Evolutionary Dynamics in a Two Sector Neo-Kaleckian Model of Growth and Distribution

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Abstract

In this paper, we focus on a two sector version of the Neo-Kaleckian model of growth and distribution by using evolutionary dynamic to model the dynamics of investment flows between the capital good and the consumption goods sectors. In such an evolutionary dynamics, not only the rates of profit but also the capital stocks of each sector play an important role in the decisions of each entrepreneur in investing in one of the sectors. With such analysis we intend to introduce explicitly a mechanism that takes into account the tendency for profit equalization, but we also study the possibility that profit equalization is prevented due to the existence of barriers to capital flows. In all cases the model is perfectly determined which is evidence of the robustness of the Neo-Kaleckian analysis when conducted in higher levels of disaggregation.

Keywords: Evolutionary game theory, two sector Neo-Kaleckian model, profit rate equalization.

1 Introduction

The Neo-Kaleckian model is one of the most ingenious formulations of the growth theory in as much as it delivers an analysis of the growth process that assigns a key role to the distributive features of the economy. Based on the seminal works of Kalecki (1954, 1971) [18, 19] and Steindl (1952) [26], such framework considers that income distribution may be an important source of growth if the economy is not working under full capacity utilization. Such possibility is relevant in an economy characterized by monopoly power, since the distribution of income that accrues from the power balance between workers and capitalists may give rise to a dynamic of capital accumulation that can be either beneficial or harmful to growth even in the long run. The origins of such analysis may be found in the growth model coined by Kaldor (1956) and Robinson (1956, 1962), who have highlighted the importance of the distributive

process to the growth performance, but who still worked on the notion of full or normal capacity utilization.

Dutt (1984) [6] and Rowthorn (1982) , working independently, have built what is known as the Neo-Kaleckian model by endogenizing the rate of capacity utilization in the lines of Steindl (1952). One of the main contributions of such extension is the possibility of disequilibrium and the presence of a stagnationist regime in which an increase in the profit share implies a reduction in the rate of capital accumulation and growth. The key assumption behind this result is that the growth rate of investment is a function not only of the profit rate, as in Kaldor-Robinson but also of the rate of capacity utilization. Bhaduri and Marglin (1990) have challenged this view by considering that the growth rate of investment is a straight function not of the profit rate but of the profit share. According to them the profit rate should be replaced by the margin of profit conveyed by the profit share in the investment equation. One of the properties of the Post-Kaleckian model, as it became known, is the possibility of a non-stagnationist regime in which eventual falls in consumption due to a lower wage share are overcompensated by an increase in investment led by a profit share expansion. Arguably, the possibility of a profit led regime is an important improvement brought by the Bhaduri and Marglin (1990) formulation since empirical evidence had shown that its occurrence is a reality in more open economies [see e.g. Hein and Vogel (2008), Ederer et al. (2009), Naastepad and Storm (2007) and Elder and Stockhammer (2008)]. But if such possibility gives more realism to the analysis, there is an increasing consensus that "it is not clear why investing entrepreneurs would care about the profit share, in contrast to the profit rate." [Lavoie (2010, p. 133)]. In short, if on one hand the Post-Keynesian view is celebrated in terms of its results on the other hand it is challenged in terms of justifying one his central assumptions, namely the substitution of the profit rate by the profit share in the investment function. Here in order to avoid this controversy we focus on the Neo-Kaleckian version of model. More specifically we focus on the debate between Park (1995) and Dutt (1997) in the nineties concerning a two-sector version of the Neo-Kaleckian model.

Park (1995) claimed to show that the Kalecki-Steindl model becomes overdetermined – i.e., has more equations than unknowns – in a multi-sector framework thus calling into attention a disaggregated version of the Neo-Kaleckian model. Dutt (1997) disputed both the claim and the approach by considering that it is inappropriate to decompose the growth of the capital stock (g) into components at the sectoral level. Moreover, he advocates that the overdetermination problem vanishes once one specifies an adjustment mechanism of profit rate equalization. The present paper adopts an evolutionary view to approach such issue and derives two interesting results: (i) that the model is not overdetermined even when disaggregated accumulation functions are adopted and (ii) profit rate equalization may, under certain conditions, be given an underlying explanation as outcomes of evolutionary dynamics. The importance of such an approach can be grasped by considering that one of the main criticisms of the Post-Keynesian economics levelled against the Neo-classical paradigm is related

to the aggregative hypothesis. In order to escape from this criticism the Neo-Kaleckian model should envisage the possibility of having its original analysis being carried out in more disaggregated levels of analysis [see Araujo and Teixeira (2015)], and in such context the two-sector version of such model plays an important role as a prototype.

