Title: Samuelson and Davidson on ergodicity: a reformulation

Authors:
Miguel Carrión Álvarez, Banco Santander Madrid
Dirk Ehnts, Free University Berlin

Abstract:

The concept of ergodicity as it is used in economics seems to have the qualities of a shibboleth – a word or saying used by adherents of a party, sect, or belief and usually regarded by others as empty of real meaning. It is in use by both neoclassical economics – after Samuelson (1965, 43), who used the term in his paper on what later became foundation of the efficient market hypothesis – and Post-Keynesian economics – after Davidson, who picked up the term in order to highlight methodological differences. Considering the origin of the concept in statistical physics and its use in the topology of dynamical systems, which most economists are not conversant with, the importance ascribed to ergodicity in economic debate seems mystifying. We deconstruct the meaning of the term in the major contributions of Samuelson and Davidson.

We suggest alternatives to the use of 'ergodicity' to discuss the nature of randomness in the real world and the social world. While neo-classical theory assumes stochastic randomness, Post-Keynesians assume non-stochastic randomness. We discuss both types of randomness and then connect them to financial contracts and balance sheets. Following Keynes (1921, 23) idea of raising insurance rates in case of 'specially large demand' when 'the book is in danger of being upset' we argue that even in an ergodic world there is a problem with the idea that stochastic randomness can be dealt with by the financial system.

Keywords:Ergodicity, stochastic, Paul Davidson, Paul Samuelson, efficient market hypothesis

JEL classification: B4, C60, G17

<u>Structure:</u>

1. Introduction
2. Ergodicity in physics
3. Ergodicity in economics: Samuelson (--> stochastic randomness)
4. Ergodicity in economics: Davidson (--> non-stochastic randomness)
5. From ergodicity to stochastic process? (--> Keynes ToP)
6. Conclusion

Samuelson and Davidson on ergodicity: a reformulation

"When I use a word," Humpty Dumpty said in rather a scornful tone,
"it means just what I choose it to mean — neither more nor less."
"The question is," said Alice, "whether you can make words mean so many different things."
"The question is," said Humpty Dumpty, "which is to be master—that's all."

Lewis Carroll (1797)

1. Introduction

The term shibboleth describes a word or saying used by adherents of a party, sect, or belief and usually regarded by others as empty of real meaning. In economics, discussions of 'ergodicity' often make one wonder whether this is not a shibboleth. Samuelson (1965, 43) used the term in a paper on what later became a foundation of the efficient market hypothesis. Years later, Paul Davidson picked up the term in order to highlight methodological differences. His discussion of Samuelson's use of the concept of ergodicity, which will be presented in more detail later, we find rather confusing. Samuelson (1965) ended his paper on a very cautious note, and a major quotation attributed to Samuelson that is used by Davidson does not seem to exist. There is a divergence between Samuelson in the original and Samuelson as presented by Davidson.

Considering the origin of the concept in statistical physics and its use in the topology of dynamical systems, which most economists are not conversant with, the importance ascribed to 'ergodicity' in economic debate seems mystifying. We deconstruct the meaning of the term in the major contributions of first Samuelson and then Davidson. In the following section, we suggest an alternative to the use of ergodicity centered on the term 'stochastic' to discuss the nature of randomness in the real world and the social world. According to our reformulation, neo-classical theory assumes stochastic randomness whereas Post-Keynesians assume non-stochastic randomness. We discuss both types of randomness and then connect them to financial contracts and balance sheets. Following Keynes' (1921, 23) idea of insurers raising rates in case of 'specially large demand' when 'the book is in danger of being upset' we argue that even in an ergodic world there is a problem with the idea that stochastic randomness can be somehow 'managed' by the financial system.

