From GDP to flow of funds: a graphical stock-flow modelling approach

Miguel Carrión Álvarez

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Abstract

This is a pedagogical/methodological paper building on the insights of Godley and Lavoie[4].

In order to build an empirical Stock-Flow Consistent formulation, we attempt to construct a model as close as possible to the system of national accounts[1]. It turns out that "GDP plus imports" corresponds to the gross inflows (equal to the outflows) of the firms current account in the associated stylised model of the macroeconomy. It is also apparent in this formulation that the national accounts do not form a closed monetary circuit, and need to be supplemented with flow of funds data in order to obtain a closed system.

As a practical application, we adapt the Spanish national accounts into the SFC framework and illustrate the dynamics of the Spanish boom and bust cycle of the past 20 years. We also indicate the kinds of difficulties encountered trying to put together national accounts and flow of funds data.

This is all supported with the graph theory apparatus presented in Carrión and Ehnts[2] which provides a natural visual representation of the monetary dynamics.

1 Introduction

Stock–flow-consistent (SFC) modelling [3] is a framework for looking at the macroeconomy from a monetary point of view. In it, the basic object of interest is the monetary flow between different sectors of the economy. It is generally accepted that James Tobin's Nobel Lecture [6] contains an enumeration of the basic elements of SFC modelling, though recent work in this area is mostly built on the 2006 book *Monetary Economics* by Wynne Godley and Marc Lavoie [4].

In this paper we look at GDP from a stock-flow consistent perspective. This requires expanding GDP into the components of the national accounts. We observe that the combination of the income and demand approaches to GDP describes the cash balance of the productive sector. However, it also becomes apparent from this analysis that the rest of the institutional sectors of the economy are out of balance if one takes into account only the system of national accounts. The rest of the institutional sectors, in the simplest formulation the state, household, and external sectors, can only be brought into balance by means of the so-called flow-of-funds or financial accounts data.

To illustrate this, including some of the methodological difficulties encountered when trying to put together the national accounts and flow-of-funds data, we look at the case of Spain.

1.1 Stock-flow-consistent analysis basics

A stock-flow-consistent model of an economy consists of a partition of economic agents into sectors, which can hold stocks of real assets and financial assets or liabilities, and which exchange cash flows as counterparts of real economic activity, or resulting from existing stocks of financial instruments or from the accumulation of additional financial instruments.

Building on our earlier work [2], we organize this with the help of cash-flow and stock tables. These can be thought of as amalgamations of the cash-flow statements and balance sheets of the institutional sectors. We summarize these tables visually with the help of graphs in a systematic way.

1.2 A note on notation

We adopt the convention that stock variables are denoted by upper-case letters, while flows are denoted by lower-case letters. This is as a visual aid to distinguish stock and flow variables at a glance, and assigning lower-case letters to the flows as they are normally used more requently than stocks. Thus, idiosyncratically from the point of view of economists' conventions, we will be writing GDP and its demand components in lower case as

$$y = c + i + g + x - m \qquad \text{rather than} \quad Y = C + I + G + X - M.$$

2 GDP and the production economy

Gross Domestic Product (GDP) is a statistical aggregate attempting to capture the value of all production within a given political unit. The estimation of GDP is part of the so-called System of National Accounts. As explained in [5, ch. 1], there are three ways to define the gross domestic product or GDP (here denoted y in lower-case as it is a flow, though it is usually written as an upper-case Y): the output approach, the demand approach and the income approach. In the output approach[5, §1.1], in order to avoid double-counting of intermediate consumption by the productive sector itself, GDP is computed as the sum of the value-added by all economic units in the productive sector. For our purposes, however, the other two approaches to GDP will be more useful.

2.1 Demand GDP

The demand approach[5, §1.4] expresses GDP as the sum of *demand aggregates*: domestic consumption, gross capital formation (investment), and exports. Domestic consumption includes private consumption and government expenditure on goods and services. Investment includes net changes in inventory and private[1] fixed capital formation (which is purchases, by the productive sector itself,

[1] One could also consider public investment, but in this formulation government expenditure only adds to consumption

of durable goods intended to assist further production). In fact, the productive sector satisfies demand not only from its output but from imports, so the actual GDP equation is

$$y = c + g + i + x - m, \tag{1}$$

where

y is *output* (GDP)

c + g is *total final consumption* expenditure

c is *household consumption*, which in the system of national accounts includes consumption by *non-profit institutions serving households*

g is *general government final consumption* expenditure

i is *gross capital formation*, consisting of gross (without amortization) *fixed capital formation* and changes in *inventories*

x is *exports*

m is *imports*

In a closed economy, exports and imports are absent. Table 1 shows how these variables are reported in the national accounts by a typical national statistical service.