Therefore, the aim of the current paper consists in introducing evolutionary dynamics in a two sector version of the Neo-Kaleckian model to provide a mechanism whereby rate of profit equalization can be attained or at least pursued. Within such approach the investors, new capitalists, decide strategically in what of the sectors they will invest. But we intend to go a step further by showing how such tendency may be prevented or at least retarded if barriers to capital flows are introduced. Such hypothesis is reasonable in the presence of monopoly power. For the best of our knowledge, although the evolutionary dynamics has been introduced to the Kaleckian model to approach other issues [see Silveira and Lima (2014)] our paper is the first one that adopts this methodology to approach the issue of investment flows in a two sector Kaleckian model. The first step to build an evolutionary approach to the Neo-Kaleckian two sector model consists in rewriting such model by using the seminal contribution by Feldman. Feldman's two-sector growth model (1928) is widely used as a benchmark to study the effects of the investment allocation on economic growth. This is a model with a consumption and an investment sector in which capital goods can be used to increase the capacity of either sectors. At any given instant of time the productive capacity of each sector is quasi-fixed and non–shiftable but over time the proportions can be continuously altered by allocating new investments to the one or the other of the two sectors. By introducing such analysis in the Neo-Kaleckian model allows us to focus on the flow of investment between the two sectors by raising the possibility of considering that the decisions of investing in each of the sectors is affected not only by the rate of return but also by the amount of investment in each of the sectors. The current analysis is developed as follows: in the next section we present a brief discussion on the controversy between Park (1995) and Dutt (1997). Section 3 presents a modified version of the two sector Kaleckian model in which we focus on investment allocation. Section 4 introduces the evolutionary dynamics and and section 5 concludes.

2 The Controversy

One of the main controversies related to the two sector Neo-Kaleckian growth model is in fact a controversy in the core of Economics [see Dutt (1990, p. 146)], and it is related to the possibility of profit rate equalization in the presence of monopoly power. Since Schumpeter, it is well known that the most important competition in the capitalist system is a non-price competition that takes the form of either new products or methods of production to grant monopoly power to their holders. Hence, the search for extra-profits given by the monopoly power gives rise to barriers such as patents, technological indivisibilities and high minimum capital requirements levels of capital that prevent capital flows

amongst sectors, and in such a context profit rate equalization cannot be granted automatically. In fact, some authors such as Bortis (2003), considers that the Smithian (classical) postulate that in the long run the rate of profit is uniform across sectors is not incompatible with the Schumpeterian view that innovation leads to monopoly power. The tendency towards a uniform rate of profits is one the basic principles of the classical view of competition – and principles are invariant in space and time, and, as such, always hold.

However, the historical-empirical realisation of principles may be enhanced or obstructed by various factors. According to this range of view, the equalization of the profit rates should not occur in presence of barriers to capital flows. But for other authors such as Duménil and Lévy (1999) the time dimension should be added to discussion in as much as barriers that prevent capital flows in the short run can be overcome in the future. According to them, while the short run is characterized by barriers that prevent capital flows, the long run such is characterized by the non-existence of such barriers. This view is somehow based on the Steindl (1952, p.17) view that "[i] prices, and consequently profits, are sufficiently high, entry of new competitors into an industry becomes feasible even where capital requirements are great." But for other authors such as Sylos-Labini (1969) and Sweezy (1942) such barriers hold even in the long run since they are an inherent component of the capitalist system. In this case, the classical laws of competition should be replaced by those of capitalist economy, which are regulated by the balance of power between workers, capitalists firms and the state [Dutt (1990, p. 146)]. Although such view has found some evidence in the empirical literature supporting profit rate differentials amongst sectors, it is disputed by authors such as Semmler (1984) who argue that differentials are not incompatible with the classical notion of competition. According to this view, the monopoly power is mainly related to the ability of firms setting prices, and not to prevent capital flows. [see Dutt (1990)].