2. Ergodicity in physics

Ergodicity is a property of dynamical systems originating with the work of physicist Ludwig Boltzmann in the latter part of the 19th century.[1] The Stanford Encyclopedia of Philosophy[2] cites a 1912 article by the Ehrenfests (1912), in German, as the first use of the term 'ergodic hypothesis', and dates the term 'Ergode' to an 1884 article by Boltzmann (1884, 79). In English, the term 'ergodic' is attested as early as the 1920's.[3] Attempting to provide a mechanical foundation for thermodynamics, Boltzmann proposed a heuristic characterization of

[1] The content of this section is broadly based on chapters 7-9 of Emch and Liu (2002), to which we refer the reader for additional details. See Horst (2008) for a shorter text written for economists.
[2] http://plato.stanford.edu/entries/statphys-Boltzmann/
[3] The Merriam-Webster dictionary claims its first known use dates from 1926, without further indications.

thermal equilibrium boiling down to the equivalence between time averages of physical quantities on the one hand, and what would today be called 'ensemble averages' of the same quantities on the other hand.[4] For this, Boltzmann also needed to introduce the concept of a statistical ensemble, which in Boltzmann (1884, 79) he called a 'Monode'.

A 'statistical ensemble' is a notional or actual collection whose members are in principle but not in practice distinguishable[5] and such that a realization of 'the' object at hand can be taken to be a randomly selected element of the ensemble.[6] Boltzmann's 'Monode' is then a statistical ensemble of identical mechanical systems with a fixed value of a single (hence 'mono') conserved quantity of motion. An 'Ergode' he defined as a monode determined by the value of energy ('erg' is the name of the centimetre-gram-second unit of energy, and the common greek root of 'energy' and 'ergode'). Typically an ensemble in statistical mechanics generalizes Boltzmann's monode in that it is defined by fixing more than one macroscopic quantity, not just the single quantity 'energy'.

When an ensemble is discrete and finite it is usually assumed that all members of the ensemble are equally likely, in the spirit of the 'principle of indifference'.[7] Famously, Laplace (1814) made this the essence of the theory of chances:

"La théorie des hasards consiste à réduire tous les événemens du même genre, à un certain nombre de cas également possibles, c'est-à-dire, tels que nous soyons également indécis sur leur existence; et. à déterminer le nombre des cas favorables à l'événement dont on cherche la probabilité. Le rapport de ce nombre à celui de tous les cas possibles, est la mesure de cette probabilité qui n'est ainsi qu'une fraction dont le numérateur est le nombre des cas favorables, et dont le dénominateur est le nombre de tous les cas possibles."

Laplace's method breaks down in the case of infinite or continuous ensembles where some sort of 'indifferent' probability distribution must be assumed.[8] Boltzmann's brilliant intuition was that a single instance of the mechanical system would, in its evolution, sample the entire collection of available microscopic states compatible with the macroscopic constraints (the 'ergode'), and that the time spent in each state would define a natural probability distribution on the ensemble. In this way, ensemble averages would equal time averages, which is what the Ehrenfests called the 'Ergodic hypothesis' and is a key working hypothesis in signal processing and the statistics of time series. However, making this approach to grounding thermodynamics on mechanics mathematically rigorous remains fraught with problems, as Emch and Liu (2001, §9.5) discuss:

---

[4] The existence of thermodynamical equilibrium is an empirical matter, codified as the *Zeroth Law of Thermodynamics*, which has humorously been summarised as ‚temperature is a good thing'. Postulates of physical theories, such as the Zeroth Law, are accepted without logical proof on the basis of a sufficient amount of suggestive evidence.

[5] Indistinguishability in principle but not in practice is behind classic statistical physics 'paradoxes' such as Maxwell's demon and the Gibbs mixing entropy.

[6] The analogy with the neoclassical 'representative agent' is patent.

[7] The Stanford Encyclopedia on Philosophy's article on Interpretations of Probability (http://plato.stanford.edu/entries/probability-interpret/) attributes the term ‚principle of indifference' to Keynes (1921, ch. IV) who credits the principle "widely adopted … under the title of *The Principle of Non-Sufficient Reason*" to James Bernoulli and devotes an entire chapter (ibid., ch.XXX) to "Laplace's Method".

[8] This is the origin of the classical paradoxes encompassed under the category of ‚geometric probability', see ???.

