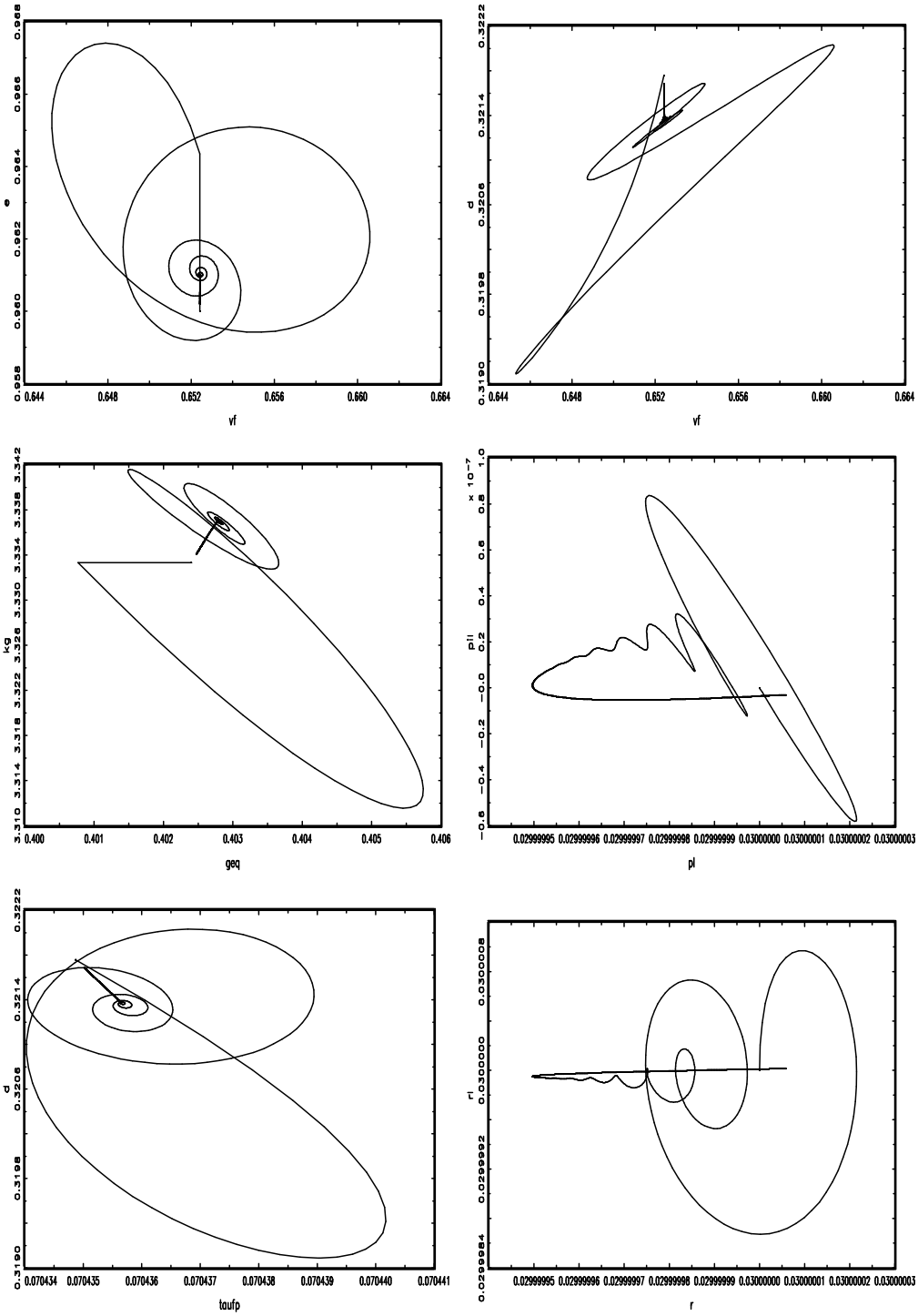


Figure 13: Switching on the market for long-term bonds, but not the monetary policy rule

8 Conclusions

This paper has been explorative in nature as it was intended to explore in a first attempt the properties of the here proposed Keynesian model of monetary growth from the mathematical perspective primarily, testing so to speak the limits of the approach we are proposing. Nevertheless it, of course, exhibits and displays the essential partial feedback structures of Keynesian macroeconomic theory as they were stressed by James Tobin primarily, while the traditional Keynes effect is covered behind the working of the so-called Taylor interest rate policy rule. Its traditional counterpart, the stabilizing Pigou effect, and its opponent, the destabilizing Mundell effect, are however not yet present in the here considered model structure, since we do not consider wealth or real rate of interest effects in this model type (which thus remain to be integrated in future research). But even without these effects the task of creating a viable economic behavior through public policy intervention was not an easy one, due to the high dimensional nature of the considered laws of motion of this macro-dynamic approach.

In a companion paper we shall then reconsider the modules of the present hierarchically structured continuous-time model of Keynesian growth extended to the case of a small open economy reformulating and modifying the type of approach introduced Charpe, Chiarella, Flaschel and Semmler (2010) such that "social protection" of worker households become the focus of interest. Their model was already sufficiently rich with respect to markets, sectors and agents in order to allow to capture the important details of actual macro-economies and was therefore well-suited to serve as the basis for modelling issues of choosing a social protection program for workers by the government which is effective and preserves the macro-efficiency of the economy.

Of primary interest was then the question how the many tax-, transfer- (unemployment benefits and pensions payments) and government-expenditure-parameters of this model could be used to improve the social protection of worker households, without loosing the efficiency of a well-performing labor market (with its partial modelling of Friedmanian supply side forces), and also without neglecting the creation of a sufficient "infrastructure" for an modern educational system, well-equipped medicare and thoughtful care for the elderly, i.e., the corner-stones for the young, the labor market participants and the retired.