The starting point of our formal discussion is the model advanced by Park (1995) in which a two sector version of the Neo-Kaleckian model is presented and scrutinized to consider the issues of over determination and profit equalization between sectors. For Park such formulation suffers from over determination , in the sense that the number of equations is higher than the number of variables to be determined in the model. Dutt (1997) has challenged this result by considering that in fact the model is perfectly determined. For this author, Park has erroneously assumed particular investment functions for each of the sectors thus leading to an excessive number of equations in the model. By considering an aggregative investment function, the author is able to establish an equal number of equations and unknowns, thus finding a closed form solution for the model. In the current paper we adopt this view which coincides with the one implicitly supported by Park (1995) and Dutt (1997) in the two sector Neo-Kaleckian model, that can be represented by the following equations:

$$p_1 = p_2 r \left(\frac{v_1}{u_1} \right) + W l_1 \tag{1}$$

$$p_2 = p_2 r \left(\frac{v_2}{u_2}\right) + W l_2 \tag{2}$$

$$1 = w\left(l_1 + l_2 x\right) \tag{3}$$

$$x = g\left(\frac{v_1}{u_1} + \frac{v_2 x}{u_2}\right) \tag{4}$$

$$W = w p_1 \tag{5}$$

$$p_1 = \left(1 + \tau_1\right) W l_1 \tag{6}$$

$$p_2 = \left(1 + \tau_2\right) W l_2 \tag{7}$$

$$g = g(r, u_1, u_2) \tag{8}$$

where subscripts 1 and 2 denote the consumption good and the investment good sectors respectively, r is the uniform rate of profits, W the uniform money wage rate, w the real wage rate in terms of good 1, g the (uniform) rate of capital accumulation, x the level of actual output of the investment good relative to the actual output of the consumption good, $p_i$the sectoral price level, $v_i$the sectoral capital output ratio, τ_i is the sectoral mark-up rate, l_i is the labour per unit of output and u_i the sectoral degree of utilization. Expressions (1) and (2) denote the usual price decomposition equations for the two sectors. Equation (3) shows that the production of consumption goods sector, namely sector 1, is entirely consumed by workers employed in sectors 1 and 2. Expression (4) shows that the production of capital goods sector is responsible for expanding the productive capacities of sectors 1 and 2. By considering that the technology is given by $X_i = \frac{u_i K_i}{v_i}$, i=1,2, we conclude after some algebraic manipulation that expression (4) means that $X_2 = gK$, where K is the total stock of capital, namely $K = K_1 + K_2$. Expression (5) defines the real wage, and expressions (6) and (7) are the usual sectoral mark-up pricing equations. In this way, the model has eight independent equation and eight variables r,w,g,x, p_i and u_i, i=1,2. consequently, the system is perfectly determined.

Equation (8) is the aggregate investment function and it is at the center of the discussion between Park (1995) and Dutt (1990, 1997). Dutt (1990, p. 117) considers that it "should properly be thought of as a reduced-form equation, which shows how investment plans are made in equilibrium, rather than as a behavioural equation". Park (1995) defends the alternative viewpoint by considering that the use of an aggregate desired accumulation function as in (8) is an inadequate representation of investment decision in a multi-sectoral model. Following Park's interpretation, if expression (8) is not "behavioural", it could not properly be regarded as representing the Keynesian autonomy of investment

and expression (8) should be substituted by two disaggregated accumulation function, namely:

$$g_1 = g_1(r_1, u_1) \tag{9}$$

$$g_2 = g_2(r_2, u_2) \tag{10}$$

Now it is necessary to introduce to conditions that define the steady state of the model, namely profit rate equalization and balanced growth:

$$r_1 = r_2 = r \tag{11}$$

$$g_1 = g_2 = g \tag{12}$$

Expression (11) refers to the classical tendency of profit rate equalization while expression (12) considers that balanced growth should hold in the long run. Now the model with disaggregated investment functions has eleven equations, namely (1), (2), (3),(4), (5), (6), (7), (9), (10), (11) and (12) but, only ten variables r_i,w, g_i,x, p_i and u_i and therefore, the model is overdetermined. In the next section we show that reconciliation between the two views is possible when more focus is given to the supply side of the model by introducing the Feldman's analysis of investment allocation in such framework. With such an approach we are able to show that while Park's view of the necessity of disaggregated investment functions should be maintained, Dutt's view that the model does not suffer from over-determination is correct. Hence, even in absence of an explicit profit equalization mechanism, the model can be perfectly determined by using disaggregated accumulation functions. Nevertheless, we will propose the use of an evolutionary dynamic as profit rate adjustment mechanism in section 4.