Yet, at this mainly mathematical stage of its development, ergodic theory is more concerned with *instantiating* specific mathematical properties, rather than asking which specific physical processes can be modelled by ergodic systems. It does not really address too closely any realistic description of matter in bulk. In particular, one should not look into this kind of ergodic theory for a sufficient justification for the foundations of statistical mechanics.

Rather, it *explores worlds that could be*. And it does it with a purpose: to confront and sharpen as yet poorly formulated ideas. Indeed, serious problems, even at the syntactic level, start when one tries to see whether the various properties of ergodic theory genuinely pertain to Hamiltonian mechanics. While it is true that even the top of the ergodic hierarchy is *compatible* with a Hamiltonian dynamics, this situation is *exceptional* [§9.1]: even the weakest level of the ergodic hierarchy is not generic among finite Hamiltonian systems. The coat of ergodic theory has become too tight for the body it is asked to cover: the idealizations involved have reached an *impasse*.

Hence, to the mythical question of whether the ergodic hypothesis justifies statistical mechanics, the answer is worse than no: it is not the right question. Indeed, the question cannot be anymore whether nature strictly obeys the demands of ergodicity; rather, the question ought to ask how good an idealization the theory really is.

Given this assessment of the place of ergodic theory in the foundations of statistical mechanics, it cannot be that ergodicity is necessary as a foundation of economics since economic systems are more complex than the physical systems to which statistical mechanics has been successfully applied. Thus, that ergodicity is neither sufficient nor necessary as a foundation of statistical mechanics would be an indictment of Samuelsonian economics, unless Samuelson ascribes importance for economics to the ergodic theorems of Birkhoff and von Neumann merely as inspiration, in which case their usefulness is limited once the theory they inspire is established on its own terms.

After all, the ergodic theorems of Birkhoff and von Neumann *assume* as a hypothesis that a dynamical system has a time-invariant measure (which can be assimilated to the probability distribution on the *ensemble* and defines the *ensemble averages*). All that these theorems prove is that, in that case, Boltzmann's time averages do exist and coincide with the ensemble averages. For applications, however, they beg the question of the existence (and uniqueness) of an invariant measure, which is not found in generic Hamiltonian systems. Ergodic theorems are best seen as part of the search for sufficient conditions for the equality of ensemble and time averages.

In fact, a careful reading of Samuelson's and others' work bears out that the hypothesis of ergodicity is never invoked even if the concept is carelessly thrown about in discussion. Davidson (1981-82) latched onto Samuelson's claims to be *inspired* by ergodicity, turned them into a claim that neoclassical economics *requires* ergodicity, and used the supposed ergodicity of neoclassical economics as a deadly constitutional flaw of the whole theory. In the next section we examine the original use of the concept of ergodicity in economics by Samuelson.

3. Ergodicity in economics: Samuelson

Samuelson (1965) in his seminal paper on the efficiency of markets tries to provide mathematical – and the following is the title of the paper – proof that properly anticipated prices fluctuate randomly. Samuelson (1965, 41) poses it as an enigma that in a competitive market a generally expected rise of the price would already have triggered an actual rise of the price. Samuelson assumes an “axiom of mathematically expected price formation”, that is, that competitive bidding causes the prices of futures contracts to be set at the expected level of terminal spot prices. From this hypothesis he proves that future prices are "fair games". Nothing is proved about the price of the underlying asset. In order to arrive at the proof, Samuelson supposes that 'there is at best a probability distribution for any future price' (p. 42). What follows on the next page is the mention of 'ergodic', the one and only that appears in the paper.

Samuelson does not claim that it makes sense to go all the way to the limit where time goes to infinity when thinking about price fluctuations. He writes, "it is possible, but not necessarily assumed, that an ergodic state for P [the probability distribution] will emerge in the limit as T [time] goes to infinity." Some lines above this sentence, Samuelson states that he does not assume 'any special Markov property' and 'nothing necessarily Gaussian or normal'. He wants to keep his mathematics as general as possible.[9] He writes "the theorem is so general that I must confess to having oscillated over the years in my own mind between regarding it as trivially obvious (and almost trivially vacuous) and regarding it as remarkably sweeping" before hedging with "the empirical question of the applicability of the model to economic reality must be kept distinct from the logical problem of what is the model's implied content".