Concept			2013Q3 (M€)		
Imports	Goods		80,333	62,355	
	Services			17,978	
GDP at market prices			247,156		
Memo			327,489		
Consumption	Hs & NPISHs	Households	194,161	147,037	144,812
		NPISHs			2,225
	Government			47,124	
Investment	Fixed assets	Tangible	42,493	43,296	39,294
		Intangible			4,002
	Inventories & valuables			-803	
Exports	Goods		90,835	58,321	
	Services			32,514	

Table 1: Demand GDP components in Spain's INE (base 2008) data

2.1.1 Self-consumption

"Gross capital formation", also know as investment, is peculiar in that it is a purchase by the productive sector from itself. Another form of self-consumption by the productive sector is purchases of intermediate goods and ancillary services, which are not included because they are a function of the organization of production into legal entities: vertical integration of industrial sectors leads to some of these intermediate purchases disappearing from the accounting, as they become internal transfers of materials within an industrial conglomerate. The

cost of intermediate consumption is supposed to be included in the final price paid by the ultimate consumer (or business[2] in the case of investment goods). All this is reflected in the value-added approach to output.

We observe that investment includes as a kind of self-consumption of the productive sector the production of unsold product which then remains as inventory. Depending on the level of detail in the modelling of product price determination, it may be necessary to separate inventory from the acquisition of fixed assets explicitly in an SFC model.

Because investment is a self-purchase by the productive sector it would seem that it cannot appear as a flow between two sectors in an SFC model, and GDP would not appear to be recoverable from the state of a stock-flow-consistent model. With a bit of massaging, however, GDP can not only be captured by the model but in the case of a closed economy it can be interpreted as the income of a sector of the economy (in an open economy, the income will be the GDP augmented by the gross value of exports).

2.1.2 The capital account

For this purpose, it is necessary to separate the productive sector into two parts: a current account to which final purchases of services, or of durable goods for consumption, are made; and a capital account which owns the fixed plant and buys it from the current account. At the micro level, it is the capital account of one firm that buys fixed plant, durable goods and production inputs from another firm and pays *investment* flows into the latter's current account. However, when aggregating all firms into a single productive sector it appears as if the firms sector is paying investment from its capital account to its own current account. This is an artifact of aggregation. Internal consumption (intermediate consumption in the productive process) nets out in the national acounts, but could be added to our scheme as a self-purchase from the capital account to itself.

The separation of the productive sector into capital and current account allows us to represent the terms of equation 1 as the first five lines (three in a closed economy) of Table 2, which is an example of what we call a cash-flow specification in [2]. The cash-flow specification defines the cash-flow matrix in

Cash Flow	**From**	**To**	**Stock**	**Price**
Consumption (c)	Households	Firm Current	N/A	N/A
Investment (i)	Firms Capital	Firms Current	N/A	N/A
Expenditures (g)	Government	Firms Current	N/A	N/A
Exports (x)	External	Firms Current	N/A	N/A
Imports (m)	Firms Current	External	N/A	N/A

Table 2: Demand GDP as a cash-flow specification

Table 3. To represent GDP we need five sectors (four in a closed economy), which appear as distinct entries in the "From" and "To" columns of the cash-flow specification, and correspond to columns of the cash-flow matrix. it is clear

[2]Again, it is a simplification to assume only firms purchase investment goods: household purchases of real estate would qualify as investment

Cash Flow	Households	Firms		Government	External
		Current	Capital		
Consumption	$-c$	c			
Investment		i	$-i$		
Expenditures		g		$-g$	
Exports		x			$-x$
Imports		$-m$			m
Balance	$-c$	GDP	$-i$	$-g$	$m-x$

Table 3: Demand GDP as a cash-flow matrix

that the cash-flow specification is a more compact format than the cash-flow table, though the cash-flow table is more visual.