3 An Extended Neo-Kaleckian Two Sector Model with Emphasis to Investment Allocation

In what follows we present an alternative version of the two sector Neo-Kaleckian model whose focus is on the supply side. We will make use of an adapted version of the Feldman two-sector growth model (1928) . In such formulation, investment goods cannot be imported, labour is surplus and the production of capital goods does not depend on the production of consumption goods sector. Here we change slightly the model by considering the possibility of under capacity utilization. With such formulation we consider that undesired excess capacity can permanently exist in the long run even in an economy in which the lack of capital goods is the main constraint to the growth process. Let us make explicit the production function for sectors 1 and 2 :

$$X_i = \frac{u_i K_i}{v_i} \tag{13}$$

Where X_i is the sectoral output sector, and K_i is the sectoral capital stock of the i-th sector. With this assumption and considering that $K = K_1 + K_2$is the total capital stock and $\frac{\dot{K}}{K} = g$ it is possible to show that expression (4) is equivalent to:

$$K = \dot{I} = X_2 \tag{14}$$

Such expression shows that in the absence of depreciation, the aggregate investment, namely I, is equal to the output of investment good sector. Here we proceed to a further decomposition, following the Feldman analysis, by assuming that the variation in stock of capital in sector 2 depends only on the proportion of the total output of this sector that is allocated to itself. We assume that a proportion of the current production of the investment sector is allocated to itself while the remaining, 1-, is allocated to sector 1 (0 1). Hence expression (14) is split in two equations highlighting the variation of capital in sectors 1 and 2:

$$\dot{K}_1 = I_1 = (1-\gamma)X_2 \tag{15}$$

$$\dot{K}_2 = I_2 = \gamma X_2 \tag{16}$$

Where $I_1$and I_2stand for the investment in the capital goods and in the consumption goods sector, respectively. By using expression (13) we can rewrite expressions (15) and (16) in growth terms as:

$$\frac{\dot{K}_1}{K_1} = g_1 = \frac{I_1}{K_1} = \frac{(1-\gamma)u_2 x}{v_2} \tag{17}$$

$$\frac{\dot{K}_2}{K_2} = g_2 = \frac{I_2}{K_2} = \frac{\gamma u_2}{v_2} \tag{18}$$

where g_i is the sectoral growth rate of output. In what follows let us assume particular and linear investment functions for each of the sectors. By following this specification we agree with Park (1995) who considers that an aggregated investment function not being "behavioural" cannot properly be regarded as representing the Keynesian autonomy of investment function. In order to obtain closed form solution for the model let us assume linear investment fuctions for sectors 1 and 2, respectively:

$$g_1 = \frac{I_1}{K_1} = \theta_1 + \alpha_1 r_1 + \beta_1 u_1 \tag{19}$$

$$g_2 = \frac{I_2}{K_2} = \theta_2 + \alpha_2 r_2 + \beta_2 u_2 \tag{20}$$

where θ_i's denotes autonomous growth rate of investment conveying the idea of animal spirits, the r_i's are the sectoral profit rates, and u_i's are the rates of capacity utilization. $\alpha_i >$ 0 measures the influence of the investment to the

profit rate in the i-th sector and $\beta_i > 0$ measures the sensibility of the growth rate of investment to the capacity utilization in the i-th sector and captures the accelerator effect: a high rate of capacity utilization induces firms to expand capacity in order to meet anticipated demand while low utilization induces firms to contract investment.After some algebraic manipulation it is possible to show that, total that the growth rate of the capital stock in this economy, denoted by g, is given by:

$$g = \frac{I}{K} = k_1 g_1 + k_2 g_2 \tag{21}$$

where $k_i = \frac{K_i}{K}$. By replacing expressions (19) and (20) into expression (21), such equation shows that expression (8) may be obtained from disaggregated investment functions, a point raised by Park (1995, p.) with respect expression (8). According to him: "[t]he above-mentioned 'reduced-form' equation must be obtained by 'reducing' sectoral investment behaviour to the aggregate counterpart." Let us assume the following saving function: $S = r_1 K_1 + r_2 K_2$, which considers that workers do not save and the propensity of savings of capitalists is equal to one. By dividing the total savings by the stock of capital K we obtain:

$$\frac{S}{K} = r_1 k_1 + r_2 k_2 \tag{22}$$

It follows that:

$$k_1 + k_2 = 1 \tag{23}$$

In equilibrium, total savings are equal to total investment:

$$\frac{S}{K} = \frac{I}{K} \tag{24}$$

In what follows let us follow a slightly different approach in relation to the Park model. From expressions (1), (2), (6) and (7) it is possible to obtain after some algebraic manipulation:

$$\frac{\tau_1}{1+\tau_1} = \frac{p_2}{p_1}\frac{r_1 v_1}{u_1} \tag{25}$$

$$\frac{\tau_2}{1+\tau_2} = \frac{r_2 v_2}{u_2} \tag{26}$$

The sectoral profit-share for sectors 1 and 2 can be written respectively as: $\pi_i = \frac{r_i K_i p_i}{p_i X_i}$. Then after some algebraic manipulation:

$$\pi_1 = \frac{p_2}{p_1}\frac{r_1 v_1}{u_1} \tag{27}$$

$$\pi_2 = \frac{r_2 v_2}{u_2} \tag{28}$$

Then it is possible to conclude that: $\pi_i = \frac{\tau_i}{1+\tau_i}$. Within such framework it is possible to conclude that balanced growth in the long run is more than a hypothesis as posed by expression (11). This is the content of the next

Proposition 1: The long run growth rate of the stock of capital in sectors 1 and 2 is determined by the growth rate of sector 2.

Proof. From expression (18) we know that $\frac{\dot{K}_2}{K_2} = \frac{\gamma u_2}{v_2}$. From expression (17) the growth rate of capital stock of sector 1 is given by: $\frac{\dot{K}_1}{K_1} = \frac{(1-\gamma)u_2 x}{v_2}$. By taking limits to $\frac{\dot{K}_1}{K_1}$ when t and considering that the rate of capacity utilization in the second sector, namely u_2 is constant in the long run leads us to an indetermination since both numerator and denominator tends to infinity. Then by using the L'Hôpital rule, we have that: lim $\frac{\dot{K}_1}{K_1} = \frac{\dot{K}_2}{K_2} = \frac{\gamma u_2}{v_2}$. Then the growth rate of the capital stock of sector 2 is given by: $lim \frac{\dot{K}_1}{K_1} = \frac{\dot{K}_2}{K_2} = \frac{\pi_2 u_2}{v_2}$, which means that in the long run $g_1 = g_2$. c.q.d.

By considering the system formed by expressions (3), (5), (11), (12), (17), (18), (19), (20), (21), (22), (23), (24), (31) and (32) we have fourteen equations and fourteen unknowns, namely k_i, g_i, r_i, u_i, p_i, g, x, w and , which is perfectly determined. After some algebraic manipulation the full solution of the model (see Appendix) may be obtained as:

$$r_1* = r_2* = \frac{\theta_2 \pi_2}{(\alpha_1 - \alpha_2)\pi_2 - \beta_2 v_2} \tag{29}$$

$$g* = g_1* = g_2* = \frac{\theta_2 \pi_2}{\pi_2(\alpha_1 - \alpha_2) - \beta_2 v_2} \tag{30}$$

$$u_1* = \frac{\pi_2(1-\alpha_1)(\theta_2 - \theta_1) - \beta_2 v_2 \theta_1}{[\pi_2(\alpha_1 - \alpha_2) - \beta_2 v_2]\beta_1} \tag{31}$$

$$u_2* = \frac{v_2 \theta_2}{\pi_2(\alpha_1 - \alpha_2) - \beta_2 v_2} \tag{32}$$

$$\gamma* = \pi_2 \tag{33}$$

$$x* = \frac{\pi_2}{1 - \pi_2} \tag{34}$$

$$k_1* = \pi_2 \tag{35}$$

$$k_2* = 1 - \pi_2 \tag{36}$$

$$w* = \frac{1 - \pi_2}{(1-\pi_2)l_1 + \pi_2 l_2} \tag{37}$$

$$p_1* = \frac{[(1-\pi_2)l_1 + \pi_2 l_2]W}{1 - \pi_2} \tag{38}$$

$$p_2* = \frac{[(1-\pi_2)l_1 + \pi_2 l_2]\,W}{1-\pi_2}\left\{\frac{\pi_1\pi_2\,[\theta_2(1-\alpha_1) - \theta_1(1-\alpha_2) + \beta_2 v_2 \theta_1 \pi_1]}{[\pi_2(1-\alpha_2) - \beta_2 v_2]\,\theta_2}\right\}$$