The theorem proved by Samuelson is but a financial interpretation of a very basic result that any random variable defined as an expectation conditioned on information available up to a given time must be a fair game, which is itself an elementary consequence of the ability to exchange the order of integration in multiple integrals. The content of the paper is all in the hypothesis that futures prices are expected values of future spot prices, of whose empirical applicability nothing is said. Samuelson does assume stationarity, and even a weak form of stationarity of the variance of the underlying price, but just in order to deduce that the volatility of futures prices decreases as the time to maturity decreases, and not for the main result.

We must conclude that ergodicity is neither sufficient nor necessary as a foundation of the efficient market hypothesis. Samuelson (1965, 68) concludes his paper with doubts about what he just did: "One should not read too much into the estabished theorem. It does not prove that actual competitive markets work well. It does not say that speculation is good or that randomness of price changes would be a good thing". Samuelson explicitly points out the weakness of his approach, namely, where the probability distribution comes from and in whose minds it exists. The concept of ergodicity is not prominent in Samuelson's paper.

Samuelson (1968, 11-12) writes about the ergodic hypothesis in the context of a discussion of "the classical mind" (our highlighting):[10]

> Finally, there was an even more interesting third assumption implicit and explicit in the classical mind. It was a belief in unique long-run equilibrium independent of initial conditions. I shall call it the "ergodic hypothesis" by analogy to the use of this term in statistical mechanics. Remember that the classical economists were fatalists (a synonym for "believers in equilibrium"!). Harriet Martineau, who made fairy tales out of economics (unlike modern economists who make economics out of fairy tales), believed that if the state redivided income each morning, by night the rich would again be sleeping in their comfortable beds and the poor under the bridges. (I think she thought this a cogent argument against egalitarian taxes.)

---

[9] It is possible for the probability function in the model to contain just a single price with a probability of one hundred percent. There is nothing ergodic about this, of course.
[10] This text is reprinted in the Clower (1969) volume.

Now, Paul Samuelson, aged 20 a hundred years later, was not Harriet Martineau or even David Ricardo; but as an equilibrium theorist he naturally tended to think of models in which things settle down to a unique position independently of initial conditions. Technically speaking, **we theorists hoped not to introduce hysteresis phenomena into our model**, as the Bible does when it says "We pass this way only once" and, in so saying, **takes the subject out of the realm of science into the realm of genuine history**. Specifically, we did not build into the Walrasian system the Christian names of particular individuals, because we thought that the general distribution of income between social classes, not being critically sensitive to initial conditions, would emerge in a determinate way from our equilibrium analysis.

The problem is that in this paper Samuelson is looking back at the classical economists and at himself in his pre-Keynesian youth 35 years before the 1965 paper was written, and saying that now as a neoclassical economist he disagrees with the classicals and, by extension, with his young self.[11] Samuelson writes about his own past, in which from 1932 to 1937 – aged 17-22 – he was a classical monetary theorist. He admits to 'having once been a jackass' and claims 'we were schizophrenics', teaching neutrality of money in one class and expansionary monetary policy in the other. Therefore, in this "jackass" paper, Samuelson is not claiming to believe in the ergodic hypothesis (or in a unique economic equilibrium independent of initial conditions, or in the quantity theory of money) as a neoclassical economist; but that the classical economists, including himself before reading Keynes, did have something of the sort as an implicit assumption. This is much weaker than what Davidson understands from Samuelson.

4. Ergodicity in economics: Davidson

In a 2006 working paper, Paul Davidson (2006) summarizes his well-rehearsed view that Paul Samuelson is singularly responsible for distorting Keynes' insights in the process of incorporating Keynesianism into neoclassical economics. Central to Davidson's argument is the focus on neoclassical economics' reliance on „the ergodic axiom". Dunn (2012, 434) writes in the Handbook of Post-Keynesian economics that Davidson was ‚building on Paul Samuelson's suggestion that economic knowledge about the future rested on the axiom of ergodicity'. There are two issues with Davidson's presentation that we will discuss: one perhaps minor or pedantic point of scholarship, and a more substantive discussion of ergodic theory. On the scholarship, Davidson (2006, 11) supports his contention that neoclassical economics relies on ergodicity on a single paraphrase of Samuelson:

Furthermore in an article published in 1969 Samuelson argued that the „ergodic hypothesis [axiom]" is a necessary foundation if economics is a hard science [Samuelson, 1969, p. 184]. (As explained in section IV infra, Keynes also rejected this ergodic axiom.) What is this ergodic hypothesis?

