A Multi-sectoral Approach to the Harrod Foreign Trade Multiplier

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Abstract

With this inquiry we seek to develop a multi-sectoral version of the static Harrod foreign trade multiplier, by showing that it can be derived from an extended version of the Pasinettian model of structural change to international trade. This new version highlights the connections between balance of payment and the level of employment and production. It is also shown that from this disaggregated version of the Harrod foreign multiplier we can drive an aggregate version of the multiplier, further developing the consistency of our analysis. By following this approach we go a step further in establishing the connections between the Structural Economic Dynamic and Balance-of-Payments Constrained Growth approaches.

Keywords: structural economic dynamics; foreign trade multiplier; balance-of-payments constrained growth.

JEL classification: O19, F12.

1. Introduction

"The causes which determine the economic progress of nations belong to the study of international trade …" Principles of Economics, Book Four, by Alfred Marshall (1890)

This article deals with the relationship between income determination and balance of payments equilibrium in a structural economic dynamic setting. In particular, the article delivers multi-sectorial versions of the static and dynamic Harrod foreign trade multiplier [Harrod (1933)] by showing that it can be derived from an extended version of the Pasinettian model (1993) that takes into account foreign trade [Araujo and Teixeira (2004)]. Besides, in order to prove the consistency of our approach we also show that departing from the multi-sectoral Harrod foreign trade multiplier we can obtain the aggregated version, with emphasis on the role played by the structure on determining the output performance. The disaggregated version of the multiplier is then shown to keep the original flavour of the aggregated version since it predicts that the output of each sector is strongly affected by its export ability, which highlights the validity of the original Harrod's insight not only at an aggregated level.

The SED framework is adopted as the starting point for our analysis. Initially this model was conceived for studying the interactions between growth and structural change in a closed economy [see Pasinetti (1981, 1993)]. However, more recently it was formally extended to take into account international flows of goods [see Araujo and Teixeira (2003, 2004)], and a balance of payments constrained growth rate was derived in this set up under the rubric of the multi-sectoral Thirlwall's law [see Araujo and Lima (2007)]. Such extensions have proven that the insights of the Pasinettian analysis remain valid for the case of an open economy: the interaction between tastes and technical change is responsible for

variations in the structure of the economy, which by its turn affect the overall growth performance.

This view is also implicit in the Balance-of-Payments Constrained Growth (BoP) approach to the extent that variations in the composition of exports and imports lead to changes in the structural of the economy and determine the output growth consistent with balance of payments equilibrium [See Thirlwall (2013)]. The BoP approach asserts that assuming that real exchange rates are constant and that trade must be balanced in the long run, there is a very close correspondence between the growth rate of output and the ratio of the growth of exports to the income elasticity of demand for imports. Indeed, this result is the prediction of a dynamic version of the Harrod trade multiplier (1933) known as the Thirlwall law [See Thirlwall (1979)].

It can also be argued that the particular dynamics due to the interaction of technical change and patterns of demand is taken into account in the BoP approach since observed differences in the income elasticities of demand for exports and imports reflect the non-price characteristics of goods and, therefore, the structure of production [Thirlwall (1997, p. 383)]. But in fact, by departing from the aggregated Keynesian model, the literature on both the static and dynamic Harrod foreign trade multiplier is advanced in terms of an aggregated economy, in which it is not possible to fully consider particular patterns of demand and productivity for different goods.

Harrod (1933) considered an open economy with neither savings and investment nor government spending and taxation. In this set-up income, *Y*, is generated by the production of consumption goods, *C*, and exports, *X*, namely: $Y = C + X$. It is assumed that all income is spent on consumption goods and imports (M), such that $Y = C + M$. The real terms of trade are constant and balanced trade is assumed: $X = M$. If we assume a linear import function

such as $M = mY$, where m is the marginal propensity to import, then we have after some algebraic manipulation:

$$Y = \frac{1}{m} X$$

(1)

Expression (1) is known as the static Harrod foreign trade multiplier[1], under which the main constraint to income determination is the level of export demand in relation to the propensity to import. McCombie and Thirlwall (1994, p. 237) claim that "Harrod put forward the idea that the pace and rhythm of industrial growth in open economies was to be explained by the principle of the foreign trade multiplier which at the same time provided a mechanism for keeping the balance-of-payments in equilibrium." Any change in X brings the balance trade back into equilibrium through changes in income and not in relative prices. According to that view the Harrod foreign trade multiplier is an alternative to the Keynesian determination of income through the investment multiplier.