(39)

One of the important outcomes of our approach is that it is possible to determine the value of rate of investment allocation , namely . A number of authors have searched to establish such value under different criteria. While Bose (1968) and Weitzman (1971) modeled Feldman's analysis in an intertemporal dynamic model a la Ramsey to determine the optimal value of rate of investment allocation that maximizes intertermporal consumption, Araujo and Teixeira (2002) have shown that it can be treated as a particular case of Pasinetti's model of structural change (1981). With such approach the authors were able to find the value of the rate of investment allocation consistent with the fulfilment of capital accumulation conditions. Here one of the outcomes of the analysis presented is that it allows us to determine the value of the rate of investment allocation consistent with the tendency of profit rate equalization, a criterion that was not taken into account for the above mentioned authors. From expression (40) we conclude that such rate should equal the profit share of sector 2. The proposition shows that in the framework adopted here $g_1 = g_2$ is not a mere hypothesis but it is a result that accrues from the fact that one of the sectors, namely the consumption goods sector, relies on the production of the capital goods sector. So it cannot grow indefinitely faster than the sector that it feeds upon. Another important result is that in such an economy we face a profit led regime. This is the content of the next

Proposition 2: The model operates under a wage led regime.

Proof. We know that the overall growth rate of the economy is given by expression (30). By taking the derivating of g* with respect to π_2 yields:

$$\frac{\partial g*}{\partial \pi_2} = \frac{\beta_2 v_2 \theta_2}{[\pi_2(\alpha_1 - \alpha_2) - \beta_2 v_2]^2} < 0 \tag{40}$$

Then we have a wage led regime in which an increase in the profit share of the sector 2 leads a smaller growth rate for the whole economy. c. q. d.

With this approach, even considering separated investment functions for sectors 1 and 2, as claimed by Park we show that the model does not suffer from over-determination as pointed out by Dutt. The reason for that is that the rate of investment allocation, a new variable that it is not taken into account by either Park or Dutt has to be found in order to guarantee the existence of a steady state solution for the model. However, although the equalization of the growth rate of sectors 1 and 2 given by expression (34) can be easily demonstrated the tendency of the profit rate equalization is assumed but not proved. In what follows, we intend to provide a rationale for this result.

4 The Evolutionary Dynamics

In order to approach the above system from an evolutionary viewpoint we assume for the sake of simplicity only that each capitalist holds only one unit of capital goods, namely N=K, where N is the number of capitalists. Accordingly, the number of capitalists investing in sectors 1 and 2, namely N_1 and N_2 , are respectively given by $N_1 = K_1$ and $N_2 = K_2$. But due to the non-shiftability assumption once the capitalist makes the investment in which of the sectors the decision cannot be reverted. This means that following the Feldman's analysis our focus is on the investment allocation and not on the capital flows as a whole since the capital goods already installed cannot be moved to other sector. Hence we are mainly concerned with the flows of new capital goods that are being produced in such an economy. From expression (14) we know that $K = \dot{I} = X_2$ and we know that in steady state the growth rate of the production of the second sector is given by g. This means that we have to assume that the population of capitalists grows at g in steady state in order to make the model consistent with the assumption that each capitalist holds exactly one unit of capital goods. In formal terms we can convey such formulation as: $\dot{N} = X_2$. In order to explicit the evolutionary dynamics let us consider that the new capitalists in each unit of time is denoted by , namely $\dot{N} = \Omega$. Let us assume that a fraction of such capitalists decide to invest in sector 2 while the remaining decide to invest in sector 1, namely $\gamma = \frac{\Omega_2}{\Omega}$ and $1 - \gamma = \frac{\Omega_1}{\Omega}$.