Despite the citation in the body Davidson (2006), its bibliography contains entries for Samuelson (1947) and a Samuelson (1948), but not for Samuelson (1969). The same citation, including the same page number, recurs in several of Davidson's publications. For instance, Davidson (1991, 133) contains a reference to Samuelson (1969, 184) in Clower (1969):

[11] See Samuelson (1968, pp. 1, 2, 5) as the source of the following quotes.

Indeed, Samuelson (1969, p. 184) has made the acceptance of the „ergodic hypothesis“ the sine qua non of the scientific method in economics. [Samuelson (1969, p. 184) indicated that he used the term ergodic „by analogy to the use of this term in [19th century] statistical mechanics“ in order to remove economics from the „realm of genuine history,“ and keep it in the „realm of science.“]

Samuelson's paper in Clower (1969) does contain the phrases quoted by Davidson, and they are identical with a 1968 paper by Samuelson (pp. 11-12), of which we have presented a quote of some length above. Samuelson states that the classicals had ‚a belief in unique long-run equilibrium independent of initial conditions’. He calls it the ‚ergodic hypothesis’ because of the resemblance with this concept from statistical mechanics. The unique long-run equilibrium arose because ‚the general distribution of income between social classes would not be critically sensitive to initial conditions’. It seems that in the background lay a determinate view of the income distribution. If only time would elapse long enough, the incomes of the highly efficient people would be high and that of the below average efficient people would be low. In this neo-classical view, wages are determined by productivity and there is no influence from institutions, like monopolies in market, labor unions or government policies like the minimum wage.

In Davidson and Davidson (1996, 65) an alternative starting point if offered with Samuelson (1968):

In 1968, MIT Professor and later Nobel Prize winner Paul Samuelson wrote that in their quest to provide a hard scientific basis for the economics discipline modern economists must believe in a ‚unique long run equilibrium [i.e., an inevitable outcome for the economy] independent of the initial conditions’.

In the light of the full quote presented above, this seems to be a misreading of Samuelson. The quote is taken out of context – Paul Samuelson in his 1968 paper looks back at himself at age 17-22 and concludes that he has ‚the great advantage of having once been a jackass’, with the ‚unique long run equilibrium’ falling into exactly this period. Samuelson is not claiming to believe in the ergodic hypothesis, and attributes it to the classicals, which had it as an implicit assumption. It is not correct to say that Samuelson in his ‚quest to provide a hard scientific basis for the economics discipline modern economists’ requires the acceptance of the ergodic theorem.[12]

5. From ergodicity to stochastic process?

If ergodicity is not required for neoclassical economics, this does not invalidate the main points of criticism made by Davidson against it. However, the use of the concept of ergodicity takes the discussion away from practical issues and into areas in which not many economists feel confortable. We do not take up the discussion of rational expectations, which has been discussed at length elsewhere. The issue of (non-)ergodicity as defined by Davidson is connected to the discussion of whether it is a good abstraction to assume that one can predict the future out of the past. The idea of this section is to find a vocabulary that would be suitable for this discussion. It should be noted that it is not the neoclassical position that answers in the affirmative and that it is not the Post-Keynesian position that answers in the negative. Rather it is pricing formulas from the finance literature that are used to predict future prices using past data, like Li’s (2000) copula formula. In the following we try to clearify

[12] In a 350 page book published by Szenberg et al. (2006) titled Samuelsonian Economics and the twenty-first century the only contributor mentioning ergodicity is Paul Davidson. It seems that the other authors do not find the topic worth of mention.

the concept of ergodicity by discussing some statements made by economists.