The subsequent development of Harrod's analysis has been to study the growth implications of his model; but as pointed out by Thirlwall (2013, p. 83), Harrod himself never managed to accomplish such task. This has been carried out by a number of authors who built on the insights of Kaldor (1975) as a starting oint. [see e.g. Thilwall (1979), McCombie (1985) and Setterfield (2010)]. Probably the main outcome of this strand is built in terms of a

[1] The dynamic Harrod foreign trade multiplier is connected to the Hicks supermultiplier. While the former considers just the straight impact of the growth rate of exports on the growth rate of output the latter also takes into account the feedbacks that a higher growth rate of exports has on other components of autonomous expenditures. According to McCombie (1985, p. 63) "(…) an increase in exports will allow other autonomous expenditures to be increased until income has risen by enough to induce an increase in imports equivalent to the initial increase in exports".

dynamic version of the Harrod foreign trade multiplier that became known in the literature as Thirlwall's law [McCombie and Thirlwall (2004)]. According to this view the Harrod multiplier was turned into a theory of balance of payments constrained growth, in which the growth process is demand led rather than supply constrained. Assuming constant real exchange rates and that trade must balance in the long run, there is a very close correspondence between the growth rate of output and the ratio of the growth of exports to the income elasticity of demand for imports, namely π:

$$\frac{\Delta Y}{Y} = \frac{1}{\pi}\frac{\Delta X}{X} \tag{2}$$

According to this expression the growth rate of ouput, namely $\frac{\Delta Y}{Y}$, is related to the growth rate of exports, that is $\frac{\Delta X}{X}$, by the inverse of the propensity to import, represented by m. Thus in a balanced trade framework with the real terms of trade constant, countries are constrained to grow at this rate, which in its continuous time version became widely known in the literature as the Thirlwall law[2]. According to this view the balance of payments position of a country is the main constraint on the overall growth rate, since it imposes a limit on demand to which supply can (usually) adapt. As it turns out, observed differences in growth performance between countries are associated with particular elasticities of demand for exports and imports.

In this context, structural change features as one of the sources for changes in the elasticity of income of exports and imports with such elasticities being seen as the weighted

[2] According to McCombie (1985, p. 71) the conciliation between Thirlwall's law and the dynamic foreign trade multiplier is not so straightforward since the former is based on a multiplicative import function while the latter is based on a linear import function.

average of sectoral elasticities. In such a view structural changes due to variations in the share of exports/imports may give rise to changes in the aggregated elasticity. Arguably, a country whose structure is concentrated on sectors that produce raw materials, for instance, will have a lower income elasticity of demand for exports than a country specialized in the production of sophisticated goods. From this perspective we may conclude that the policy implications from the SED and the BoP approaches are similar: underdeveloped countries should pursue structural changes in order to produce and export goods with higher income elasticity of demand.

Previous attempts to establish connections between these two strands have been proven fruitful. Results such as the multi-sectoral version of Thirlwall's law [Araujo and Lima (2007)] and the disaggregated version of the cumulative model [Araujo (2013) and Araujo and Trigg (2015)] have shown that demand, captured mainly by income elasticities, plays a central role in determining the growth rates even in the long run. These developments have shown that disaggregated assessments of well establish results of that literature can give rise to new insights.

Kaldor himself has abandoned aggregated view in search of sectoral and regional approaches that emphasize divergence in growth rates, dynamic returns of scale, cumulative causation and path dependence in economic development [see e.g. Hein (2014)]. Taking a disaggregated analysis led him to conclude that the manufacturing sector plays a key role in establishing the pace of economic growth due to its positive effects on overall labour productivity growth. Such effects are related to the existence of significant forward and backward linkages in the production chain of the manufacturing sector whereby a productivity gain in one industry may be spread to others due to such linkages. Following such developments, the so-called 'Kaldor growth laws' [Kaldor (1966) and Thirlwall (1987)] convey a strong sectoral flavour in as much as the manufacturing sector is seen as the 'engine

of growth'. In such a view the process of economic development is conceived not only as economic growth but also as structural change in which the transfer of labour from low to high productivity sectors play an important role in the determination of the overall productivity.

However, despite the importance given by Kaldor to a disaggregated analysis the formal model employed to support his verbal reasoning [see Dixit and Thirlwall (1975) and Thirlwall (1987)] is built in terms of an aggregated economy. And one the main component of such model is a dynamic version of the Harrod foreign trade multiplier, as derived in Araujo and Trigg (2015).

In order to carry out the present analysis we have adopted a procedure analogous to the one advanced by Trigg and Lee (2005) and extended by Araujo and Trigg (2015) to consider international trade. The former work explores the relation between the Keynesian multiplier and Pasinetti's model of pure production in a closed economy, by showing that indeed it is possible to derive a simple multiplier relationship from multisectoral foundations in a closed version of the Pasinetti model; hence a scalar multiplier can legitimately be applied to a multisector economy. By departing from this result, Araujo and Trigg (2015) have derived an initial formulation of the disaggregated Harrod foreign trade multiplier. Here we intend to go a step further by showing the consistency of such formulation by proving that through its aggregation we can obtain the original Harrod foreign trade multiplier. Such formulation requires the introduction of the price system: a task not performed by Araujo and Trigg (2015). With this new direct translation to the Harrod multiplier additional insights are madeto better understand the connections between balance of payments and real income and employment in a pure labour economy.

Following this approach we show, for instance that the equilibrium Pasinettian solution for the system of physical quantities may be obtained as a particular case of the

solution given by multi-sectoral Harrod foreign trade multiplier derived here when the condition of trade balance is satisfied. Besides imposing that such condition must hold in the long run it is possible to derive a matrix version of the multi-sectoral Thirlwall's law akin to Araujo and Lima (2007). With this analysis, we intend to emphasize the view that in the presence of a favorable economic structure a country may enjoy a higher level of output, which may be reached through relaxing the balance of payments constraint.