2.2 Income GDP

The GDP is distributed as wages and profits, which we assume at this stage are not retained and are instead fully distributed to the capital owners in each period. In addition[5, §1.5] the GDP includes taxes paid on production and imports. The reason taxes on imports needs to be included is that, as explained in section 2.1, the productive sector satisfies its demand by means of output plus imports:

$$c + i + g + x = y + m.$$

The taxes paid by the productive sector are thus interpreted as taxes on production (y) plus imports (m). Because both wage earners and business owners are part of the same private sector at this level of aggregation, at this stage wages (w) and profits (p) are indistinguishable in that they both flow from the productive sector to the private sector. The basic income and demand equations are:

$$c + i + g + x - m = y = w + p + t \tag{2}$$

where

w represents *compensation of employees*, both monetary and in kind

p includes *gross operating profit* and *mixed income*

t is tax (net of subsidies) on production and imports

Taxes as defined in the national accounts are peculiar. They (t) refer only to producer taxes. Income taxes and social security contributions, are included in the compensation of employees w, regardless of whether income is taxed at source as it is partially done in most countries, or is paid by the employee.

We summarize the situation with the following (partial) specification matrix:

Cash Flow	**From**	**To**	**Stock**	**Price**
Consumption (c)	Households	Firm Current	N/A	N/A
Investment (i)	Firms Capital	Firms Current	N/A	N/A
Expenditures (g)	Government	Firms Current	N/A	N/A
Exports (x)	External	Firms Current	N/A	N/A
Imports (m)	External	Firms Current	N/A	N/A
Wages (w)	Firms Current	Households	N/A	N/A
Profits (p)	Firms Current	Households	N/A	N/A
Taxes (t)	Firms Current	Government	N/A	N/A

(3)

Note that there are no financial asset stocks, and no money. This defines the following (partial) cash flow matrix:

Cash Flow	**Households**	**Current**	**Capital**	**Government**	**External**
Consumption	$-c$	c			
Investment		i	$-i$		
Expenditures		g		$-g$	
Exports		x			$-x$
Imports		$-m$			m
Wages	w	$-w$			
Profits	p	$-p$			
Taxes		$-t$		t	
Total	$w+p-c$	0	$-i$	$t-g$	$m-x$
	Saving		$(-)$ Investment	Surplus	Deficit

(4)

2.2.1 Saving and Investment

As every row of the previous cash flow matrix (equation 4) adds up to zero, we obtain the classical national income relation:

$$(\text{saving}) - (\text{investment}) = (\text{government deficit}) + (\text{external surplus})$$

The familiar full form of the sector balance identity including an external sector would be

$$(s-i)+(t-g)=(x-m).$$

2.2.2 Graphical representation

3 Beyond GDP: flow of funds

On the fact that all columns of the cash flow matrix don't add up to zero Godley and Lavoie [4] comment

> But now it is easy to see that this system of concepts is seriously incomplete. Consideration of the matrix immediately poses the following questions. What form does personal saving take? Where does any excess of sectoral income over expenditure actually go to for it must all go somewhere? Which sector provides the counterparty to

> every transaction in assets? Where does the finance for investment come from? And how are budget deficits financed?
>
> There is an obvious answer to these questions, which follows from an elementary knowledge of the way the real world works and which can be quickly verified by inspecting the Flow-of-Funds tables published by the Federal Reserve in the United States, which provide data relating to every quarter since 1952.

In other words, the traditional national income relation may correspond to the state of the art in national accounting between the 1920s or 30s and the 1950s, but since the 1950s our picture of the economy should include the full flow of funds rather than just sectoral income. This is a transition from a pure production and consumption view of the economy to a monetary view of the economy.

The previous cash flow specification can be completed as follows. First, we replace profit distribution to households with a combination of retained earnings, seen as a transfer from the current to the capital account; and dividends, distributed to equity owners. Equity is the first financial stock in our model, and it is a liabiity of the firms and an asset of the households. Equity pays no interest, but rather a divident cash flow. Profit is not distributed from the Firms' Current account to the Households but to the Capital account. Retained earnings is the difference between the profit income of the Firms' Capital account and its dividend outlay. Firms also finance themselves with bank loans, which requires the explicit addition of a financial sector. But at least we have now answered the question of how investment is financed by the productive sector: as a combination of retained earnings, issue of securities, and bank loans.

Note that, because they are not tradeable, loans (L) are carried on the balance sheet at notional value and so their price is identically 1 and they give rise to no capital gains. However, they do pay interest at the end of each period at a rate (possibly a constant) determined at the start of the period.

We have now closed the cash flows of the productive sector and introduced securities issued by firms as one possible outlet for private savings. The unaccounted-for investment is financed by a stock of new bank loans which the financial sector must now somehow balance.