One important relation is that of ergodicity to uncertainty. Davidson (2007, 87) writes: ‚Important decisions involving production, investment and consumption activities are often taken in an uncertain (nonergodic) environment.' By ‚uncertain' most economists, including Keynes and Davidson, mean that ‚it is not possible to give a well defined probability distribution' (Sauter 2014, 39).[13] Davidson sometimes refers to it as ‚true uncertainty' to distinguish it from risk.[14] Keynes (1936, 114) wrote about uncertain things as those ‚we simply do not know'.[15] The term ‚uncertain' as used by Davidson is problematic because in chaos theory paths can diverge exponentially and so the predictive precision is lost, even though the system is completely deterministic.[16] It always produces the same result given that the initial conditions are the same, but it is not possibly to give a well defined probability distribution. This chaotic system is ergodic and yet uncertain. A Brownian motion is also uncertain. Unconstrained Brownian motion is non-ergodic as it does not have a stationary limiting distribution and a Brownian path will almost surely escape to infinity. Mean-reverting Brownian motion does have a stationary limiting distribution and is ergodic, but it is still uncertain. The uncertainty cannot be reduced below a certain finite level and increasing the amount of available information does not allow more precise prediction of future path behaviour. Determining the mean and standard deviation of future values does not qualify as ‚predictability'.

Uncertain and non-ergodic are not the same thing. A better term than ‚uncertain' might be ‚non-stochastic'. The term ‚stochastic' is not the opposite of ‚deterministic'.[17] ‚Stochastic (adj.)' has the following entry in the Online Etymology Dictionary:

1660s, "pertaining to conjecture," from Greek stokhastikos "able to guess, conjecturing," from stokhazesthai "to guess, aim at, conjecture," from stokhos "a guess, aim, target, mark," literally "pointed stick set up for archers to shoot at," from PIE *stogh-, variant of root *stegh- "to stick, prick; pointed" (see sting (v.)). The sense of "randomly determined" is from 1934, from German stochastik (1917).

‚Stochastic' seems to be quite a good fit for Davidson's purpose. ‚Randomly determined' can be interpreted that there might exist a probability function – based on past data – which allows one to be ‚able to guess'. In the context of randomness this quote from Kolmogorov from Ivanenko (2010, 74) sheds some light on the issue:

Speaking of randomness in the ordinary sense of this word, we mean those phenomena in which we do not find regularities allowing us to predict their behavior. Generally speaking, there are no reasons to assume that random in this sense phenomena are subject to some probabilistic laws. Hence, it is necessary to distinguish between randomness in this broad sense and stochastic randomness (which is the subject of probability theory).

[13] An alternative definition of uncertainty from the natural sciences would define it as the difficulty of making a precise measurement. Uncertainty depends on accuracy and precision of the instrument used to make the measurement. O'Donnell (2013, 125) describes another (Post-Keynesian) alternative to uncertainty which is based on the Human Abilities/ Characteristics Approach. It is mostly epistemological.
[14] See Davidson (1991, 130).
[15] O'Donnell (2013) distinguishes probabilistic and non-probabilistic uncertainty. With the latter, probabilities are not known. O'Donnell uses the term ‚irreducible uncertainty' to highlight the human limitations to reduce the uncertainty.
[16] The parameter characterizing this behaviour is called a Lyapunov exponent.
[17] Davidson (2007, 31) writes: ‚If one conceives of the economy as a stochastic (probability) process, then the future outcome of any current decision is determined via a probability distribution.' It is utterly confusing to have something ‚determined' by a stochastic process.

In the context of the discussion of Davidson's ergodic axiom it might be worthwile to use the distinction of stochastic versus non-stochastic randomness. Only in stochastic system one can hope to predict the future at all, while systems described by non-stochastic randomness do not allow forecasts to be made. That does not mean that they won't be made or that people won't listen to those forecasts. Danielsson and Macrae (2011) write that ‚users need numbers they can use to convince their boss, client or regulator, so users of risk models prefer ‚objective' risk forecasts whereas forecasts accompanied by qualifications and uncertainties appears [sic!] less objective.' If this is the relevant frame for the discussion of whether forecasts are precise and accurate then it does not seem to matter whether the system is ergodic as we move towards infinity. Financial market actors are not famous for thinking about the long run – quite the opposite.