In order to establish the pay-offs we have to take into account that they depend not only on the interest rate paid in each sector but also to the existing entry cost. Such hypothesis is reasonable because as point out in section 3, one of the main characteristics of capitalism that is monopoly power, which is translated in terms of barriers to investment, flows. Then in what follows we consider that without entry barriers cost the pay-off of investing in each of the sectors is given by the rate of return of this investment, namely the rate of interest. But we assume that there is an entry cost represented by δ_1 if the new capitalist decides to invest in sector 1. Accordingly, we assume that there is an entry cost represented by δ_2 if the capitalist decides to invest in sector 2. Then we consider the following notation: $L_1 = r_1 - \delta_1$ is the pay-off of investing in the first sector 1, while $L_2 = r_2 - \delta_2$is the pay-off of investing in the second sector. It is worth to mention that capitalists already investing in either sectors 1 or 2 do not face the entry but they are subject to the evolutionary dynamics presented here. Following the literature on evolving game theory, we assume that the frequency distribution of strategies in the population of new capitalists changes in time according to an evolutionary dynamic. Here, the evolutionary dynamics can be interpreted as a learning process, determined by the intertemporal comparison of pay-offs [see Gintis (1990, p. 190)]:

$$Ek_1^{t+dt} = k_1^t + dt\kappa^t k_1^t k_2^t (L_1 - L_2) \tag{41}$$

$$Ek_2^{t+dt} = k_2^t + \alpha dt\kappa^t k_1^t k_2^t \beta (L_2 - L_1) \tag{42}$$

where $\kappa = \frac{\Omega}{N}$. According to expression (41) the expected proportion of capitalists investing in the first sector in period t+dt, namely Ek_1^{t+dt}, is equal to the proportion investing in such sector in period t, namely k_1^t, plus the number of new capitalists over total population of capitalists that decide to invest in the first sector, namely $dt\kappa k_1^t k_2^t(L_1 - L_2)$, that can be both negative or postive. If $L_1 > L_2$is positive then $dt\kappa k_1^t k_2^t(L_1 - L_2)$ is positive and the proportion of capitalists investing in the first sector will increase since the pay-off in such a sector is higher then the pay off in the second sector. The converse is also true, namely if $L_2 > L_1$then $dt\kappa k_1^t k_2^t(L_1 - L_2)$is negative and the proportion of capitalists investing in the first sector will decrease with a consequently increase in the proportion of capitalists investing in the second sector. A similar reasoning can be applied to expression (42). Expressions (41) and (42) show that the proportions of capitalists investing in sectors 1 and 2 depend on the comparisons of pay-offs in sectors 1 and 2. After some algebraic manipulation, expresssions (41) and (42) may be written as:

$$\dot{k}_1 = \alpha\kappa k_1 k_2 \beta(L_1 - L_2) \tag{43}$$

$$\dot{k}_2 = \alpha\kappa k_1 k_2 \beta(L_2 - L_1) \tag{44}$$

In equilibrium, solutions $k_1 = 0$ and $k_2 = 0$ are meaningless so the only possible solution corresponds to the case in which:

$$r_1 - r_2 = \delta_1 - \delta_2 \tag{45}$$

Such expression shows us that by introducing evolutionary dynamics in a two sector version of the Neo-Kaleckian model allows us to introduce a mechanism whereby the tendency of profit rate equalization is taken into account. But such tendency is confirmed only in the case in which $\delta_1 = \delta_2$, where expression (11) may be viewed as a particular case of expression (49). Hence, we show that the existence of monopoly power conveyed by the entry costs is not incompatible with profit rate equalization, a point emphasized by Dutt (1997). But we also show that under some strict hypothesis, namely $\delta_1 = \delta_2$ such equalization will occur. It is also possible to conclude that the system formed by expressions (12), (17), (18), (19), (20), (21), (22), (23), (24), (31), (32) and (45) also has twelve equations and twelve unknowns and it is perfectly determined as also pointed out by Dutt (1997).

5 Concluding Remarks

This study was initially conceived in order to introduce evolutionary dynamics in a two sector version of the Neo-Kaleckian model. By following this approach we intend to present a new method whereby the tendency of profit rate equalization is considered in that framework. Park has taken into account such tendency in his scheme but he did not explain the mechanism whereby such equalization would occur. Here we have shown that by introducing the Feldman analysis of

investment allocation in two sector Neo-Kaleckian model it is possible to show that over-determination problem raised by Park does not hold since the rate of investment allocation, a new variable, has to be found in order to guarantee the existence of a steady state solution for the model. Besides, by using an evolutionary game approach we derive a differential equation from the satisficing evolutionary dynamics that accounts not only for the pay-offs but also for the fraction of entrepreneurs investing in one the sectors. We also take into account barriers to capital flows as a cost incurred by the entrepreneurs when they decide to change the sector they are investing. With such an approach we are able to deliver the following results: (i) As claimed by Dutt and contrary to Park's view the model does not suffer from over-determination; (ii) Even in the case where there are barriers to capital flows, the tendency for profit equalization may be one of the possible outcomes; this result is true if the barrier to capital movement captured by a cost in changing investment from one sector the other, is equal for both sectors; and (iii) if such costs are particular for each sector, then the tendency for profit equalization will not materialize even in the long run, but the model is still perfectly determined.