This article is structured as follows: in the next section we derive and highlight the relevance of the scalar Harrod foreign trade multiplier. Section 3 performs the derivation of this scalar multiplier from multisectoral foundations, exploring how this relates to the Harrod matrix multiplier. And in Section 4 some conclusions are provided.

2. Systems of physical and monetary quantities in an extended version of the Pasinettian Model to International Trade

Let us consider an extended version of the pure labour Pasinettian model to foreign trade as advanced by Araujo and Teixeira (2004). Demand and productivity vary over time at a particular rate in each sector of the two countries – the advanced one is denoted by *A* and the underdeveloped one by *U*. Assume also that both countries produce $n - 1$ consumption goods in each vertically integrated sector, but with different patterns of production and consumption. In order to establish the basic notation, it is useful to choose one of the countries, let us say *U*, to express physical and monetary flows. The system of physical quantities may be expressed as:

$$\begin{bmatrix} \mathbf{I} & -(\mathbf{c}+\xi\mathbf{c}^{e}) \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix} \tag{3}$$

where **I** is an (n–1)x(n–1) identity matrix, **0** is an (n–1) null vector, $\mathbf{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix}$ is the (n–1) column vector of physical quantities, $\mathbf{c} = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{n-1,n} \end{bmatrix}$ is the (n–1) column vector of consumption coefficients, $\mathbf{c}^{\mathrm{e}} = \begin{bmatrix} a_{1\hat{n}} \\ \vdots \\ a_{n-1,\hat{n}} \end{bmatrix}$ refers to the (n–1) column vector of foreign demand coefficients, and $\mathbf{a} = \begin{bmatrix} a_{n1} & \cdots & a_{n-1,n} \end{bmatrix}$ is the (n–1) row vector of labour coefficients. X_n denotes the quantity of labour in all internal production activities. The family sector in country A is denoted by $\hat{n}$ and the population sizes in both countries are related by the coefficient of proportionality ξ. According to Pasinetti (1993), system (3) is a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions of the system for physical quantities is:

$$\det \begin{bmatrix} \mathbf{I} & -(\mathbf{c} + \xi \mathbf{c}^{\mathrm{e}}) \\ -\mathbf{a} & 1 \end{bmatrix} = 0 \tag{4}$$

Condition (4) may be equivalently written as [see Araujo and Teixeira (2004)]:

$$\mathbf{a}(\mathbf{c} + \xi \mathbf{c}^{\mathrm{e}}) = 1 \tag{4'}$$

If condition (4)' is fulfilled then there exists solution for the system of physical quantities in terms of an exogenous variable, namely $\overline{X}_n$. In this case, the solution of the system for physical quantities may be expressed as:

$$\begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} (\mathbf{c} + \xi \mathbf{c}^{\mathrm{e}})\overline{X}_n \\ \overline{X}_n \end{bmatrix} \tag{5}$$

From the first n – 1 lines of (5), we conclude that in equilibrium the physical quantity of each tradable commodity to be produced in country U, that is X_i , $i = 1,...,n-1$, will be determined by the sum of the internal and foreign demand, namely $a_{in}\overline{X}_n$ and $\xi a_{in}\overline{X}_n$ respectively. The last line of (5) shows that the labour force is fully employed. It is important to emphasize that solution (5) holds only if condition (4)' is fulfilled. If (4)' does not hold, then the non-trivial solution of physical quantities cannot be given by expression (5). The economy depicted by system (3) may also be represented by a system of monetary quantities, where total wages are spent on domestic consumption goods (represented by domestic coefficients, $\mathbf{c}$) and imports of foreign goods (represented by import coefficients, $\mathbf{c^m}$. The monetary system may be written as:

$$\begin{bmatrix}\mathbf{p} & w\end{bmatrix}\begin{bmatrix}\mathbf{I} & -(\mathbf{c}+\mathbf{c}^{\mathrm{m}}) \\ -\mathbf{a} & 1\end{bmatrix} = \begin{bmatrix}\mathbf{0} & 0\end{bmatrix} \tag{6}$$

where $\mathbf{p} = \begin{bmatrix}p_1 & \cdots & p_{n-1}\end{bmatrix}$ is the (n–1) row vector of prices, $\mathbf{c}^{\mathrm{m}} = \begin{bmatrix}a_{\hat{1}n} \\ \vdots \\ a_{\hat{n}-1,n}\end{bmatrix}$ is the (n–1) column vector of consumption import coefficients, and w is the uniform wage. Like system (3), system (6) is also a homogenous and linear system and, hence a necessary condition to ensure non-trivial solutions for prices should be observed, that is:

$$\det\begin{bmatrix}\mathbf{I} & -(\mathbf{c}+\mathbf{c^m}) \\ -\mathbf{a} & 1\end{bmatrix} = 0 \tag{7}$$