Davidson might disagree with the proposal to embrace the word ‚stochastic'. He has from time to time avoided the use of this concept explicitly. In Davidson (2007, 102) he writes: ‚In a wider sense, however, ergodicity means the presumption of a preprogrammed stable, conservative system where the past, present, and future reality are predetermined whether the system is stochastic or not.'[18] Words mean so different things, and here Davidson is giving his very own definition of ergodicity. Since ergodicity requires an underlying stochastic process, it should be impossible to find a non-stochastic system that can be described as ergodic. Davidson's use of the word ‚predict' is also different from common usage. In Davidson (2007, 32), he writes: ‚The ergodic axiom therefore assures that the outcome associated with any future date can be reliably predicted by a statistical analysis of already existing data' so that ‚the future is therefore never uncertain.' One has to wonder what the definition of ‚reliably predicted' would be. With ergodic systems parameters can be estimated, but usually that does mean that the system can be ‚reliably predicted'.[19]

Kauffman et al. (2012, 1) use the term ‚unprestatable' to argue ‚that the evolution of life marks the end of a physics world view of law entailed dynamics'. The interactions between organisms, biological niches and ecosystems would be ever changing and hence indeterminate. The mathematical unprestatability of the space of possibilities means that we cannot form laws of motion. If the space of possibilities is changing, the idea of time averages is impossible to implement. Ergodic systems, which are part of the physics world view that Kauffman et al. (2012) mention, are incompatible with unprestatability, whereas non-ergodic systems are. This approach seems to be another possibilty to describe the problem of predetermination, as Davidson put it. Unprestatable is equivalent to uncertain, non-ergodic and non-stochastic.

Another approach yet leading to the same result is the theory of reflexivity put forward by George Soros (1987). He focuses on what Kauffman et al. (2012) named interactions between different entities that are everchanging and indeterminate. The idea of Soros is that there are two functions: the cognative function and the manipulating function.[20] Applying his theory to markets, market participants try to understand the market through observation

[18] We are at a loss to understand what Davidson means by ‚predetermined', ‚stable' and ‚conservative' – these terms are have precise meanings in physics and mathematics, but not necessarily in economics. Again, to speak of something ‚predetermined' in a stochastic world is a misuse of language.
[19] See the discussion of the Brownian motion above.
[20] Soros called the manipulative function the participating function before changing terms in Soros (2008).

with the potential goal of using the market to reach their goals. The two functions interact, and this is what Soros calls ‚reflexivity'.[21]

Keynes (1921, 23) pointed out another problem that would make the use of probabilities difficult in the real world. He imagines a coin toss which has a 50-50 chance of showing heads and tails. Bets to insure against either result would first be insured at a rate of 1:1, but if the insurance company would experience a ‚specially large demand' it puts itself in a position in which it is heavily dependend on the outcome of one coin toss. As Keynes put it, ‚the book is in danger of being upset', meaning that the outcome of one insured event has the potential to bankrupt the insurance company. Even though the probability is 50-50, an insurance company therefore will raise the price of insurance against an event as the demand for that insurance is getting larger.

6. Conclusion

The seminal paper of Samuelson (1965) introduced the term ergodicity into the economics literature. Samuelson was very cautious with the result that if there exists a probability distribution of future prices then a single investor cannot 'beat the market'. He explicitly stated that his paper should not be read as 'competitive markets work well'. In other writings, he does not support the idea that a social system settles to a equilibrium position whatever the initial values had been. Davidson read Samuelson to the contrary, stating that ergodicity is an important axiom for neo-classical economics. Having compared Davidson's interpretation of Samuelson's writings, we have concluded that the latter do not vindicate his views.