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Appendix

The solution of the model can be obtained as follows. From expressions (27) and (28) it is possible to isolate the rate of capacity ulization for sectors 1 and 2 as:

$$u_1 = \frac{p_2}{p_1}\frac{r_1 v_1}{\pi_1} \tag{46}$$

$$u_2 = \frac{r_2 v_2}{\pi_2} \tag{47}$$

By substituing (46) and (47) into expressions (19) and (20) allows us to obtain after some algebraic manipulation:

$$g_1 = \theta_1 + \left(\alpha_1 + \beta_1 \frac{p_2}{p_1}\frac{v_1}{\pi_1}\right) r_1 \tag{48}$$

$$g_2 = \theta_2 + \left(\alpha_2 + \beta_2 \frac{v_2}{\pi_2}\right) r_2 \tag{49}$$

By equalizing (48) to (17) and (49) to (18) and considering profit rate equalization we obtain:

$$\frac{(1-\gamma)u_2 x}{v_2} = \theta_1 + \left(\alpha_1 + \beta_1 \frac{p_2}{p_1}\frac{v_1}{\pi_1}\right) r \tag{50}$$

$$\frac{\gamma u_2}{v_2} = \theta_2 + \left(\alpha_2 + \beta_2 \frac{v_2}{\pi_2}\right) r \tag{51}$$

by inserting (46) and (47) into (50) and (51) respectively it yields after some algebraic manipulation:

$$\frac{(1-\gamma)rx}{\pi_2} = \theta_1 + \left(\alpha_1 + \beta_1 \frac{p_2}{p_1}\frac{v_1}{\pi_1}\right) r \tag{52}$$

$$\frac{\gamma r}{\pi_2} = \theta_2 + \left(\alpha_2 + \beta_2 \frac{v_2}{\pi_2}\right) r \tag{53}$$

In steady state we can equalize (17) to (18) to obtain after some algebraic manipulation:

$$x = \frac{\gamma}{1-\gamma} \tag{54}$$

By substituing (54) into (52) we obtain:

$$\frac{\gamma r}{\pi_2} = \theta_1 + \left(\alpha_1 + \beta_1 \frac{p_2}{p_1}\frac{v_1}{\pi_1}\right) r \tag{55}$$

The left hand sides of expressions (53) and (55) are equal so by equalizing their right hand sides allows us to obtain:

$$r = \frac{\theta_1 - \theta_2}{\alpha_1 - \alpha_2 + \beta_1 \frac{p_2}{p_1}\frac{v_1}{\pi_1} - \beta_2 \frac{v_2}{\pi_2}} \tag{56}$$

But we know from expression (25) or (46) that:

$$\frac{p_2}{p_1} = \frac{\pi_1 u_1}{r v_1} \tag{57}$$

From the market clearing condition: g = r. By substituing this result into expression (19) allows us to obtain after some algebraic manipulation that:

$$u_1 = \frac{(1-\alpha_1)r - \theta_1}{\beta_1} \tag{58}$$

By substituing (58) into (57) allows us to obtain:

$$\frac{p_2}{p_1} = \frac{\pi_1 \left[(1-\alpha_1)r - \theta_1\right]}{r v_1 \beta_1} \tag{59}$$

By substuting expression (59) into (56) it yields after some algebraic manipulation the steady state value for r, namely:

$$r* = \frac{\theta_2 \pi_2}{(\alpha_1 - \alpha_2)\pi_2 - \beta_2 v_2} \tag{60}$$

Hence by the marketing clearing condition $r = g$, which yields:

$$g* = \frac{\theta_2 \pi_2}{(\alpha_1 - \alpha_2)\pi_2 - \beta_2 v_2} \tag{61}$$