Condition (7) may be equivalently written as [see Araujo and Teixeira (2004)]:

$$\mathbf{a}(\mathbf{c}+\mathbf{c}^{\mathrm{m}}) = 1 \tag{7'}$$

If condition (7)' is fulfilled then there exists a solution for the system of monetary quantities in terms of an exogenous variable, namely $\overline{w}$. In this case, the solution of the system for monetary quantities may be expressed as:

$$[\mathbf{p} \quad w] = [\mathbf{a}w \quad \overline{w}] \tag{8}$$

From the first n – 1 lines of (8), we conclude that in equilibrium the price of each tradable commodity is given by amount of labour employed in its production, that is $p_i = a_{in}\overline{w}$, $i = 1,..., n-1$. If expressions (5) and (8) hold simultaneously it is possible to show after some algebraic manipulation that they express a new condition, which can be viewed as embodying a notion of equilibrium in the trade balance. If $\mathbf{a}(\mathbf{c} + \xi\mathbf{c}^{\mathrm{e}}) = 1$ and $\mathbf{a}(\mathbf{c} + \mathbf{c}^{\mathrm{m}}) = 1$ then by equalizing the left hand side of both expressions we obtain:

$$\mathbf{a}(\xi\mathbf{c}^{\mathrm{e}} - \mathbf{c}^{\mathrm{m}}) = 0 \tag{9}$$

The fulfilment of conditions (4)' and (7)' implies equilibrium in the trade balance but the reverse is not true. Note for instance that if $\mathbf{a}(\mathbf{c} + \xi\mathbf{c}^{\mathrm{e}}) = 0.9$ and $\mathbf{a}(\mathbf{c} + \mathbf{c}^{\mathrm{m}}) = 0.9$ the trade balance condition will also be fulfilled by equalizing the right hand side of both expressions but this situation corresponds to unemployment and under expenditure of national income. That is, the equilibrium in trade balance does not imply neither full employment of the labour force nor full expenditure of national income. This possibility has been somewhat emphasized by the BoP constrained growth approach. According to this view the main constraint on the performance of a country is related to the balance of payments that must be balanced in the long run. In this set up a poor export performance may lead to low levels of employment and national output, thus showing that the external constraint may be more relevant that shortages in savings and investment mainly for developing economies. In this context the Harrod foreign trade multiplier plays a decisive role since it changes the focus of determination of national income from investment to exports.

From the first line of expression (8), we know that $\mathbf{p} = \mathbf{a}w$. Hence by assuming a wage unit, namely $w = 1$, money prices equal to labour coefficients, and the equilibrium in the trade balance may be rewritten as:

$$\mathbf{p}(\xi\mathbf{c}^{\mathrm{e}} - \mathbf{c}^{\mathrm{m}}) = 0 \qquad (9)'$$

In the next section a disaggregated version of the Harrod foreign trade multiplier is derived from the system of physical quantities. The system of monetary quantities will be employed to show the consistency of this disaggregated version since departing from it we arrive at the aggregated version of the static Harrod foreign trade multiplier.

3. The Derivation of the Multi-sectoral static Harrod Foreign Trade Multiplier

The idea of developing a multi-sectoral version of the Keynesian multiplier dates back to derivations by Goodwin (1949) and Miyazawa (1960) of a disaggregated version of the income multiplier in Leontief's framework from a relatively simple Keynesian structure. Both authors emphasized that although there are important differences between the Keynes and Leontief approaches, a bridge between them, namely a disaggregated version of the multiplier, is an important development for both views. In order to derive a multi-sectoral version of the Harrod foreing trade multiplier, let us adopt a procedure similar to the one advanced by Trigg and Lee (2005) and extended by Araujo and Trigg (2015). Dealing with the original Pasinettian model, Trigg and Lee (2005) had to assume that investment in the current period becomes new capital inputs in the next period and that the rate of depreciation is 100% (that is, all capital is circulating capital) in order to derive the Keynesian multiplier. By considering an economy extended to foreign trade, however, we do not need this hypothesis. Let us rewrite the system of physical quantities in (3) as:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ -\mathbf{a} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ X_n \end{bmatrix} = \begin{bmatrix} \mathbf{E} \\ 0 \end{bmatrix} \tag{3'}$$

Note that the difference between expression (3) and (3)' is that in the latter we isolate the vector of sectoral exports $\mathbf{E} = \xi \overline{X}_n \mathbf{c}^{\mathrm{e}}$ on the right hand side. We may rewrite system (3)' as:

$$\begin{cases} \mathbf{X} - \mathbf{c}X_n = \mathbf{E} \\ -\mathbf{aX} + X_n = 0 \end{cases} \tag{10}$$