Cash Flow	**From**	**To**	**Stock**	**Price**
Consumption (c)	Households	Firm Current	N/A	N/A
Investment (i)	Firms Capital	Firms Current	N/A	N/A
Expenditures (g)	Government	Firms Current	N/A	N/A
Exports (x)	External	Firms Current	N/A	N/A
Imports (m)	External	Firms Current	N/A	N/A
Wages (w)	Firms Current	Households	N/A	N/A
Profits (p)	Firms Current	Firms Capital	N/A	N/A
Taxes ($t^{(F)}$)	Firms Current	Government	N/A	N/A
Taxes ($t^{(H)}$)	Households	Government	N/A	N/A
New Equity (H)	Households	Firms Capital	Equity ($E^{(H)}$)	$\pi^{(E)}$
Dividends (d)	Firms Capital	Households	N/A	N/A
New Loans (B)	Banks	Firms Capital	Loans ($L^{(B)}$)	1
Loan ($L^{(B)}$) interest	Firms Capital	Banks	N/A	N/A

(5)

Stock	Households	Capital	Banks	Government	External
Equity	$\pi^{(E)}E^{(H)}$	$-\pi^{(E)}E^{(H)}$			
Loans		$-L^{(B)}$	$L^{(B)}$		

(6)

Cash Flow	Households	Current	Capital	Banks	Government	External
Consumption	$-c$	c				
Investment		i	$-i$			
Expenditures		g			$-g$	
Exports		x				$-x$
Imports		$-m$				m
Wages	w	$-w$				
Profits		$-p$	p			
Taxes	$-t^{(H)}$	$-t^{(F)}$			t	
New Equity	$-\pi^{(E)}\Delta E^{(H)}$		$\pi^{(E)}\Delta E^{(H)}$			
Dividends	d		$-d$			
New Loans			$\Delta L^{(B)}$	$-\Delta L^{(B)}$		
Loan interest			$-\tau\upsilon_0^{(L)}L_0^{(B)}$	$\tau\upsilon_0^{(L)}L_0^{(B)}$		

(7)

In this model, the loans made by the financial sector to the production sector are balanced solely by deposits from the household sector (there is no bank equity!). Any excess of deposits over loans will be invested in government debt, or held in as currency, which is an undated, non-interest-bearing liability of the government.

Cash Flow	From	To	Stock	Price
Consumption (c)	Households	Firm Current	N/A	N/A
Investment (i)	Firms Capital	Firms Current	N/A	N/A
Expenditures (g)	Government	Firms Current	N/A	N/A
Exports (x)	External	Firms Current	N/A	N/A
Imports (m)	External	Firms Current	N/A	N/A
Wages (w)	Firms Current	Households	N/A	N/A
Profits (p)	Firms Current	Firms Capital	N/A	N/A
Taxes ($t^{(F)}$)	Firms Current	Government	N/A	N/A
Taxes ($t^{(H)}$)	Households	Government	N/A	N/A
New Equity (H)	Households	Firms Capital	Equity ($E^{(H)}$)	$\pi^{(E)}$
Dividends (d)	Firms Capital	Households	N/A	N/A
New Loans (B)	Banks	Firms Capital	Loans ($L^{(B)}$)	1
Loan ($L^{(B)}$) interest	Firms Capital	Banks	N/A	N/A
New Deposits (H)	Households	Banks	Deposits ($D^{(H)}$)	1
Deposit ($D^{(H)}$) interest	Banks	Households	N/A	N/A

(8)

Stock	**Households**	**Capital**	**Banks**	**Government**	**External**
Equity	$\pi^{(E)}E^{(H)}$	$-\pi^{(E)}E^{(H)}$			
Loans		$-L^{(B)}$	$L^{(B)}$		
Deposits	$D^{(H)}$		$-D^{(H)}$		

(9)

Cash Flow	**Households**	**Current**	**Capital**	**Banks**	**Government**	**External**
Consumption	$-c$	c				
Investment		i	$-i$			
Expenditures		g			$-g$	
Exports		x				$-x$
Imports		$-m$				m
Wages	w	$-w$				
Profits		$-p$	p			
Taxes	$-t^{(H)}$	$-t^{(F)}$			t	
New Equity	$-\pi^{(E)}\Delta E^{(H)}$		$\pi^{(E)}\Delta E^{(H)}$			
Dividends	d		$-d$			
New Loans			$\Delta L^{(B)}$	$-\Delta L^{(B)}$		
Loan interest			$-\tau\upsilon_0^{(L)}L_0^{(B)}$	$\tau\upsilon_0^{(L)}L_0^{(B)}$		

(10)

The household sector can also hold bills and currency.