Concerning the topics just enumerated we have provided a wide range of numerical answers showing the macro-advantages of the development of an modern type of capitalist economy for the "social protection" of workers, where high output-capital ratios and high productivity as well as work-related income growth was based on public investments into the "infrastructure" of the country, various types of income transfers, and counter-cyclical fiscal and monetary policies. We have however also seen some obstacles in the promotion of such a development, preventing the creation of ideal situation of what is called a "free lunch" by mainstream economics.

These aspects will be illustrated through numerous simulations of the laws of motion of the formulated macro-dynamical system in the sections that follow the detailed determination of the reference balanced growth path, used in this paper to start the dynamics with a situation, where the intensive form state variables are constant, but then disturbed at time t=1 by a unit-wage cost shock for example, with convergence to a new or the old balanced growth path thereafter. The reference balance growth path could therefore in particular be shown to be non-uniquely determined from the global perspective, due to nonlinearities though positive externalities as they were created by the public investment into the public capital stock run by the government.

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Notation

Steady state or trend values are indicated by a subscript 'o'. A dot over a variable $x = x(t)$ denotes the time derivative, a caret its growth rate; $\dot{x} = dx/dt$, $\hat{x} = \dot{x}/x$. In the numerical simulations, flow variables are measured at annual rates.

As far as possible, the notation tries to follow the logic of using capital letters for level variables and lower case letters for variables in intensive form, or for constant (steady state) ratios. Greek letters are most often constant coefficients in behavioural equations (with, however, the notable exceptions being π, ρ).

A. Statically or dynamically endogenous variables:

Y	Output of the domestic good
Y^d	Aggregate demand for the domestic good
Y^p	Potential output of the domestic good
Y^e	Expected sales for the domestic good
Y_w^{Dn}, Y_c^{Dn}	Nominal disposable income of workers and asset-holders
$u = Y/Y^p$	Rate of capacity utilisation of firms
Y_f	Income of firms
L	Population aged 17 – 64 (48=3*16 years)
L_r	Population aged 65 – 80
$L_{k(ids)}$	Population aged 0 – 16
L^d	Total employment of the employed
L_f^d	Total employment of the work force of firms
$L_g^d = L_g^w$	Total government employment (= public work force)
L_f^w	Work force of firms (g: Government)
L^w	Total active work force
u_f^w ($\bar{u}_f^w$)	(Normal) Employment rate of those employed in the private sector
α_r	Participation rate of the potential work force
$e = L^d/L$	Rate of employment ($\bar{e}$ the employment–complement of the NAIRU)
C_w	Real goods consumption of workers
C_c	Real goods consumption of asset owners
C_h^s	Supply of dwelling services
C_h^d	Demand for dwelling services
I	Gross business fixed investment
I_h	Gross fixed housing investment
D	Debt of government
$\mathcal{I}$	Planned inventory investment
N	Actual inventories
N^d	Desired inventories
r	Nominal short-term rate of interest (price of bonds $p_b = 1$)
r_l	Nominal long-term rate of interest (price of bonds $p_l = 1/r_l$)

$\pi_l = \hat{p}_l^e$	expected appreciation in the price of long-term domestic bonds
$T^n(T)$	Nominal (real) taxes
G	Real government expenditure
ρ^e	Expected short-run rate of profit of firms
ρ_h	Actual rate of return for housing services
K	Capital stock
K_h	Capital stock in the housing sector
w	Nominal wages including income and payroll tax
$\omega = w/p$	real wages $[\omega l_f]$ the wage share of production workers
w^u	Unemployment benefit per unemployed
w^r	Pension rate
p	Price level of domestic goods including value added tax
p_y	Price level of goods net of value added tax
p_h	Rent per unit of dwelling
$\pi^c = \hat{p}^e$	Expected rate of inflation or inflation climate
L	Labor supply
l^e	Labor supply in efficiency units per unit of capital
B	Stock of domestic short-term bonds (index d: stock demand)
B_w	Short-term debt held by workers
B_c	Short-term debt held by asset owners
B_c^l	Stock of long-term bonds of asset-holders (index d: demand)
$1/l_y = Y/L^d$	Labor productivity
$-\hat{l}_y$	-Rate of Harrod neutral technical change
τ_w	tax rate on wages
t	total real taxes per value unit of capital
g_k, g_k	Actual rate of growth of the capital stock K
g_h, g_h	Actual rate of growth of the housing capital stock K_h
d	Actual debt to capital ratios of the government

B. Parameters of the model

δ	Depreciation rate
α	All α-expressions (behavioural or other parameters)
β	All β-expressions (adjustment speeds)
$\bar{\gamma}$	Steady growth rate in the investment functions
$\bar{e}$	NAIRU employment rate (NAIRE)
$\bar{u}$	Normal rate of capacity utilisation of firms
$\bar{u}_h$	Normal rate of capacity utilisation in housing
κ_w, κ_p	Weights of short– and long–run inflation ($\kappa_w \kappa_p \neq 1$)
y^p	Output–capital ratio
l_y	Labor-output ratio
$\bar{d}$	Desired public or firm debt/output ratio
τ_c	Tax rates on profit, rent and interest
τ_v	Value added tax rate
τ_p	Payroll tax
c_{wc}	Propensity to consume goods (out of wages)
c_{wh}	Propensity to consume housing services (out of wages)

C. Further notation

$\dot{x}$	Time derivative of a variable x
$\hat{x}$	Growth rate of x
$r_o, etc.$	Steady state values
$y = Y/K, etc.$	Real variables in intensive form
$b = B/(pK), etc.$	Nominal variables in intensive form