From the last line of system (10), it follows that:

$$X_n = \mathbf{aX} \tag{11}$$

Note that now the employment level, namely X_n, is not exogenous as in (5) since we are solving the system by considering the possibility of unemployment. That was not admissible for the solution of (5) since there the existence of full employment is a necessary condition for the existence of non-trivial solutions. By pre-multiplying throughout the first line of (10) by $\mathbf{a}$ and using (11), yields: $\mathbf{aX} = \mathbf{acaX} + \mathbf{aE}$. By isolating $\mathbf{aX}$, we obtain the employment multiplier relationship:

$$\mathbf{aX} = \frac{1}{1 - \mathbf{ac}} \mathbf{aE} \tag{12}$$

where $1/1 - \mathbf{ac}$ is a scalar employment multiplier [Trigg and Lee (2005)]. This is an employment multiplier relationship between the employment level $\mathbf{aX}$ and the total labour embodied in exports $\mathbf{aE}$, where the scalar employment multiplier is $1/1 - \mathbf{ac}$. Since $\mathbf{E} = \xi \overline{X}_n \mathbf{c}^{\mathrm{e}}$ expression (12) may be rewritten as:

$$\mathbf{aX} = \frac{\xi \mathbf{ac}^{\mathrm{e}}}{1-\mathbf{ac}} \overline{X}_n \qquad (12)'$$

From expression (7)', $1-\mathbf{ac} = \mathbf{ac}^{\mathrm{m}}$. It is worth remembering that implicit in this expression is the notion of full expenditure of national income. By substituting this result into expression (12)' we can rewrite it as:

$$\mathbf{aX} = \frac{\xi \mathbf{ac}^{\mathrm{e}}}{\mathbf{ac}^{\mathrm{m}}} \overline{X}_n \qquad (12)''$$

This result shows that if the balance of payment equilibrium condition conveyed by expression (9) is fulfilled, namely $\xi \mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$ then the employment level is equal to the full employment level, namely $\mathbf{aX} = \overline{X}_n$.

Further scrutiny of this result allows us to conclude that the full employment of the labour force will be reached when both the condition of full expenditure of national income and the balance of payments equilibrium are simultaneously satisfied. Another way of showing this result is to note that if $\xi \mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$ and $1-\mathbf{ac} = \mathbf{ac}^{\mathrm{m}}$ then $1-\mathbf{ac} = \xi \mathbf{ac}^{\mathrm{e}}$, which is the full employment condition given by expression (7)'. The rationale for this result may be grasped considering two main possibilities. Assume first that the condition of full expenditure is satisfied, namely $1-\mathbf{ac} = \mathbf{ac}^{\mathrm{m}}$, but there is a trade imbalance in the sense that imports are higher than exports, that is $\xi \mathbf{ac}^{\mathrm{e}} < \mathbf{ac}^{\mathrm{m}}$. In this case, $1-\mathbf{ac} > \xi \mathbf{ac}^{\mathrm{e}}$ which implies that $\mathbf{a}(\mathbf{c}+\xi \mathbf{c}^{\mathrm{e}}) < 1$, meaning unemployment. In this case, although the national income is fully expended the content of labour in the exports is lower than the content of labour in the imports, which gives rise to unemployment.

The other possibility is connected to the case in which the trade is balanced but the national income is not fully expended. Hence $\xi \mathbf{ac}^{\mathrm{e}} = \mathbf{ac}^{\mathrm{m}}$ but $\mathbf{a}(\mathbf{c}+\mathbf{c}^{\mathrm{m}}) < 1$. It is easy to show

that this case also leads to: $\mathbf{a}(\mathbf{c}+\xi\mathbf{c}^{\mathrm{e}})<1$, also meaning unemployment. Then it is proven that the full employment of the labour force depends on the conjunction of two other conditions, namely full expenditure of national income and balance of payments equilibrium.

This result shows that if the effective demand condition given by expression (5) is fulfilled then the employment level is equal to the full employment level, namely $\mathbf{aX}=\overline{X}_n$. While expression (12)' generates different levels of employment, only one of them will be the full employment level that corresponds to the Pasinettian solution. Through further decomposition [see Trigg (2006, Appendix 2)], (12) can be substituted into the first line of (10) to yield:

$$\mathbf{X}=\left(\mathbf{I}+\frac{\mathbf{ca}}{1-\mathbf{ac}}\right)\mathbf{E} \tag{13}$$

From expression (7)' $\mathbf{ac}^{\mathrm{m}}=1-\mathbf{ac}$. Hence:

$$\mathbf{X}=\left(\mathbf{I}+\frac{\mathbf{ca}}{\mathbf{ac}^{\mathrm{m}}}\right)\mathbf{E} \tag{14}$$

This is a multiplier relationship between the vector of gross outputs, **X**, and the vector representing foreign demand $\mathbf{E}$, where $\left(\mathbf{I}+\frac{\mathbf{ca}}{\mathbf{ac}^{\mathrm{m}}}\right)$ is the output multiplier matrix. This result is a multi-sectoral version of the Harrod foreign trade multiplier whereby the output of each sector is related to the export performance of that sector. One of the main differences between this multi-sectoral multiplier for an open economy and the one derived by Trigg and Lee is that the latter is a scalar, and the former is a matrix.