Finally, the external sector must recycle its balance into domestic assets. Equity, loans, cash and debt can all be held by the external sector. This completes

the cash flow specification:

Cash Flow	**From**	**To**	**Stock**	**Price**
Consumption (c)	Households	Firm Current	N/A	N/A
Investment (i)	Firms Capital	Firms Current	N/A	N/A
Expenditures (g)	Government	Firms Current	N/A	N/A
Exports (x)	External	Firms Current	N/A	N/A
Imports (m)	External	Firms Current	N/A	N/A
Wages (w)	Firms Current	Households	N/A	N/A
Dividends (d)	Firms Current	Households	N/A	N/A
Retained earnings (r)	Firms Current	Firms Capital	N/A	N/A
Taxes ($t^{(F)}$)	Firms Current	Government	N/A	N/A
Taxes ($t^{(H)}$)	Households	Government	N/A	N/A
New Equity (H)	Households	Firms Capital	Equity ($E^{(H)}$)	$\pi^{(E)}$
New Loans (B)	Banks	Firms Capital	Loans ($L^{(B)}$)	1
Loan ($L^{(B)}$) interest	Firms Capital	Banks	N/A	N/A
New Deposits	Households	Banks	Money (M)	1
New Cash (H)	Households	Government	Currency ($C^{(H)}$)	1
New Cash (B)	Banks	Government	Currency ($C^{(B)}$)	1
New Debt (H)	Households	Government	Bills ($B^{(H)}$)	1
New Debt (B)	Banks	Government	Bills ($B^{(B)}$)	1
New Equity (X)	External	Firms Capital	Equity ($E^{(X)}$)	$\pi^{(E)}$
New Loans (X)	External	Firms Capital	Loans ($L^{(X)}$)	1
New Cash (X)	External	Government	Currency ($C^{(X)}$)	1
New Debt (X)	External	Government	Bills ($B^{(X)}$)	1

(11)

The associated balance sheet matrix is

Stock	**Households**	**Capital**	**Banks**	**Government**	**External**
Equity	$\pi^{(E)}E^{(H)}$	$-\pi^{(E)}E$			$\pi^{(E)}E^{(X)}$
Loans		$-L$	$L^{(B)}$		$L^{(X)}$
Deposits	M		$-M$		
Cash	$C^{(H)}$		$C^{(B)}$	$-C$	$C^{(X)}$
Debt	$B^{(H)}$		$B^{(B)}$	$-B$	$B^{(X)}$

(12)

And the cash flow matrix is as follows:

Cash Flow	**Households**	**Current**	**Capital**	**Banks**	**Government**	**External**
Consumption	$-c$	c				
Investment		i	$-i$			
Expenditures		g			$-g$	
Exports		x				$-x$
Imports		$-m$				m
Wages	w	$-w$				
Profits	d	$-(d+r)$	r			
Taxes	$-t^{(H)}$	$-t^{(F)}$			t	
New Equity	$-\pi^{(E)}\Delta E^{(H)}$		$\pi^{(E)}\Delta E$			$-\pi^{(E)}\Delta E^{(X)}$
New Loans			ΔL	$-\Delta L^{(B)}$		$-\Delta L^{(X)}$
New Deposits	$-\Delta M$			ΔM		
New Cash	$-\Delta C^{(H)}$			$-\Delta C^{(B)}$	ΔC	$-\Delta C^{(X)}$
New Debt	$-\Delta B^{(H)}$			$-\Delta B^{(B)}$	ΔB	$-\Delta B^{(X)}$
Total	0	0	0	0	0	0

(13)

There are some peculiarities of this scheme. First, all government debt is in the form of short-term bills. Also, the financial sector does not distribute profits nor issue equity. This means that it is impossible for banks to have an excess of loans over deposits, as deposits are the only assets liabilities banks can issue. In addition, it is impossible to address questions about bank capitalization, or to meaningfully ask whether bank lending is equity-constrained, reserve-constrained, or something else. A further oversimplification is the absence of inventory or fixed capital stocks: all investment is a cash flow for purchases of intermediate inputs of production.

4 The case of Spain

5 Conclusion

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