The derivation of the multi-sectoral Harrod foreign trade multiplier allows us to better understand the connection between the balance of payments and the level of employment and production. Expression (12)' and (14) shows that balance of payments equilibrium may be

associated with levels of employment and production lower than those related to full employment and equilibrium. In order to show this let us rewrite expression (14) by considering that $\mathbf{E} = \xi \overline{X}_n \mathbf{c}^{\mathrm{e}}$. After some algebraic manipulation it yields:

$$\mathbf{X} = \left(\xi \mathbf{c}^{\mathrm{e}} + \mathbf{c} \frac{\xi \mathbf{a}\mathbf{c}^{\mathrm{e}}}{\mathbf{a}\mathbf{c}^{\mathrm{m}}} \right) \overline{X}_n \qquad (14)'$$

Expression (14)' plays a central role in our analysis. It shows that if $\mathbf{a}\mathbf{c}^{\mathrm{m}} = \xi \mathbf{a}\mathbf{c}^{\mathrm{e}}$ then the solution given by (14)' sums up to the solution given by the first line of (5). In this vein, the equilibrium Pasinettian solution given by the first lines of expression (5) is a particular case of the solution given by multi-sectoral Harrod foreign trade multiplier (14)' when there is equilibrium in the trade balance $\mathbf{a}\mathbf{c}^{\mathrm{m}} = \xi \mathbf{a}\mathbf{c}^{\mathrm{e}}$.

Hence the solution put forward by Araujo and Teixeira (2004) for an open version of the Pasinetti model is in fact a particular case of the solution obtained here. That result is of key importance. Note that if $\xi \mathbf{a}\mathbf{c}^{\mathrm{e}} > \mathbf{a}\mathbf{c}^{\mathrm{m}}$, such that $\frac{\xi \mathbf{a}\mathbf{c}^{\mathrm{e}}}{\mathbf{a}\mathbf{c}^{\mathrm{m}}} > 1$, a situation in which the country is running trade surpluses, we should expect that the levels of output given in the Harrodian solution given by (14)' are higher than the Pasinettian solution given by the first line of (5). Otherwise, if the country is running trade deficits, that is $\xi \mathbf{a}\mathbf{c}^{\mathrm{e}} < \mathbf{a}\mathbf{c}^{\mathrm{m}}$, this implies that $\frac{\xi \mathbf{a}\mathbf{c}^{\mathrm{e}}}{\mathbf{a}\mathbf{c}^{\mathrm{m}}} < 1$, and outputs in the Pasinettian solution arehigher than in the Harrodian solution. In sum, we should expect that the sectoral outputs given by the Harrod foreign trade multiplier deviate from the equilibrium Pasinettian output in the presence of trade deficits and surpluses.

But one of the main arguments of BoP constrained growth theory is that in the long run trade should be balanced, namely $\xi \mathbf{a}\mathbf{c}^{\mathrm{e}} = \mathbf{a}\mathbf{c}^{\mathrm{m}}$, since a country cannot run permanent

deficits. While the case $\xi\mathbf{ac}^{e} < \mathbf{ac}^{m}$ is unsustainable from the viewpoint of country *U* in the long run, the reverse $\xi\mathbf{ac}^{e} > \mathbf{ac}^{m}$ is unsustainable from the viewpoint of country *A*. Hence we may conclude that the Harrodian solution gravitates around the long run Pasinettian solution.

4. From the Multi-sectoral to an Aggregate version of the static Harrod Foreign Trade Multiplier

In order to demonstrate the consistency of our approach it may be shown how the aggregated version of the static foreign trade multiplier can be devived from the analysis developed in the previous section. Now under a pure labour theory of value, as assumed by Pasinetti, let us say that there is a wage unit, $w = 1$ such that money prices are equal to labour coefficients. From the first line of system (8) we conclude that: $\mathbf{p} = \mathbf{a}$. By substituting this result into expression (12), a scalar output multiplier relationship can be specified as follows:

$$\mathbf{pX} = \frac{1}{1-\mathbf{pc}}\mathbf{pE} \tag{15}$$

Note that $\mathbf{pX}$ amounts for total output, namely $Y = \mathbf{pX}$, and $\mathbf{pE}$ represents for total exports, that is $E = \mathbf{pE}$. Hence, expression (15) takes the form:

$$Y = \frac{1}{1-\mathbf{pc}}E \tag{16}$$

This is an aggregate multiplier equation in which $\mathbf{pc}$ is the propensity to consume domestically produced goods. Expression (16) is analogous to the aggregated Harrod foreign trade multiplier since it relates output to total exports. But in order to prove that this is the Harrod multiplier it is necessary to show that the denominator embodies the propensity

to import. By also substituting $\mathbf{p} = \mathbf{a}$ into expression (4)' one obtains: $\mathbf{p}(\mathbf{c} + \xi\mathbf{c}^{\mathrm{e}}) = 1$, which yields: $\xi\mathbf{p}\mathbf{c}^{\mathrm{e}} = 1 - \mathbf{p}\mathbf{c}$. By substituting this result into expression (16) one obtains:

$$Y = \frac{1}{\xi\mathbf{p}\mathbf{c}^{\mathrm{e}}} E \tag{17}$$

A key assumption to derive the static Harrod foreign trade multiplier is that of trade balance. By also substituting $\mathbf{p} = \mathbf{a}$ into expression (9) one obtains the trade balance equation in terms of prices, meaning that in a pure labor economy there is equivalence between the trade balance equilibrium in terms of prices and in terms of labour: $\mathbf{p}(\xi\mathbf{c}^{\mathrm{e}} - \mathbf{c}^{\mathrm{m}}) = 0$, which yields: $\xi\mathbf{p}\mathbf{c}^{\mathrm{e}} = \mathbf{p}\mathbf{c}^{\mathrm{m}}$. By substituting this result into expression (17) one obtains:

$$Y = \frac{1}{\mathbf{p}\mathbf{c}^{\mathrm{m}}} E \tag{18}$$

The denominator of this multiplier is the scalar $m = \mathbf{p}\mathbf{c}^{\mathrm{m}}$, representing the propensity to consume imports, as first introduced in (1). The main contribution here is that this propensity to consume is derived from Pasinettian multisectoral foundations – instead of from an aggregate national income equation, as in the original Harrod formulation reported in Section 2. Though the propensity to import is a scalar magnitude, it is aggregated by pre-multiplying the column vector of import consumption coefficients $(\mathbf{c}^{\mathrm{m}})$ by the row vector of money prices $(\mathbf{p})$. This aggregate relationship holds regardless of the form the number of sectors (number of vector elements).

Expression (18) represents a Harrod trade multiplier that, to use a notion introduced by Pasinetti (1981, p. 35) is 'truly macroeconomic'. He writes: 'There are relations in economic analysis which take up a macro-economic form only when the analysis is carried out at a macro-economic level. They cease to be macro-economic as soon as the analysis is

carried out at a more diasaggregated level. But there are other relations which maintain a macro-economic form quite irrespective of the degree of disaggregation at which the analysis is carried out. It is these relations only that may be termed as *truly* macro-economic' (Pasinetti 1981, p. 35). On this basis it can be argued that the original Harrod aggregate equation suffers from Pasinetti's critique,that use of such an aggregate model, solely from macro-economic foundations, is somewhat artificial, compared to our alternative multiplier derived here from Pasinetti's system.

This derivation also contributes to another dimension of the Pasinetti research programme, which is to use his multisectoral foundations as a basis for synthesis between different strands of economic theory. The 'basic elements…can be traced back to various stages in the development of economic thought' (Pasinetti 1981, p. 19). One such basic element is the the Kahn employment multiplier, developed by Kahn (1931), which in the *General Theory* Keynes (1936) acknowledged to be the first formal multiplier framework. We will derive this multiplier, and show how it relates to the Pasinetti system. [insert footnote Ricardo: We are very grateful to an anonymous referee that we examine the relationship between the Kahn and Harrod multipliers]

Assume that an economy produces only consumption goods and investment goods (in contrast to the Harrod system where only goods for consumption and export are producted). Define $\mathbf{A}$ as a column vector of physical new investment goods. Kahn was interested in the primary employment generate by new investment; this can be measured by pre-multiplying the investment vector by the row vector of employment coefficients to give $\mathbf{aA}$. Using domestic consumption coefficients , $(\mathbf{c})$ to relate consumption to employment $(\mathbf{aX})$ the labour required to produce total consumption is defined as $\mathbf{ac}(\mathbf{aX})$. Hence total employment is defined by the relationship:

$$\mathbf{aX} = \mathbf{ac(aX)} + \mathbf{aA} \qquad (19)$$

from which the Kahn multiplier relationship

$$\mathbf{aX} = \frac{\mathbf{1}}{\mathbf{1} - \mathbf{ac}}\mathbf{aA} \qquad (20)$$

is defined. The Kahn multiplier – a genuinely macro-economic version – is equal to $1/1 - \mathbf{ac}$. Now since in the Pasinetti system, as we have seen, $\mathbf{p} = \mathbf{a}$, by comparing (20) with equations (16) to (18) we can see that the Kahn and Harrod multipliers are identical. Though there is no role for exogenous investment in the Pasinetti pure labour system, the investment-employment multiplier developed by Kahn, in a different system from that of Harrod, is nested in the Pasinetti system – further testament to the remarkable synthetic potential of Pasinetti's system as a foundation for different modelling approaches.

It should also be noted, by inspection of (13), that this Harrod-Kahn aggregate multiplier is integral to the matrix multiplier developed in Araujo and Trigg (2015Far from being an aggregate alternative to multisectoral structural change analysis, the aggregate multiplier is nested as a constituent part of the full blown disaggregated model. For an analysis of the impact, for example, of export expansion in a particular sector i, the impact on other sectors consists of an aggregate multiplier component $(1/1 - \mathbf{ac})$, and a disaggregated component using the first column of the matrix $\mathbf{ca}$ (see equation (13).

This decomposition of the Harrod matrix multiplier offers the basis for further extensions. Though as an abstract starting point the model developed here is based on pure labour foundations, Pasinetti (1981) has shown how this framework can be translated into an input-output framework which models intermediate capital flows. Since world input-output tables have become in recent years readily available to researchers and policymakers, the

possibility is opened up of estimating Harrod multipliers, in matrix and aggregate form. Whilst the truly macroeconomic Harrod multiplier provides a headline indicator of the overall macroeconomic impact of exports – of accessible macroeconomic appeal for policymakers – this can also be nested in a more disaggregated framework which looks at structural change. The decomposition of the multiplier framework suggested here, though firmly theoretical in its objectives, provides a possible starting point for tailoring the modelling approach to empirical research.

It is also possible to show that (18) conveys the taste of the static Harrod foreign trade multiplier since the denominator in the right hand side includes the income elasticities of demand. But by considering a disaggregated version of the linear import function, given by *M* = *mY*, it is possible to show that expression (18) may be made even closer to the Harrod foreign trade multiplier. Let us also assume a disaggregated linear import function, given by: $x_{\hat{i}n} = m_i X_i$, where $x_{\hat{i}n}$ stands for the amount of imported good *i*, and, from expression (5) $X_i = (a_{in} + \xi\, a_{i\hat{n}})X_n$. By dividing $x_{\hat{i}n}$ by X_n we obtain the per capita import coefficient for the *i*-th sector: $a_{\hat{i}n} = m_i(a_{in} + \xi a_{i\hat{n}})$. By considering that $\mathbf{p} = \mathbf{a}$, which yields $p_i = a_{ni}$, $\mathbf{pc}^{\mathrm{m}} = \sum_{i=1}^{n-1} p_i a_{\hat{i}n} = \sum_{i=1}^{n-1} p_i m_i (a_{in} + \xi\, a_{i\hat{n}}) = \sum_{i=1}^{n-1} a_{ni} m_i (a_{in} + \xi\, a_{i\hat{n}})$. By substituting this result into expression (18) we obtain:

$$Y = \frac{1}{\sum_{i=1}^{n-1} m_i (a_{in} + \xi\, a_{i\hat{n}}) a_{ni}} E \qquad (18)'$$

Note that $(a_{in} + \xi\, a_{i\hat{n}})a_{ni}$ measures the share of the *i*-th sector in the national income. Then by denoting $sh_i = (a_{in} + \xi\, a_{i\hat{n}})a_{ni}$ we can rewrite expression (18)' as:

$$Y = \frac{1}{\sum_{i=1}^{n-1} m_i sh_i} E \tag{21}$$

This expression is closer to the Harrod foreign trade multiplier since we can consider that: $m = \sum_{i=1}^{n-1} m_i (a_{in} + \xi\, a_{i\hat{n}}) a_{ni}$. Then, expression (21) may be rewritten as:

$$Y = \frac{1}{m} E \tag{21'}$$

which is the static version of the Harrod foreign trade multiplier. Expression (21) shows that analogous to the Multi-Sectoral Thirlwall's law, changes in the composition of demand or in the structure of production also matter for income determination. In this vein, a country GDP may be higher if it shifts resources away from sectors with a high income elasticity of demand for imports. Hence, according to the approach presented here one of the main barriers to favourable structural changes is also given by the balance of payments constraint. A country with access to foreign markets may induce changes in the structure of production that will allow the reallocation of resources from the low to high productivity sectors, thus giving rise to a propitious economic structure that will lead to higher output.

5. Concluding Remarks

The SED and the BoP-constrained growth approaches embody a shared view that demand plays an important role in the growth process but with different degrees of emphasis. While the SED framework focuses on structural changes accruing from the existence of particular growth rates of demand and technical change for each sector, the BoP literature considers that

elasticities of demand for exports and imports are responsible for explaining particular growth experiences.

On the one hand, the BoP constrained growth approach emphasizes the notion of equilibrium in the long run by considering that a country cannot permanently permanently growth rates higher than those consistent with the long run balance of payments equilibrium. On the other hand, the incorporation of balance of payments in the SED approach has shown that this equilibrium is subject to the particular dynamics of technical change and patterns of demand.

A common feature of both approaches is that the notion of equilibrium plays a central role. While in the BoP approach, equilibrium in the balance of payments is a required condition of sustainability in the long run, the SED approach shows that the most probable macroeconomic consequence of the growth process is disequilibria which is translates into structural unemployment. But it is undeniable that even in the SED approach the equilibrium in the balance of payments should be observed in the long run. The direct consequence of this characteristic is that the evolving patterns of technical change and preferences cannot be exogenous but will be subject to the external constraint as pointed out by the BoP approach. An important feature of the SED approach is that it can establish normative conditions for full employment of the labour force and conditions for equilibrium in the balance of payments,, although it is easy to prove that the former will not be generally satisfied.

Here we provide foundationalmore foundations to the connections between the SED and BoP constrained approach by showing that a disaggregated version of the static Harrod foreign trade multiplier may be derived from an open version of the Pasinettian model. In addition, we show that the equilibrium Pasinettian solution for the system of physical quantities may be obtained as a particular case of the solution given by multi-sectoral Harrod foreign trade multiplier, derived when the full employment condition is satisfied. Finally, in

order to prove the consistency of our approach we show that departing from this disaggregated version of the Harrod foreign trade multiplier we can obtain the aggregated version. With the approach developed here the outcomes from a cross-fertilization between the two approachesextends beyond the disaggregated version of the Thirlwall's law.

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