Nevertheless, the question of whether the future can be predicted is important, and we believe Davidson was right in stressing that issue. We suggest reframing the discussion, using the terms 'stochastic' and 'non-stochastic'. Whereas the latter expresses the idea that the future cannot be predicted, for whatever reasons, the former does allow it. The terms also leave enough space to ensure that stochastic systems can be uncertain as well as non-stochastic systems.

The proper use of these terms matters mostly for discussions of (financial) prices, where mathematical models are used to generate (expected) prices. Social systems that exhibit reflexivity in the sense of Soros like financial markets are

Hicks in a letter to Davidson: ‚I have missed a chance, of labelling my own point of view as non-ergodic. One needs a name to ram a point home. I had tried to read a book on stochastic processes, but I was not sharp enough to see the connections'.[22]

Implications for modeling?

---

[21] Soros uses the correspondance theory of truth to make his point, which we have discussed in Carrión Álvarez and Ehnts (2012, 3).

[22] Source for the quote is Dunn (2012, 437-438).

Literature

Boltzmann, Ludwig (1884), ‚Über die Eigenschaften Monocyklischer und anderer damit verwandter Systeme', Journal für die reine und angewandte Mathematik (Journal de Crelle), 98, 68—94

Carrión Álvarez, Miguel and Dirk H. Ehnts (2012), ‚The theory of reflexivity – a non-stochastic randomness theory for business schools only? IPE working papers 28/2013.

Davidson, Paul (1981-82), ‚Rational expectations, a fallacious foundation for studying crucial decision-making processes', *Journal of Post-Keynesian Economics*, 5(2), 182—197

Davidson, Paul (2006), Samuelson and the Keynes/Post Keynesian revolution: the evidence showing who killed Cock Robin, New School working paper

Davidson, Paul (2007), John Maynard Keynes, New York: Palgrave Macmillan

Dunn, Stephen P. (2012), ‚Non-ergodicity', in: J.E. King (ed.), The Elgar Companion to Post-Keynesian Economics (2nd ed.), Cheltenham, UK and Northamptom, MA: Edward Elgar, 434—439

Ehrenfest and Ehrenfest (1912),

Emch and Liu (2001), 'The Logic of Thermo-Statistical Physics', Berlin: Springer

Horst, Ulrich (2008), "Ergodicity and Nonergodicity in economics." The New Palgrave Dictionary of Economics (2nd ed.). Steven N. Durlauf and Lawrence E. Blume (eds.). Palgrave Macmillan. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan. 01 September 2014 <http://www.dictionaryofeconomics.com/article?id=pde2008_O000096> doi:10.1057/9780230226203.0499

Kauffman, Stuart, Giuseppe Longo and Mael Montévil (2012), ‚No entailing laws, but enablement in the evolution of the biosphere', arXiv:1201.2069 [q-bio.OT]

Keynes, John M. (1921), A Treatise on Probability, London: McMillan and Co.

Keynes, John M. (1936), The General Theory of Employment, Interest and Money. London: Macmillan and Co.

Li, David X. (2000), ‚On Default Correlation – A Copula Function Approach', *Journal of Fixed Income*, 9(4), 43—54

O'Donnell, Rod (2013), , Two Post-Keynesian Approaches to Uncertainty and Irreducible Uncertainty', in: G. C. Harcourt and Peter Kriesler (eds.), The Oxford Handbook of Post-Keynesian Economics, Volume 2: Critiques and Methodology

Samuelson, Paul A. (1965), ‚Proof that properly anticipated prices fluctuate randomly', *Industrial Management Review*, 6(2), 41—49

Samuelson, Paul A. (1968), ‚What Classical and Neoclassical Monetary Theory Really was', *Canadian Journal of Economics*, 1(1), 1—15

Samuelson, Paul A. (1969), Classical and Neoclassical Theory, in: Clower, R. W. (ed.), Monetary theory: selected readings, Harmondsworth: Penguin modern economics readings

Sauter, Oliver (2014), *Monetary Policy under Uncertainty: Historical Origins, Theoretical Foundations, and Empirical Evidence*, Wiesbaden: Springer Gabler

Soros, George. 1987. The Alchemy of Finance. New York: John Wiley & Sons

Soros, George (2008). *The new paradigm for financial markets: the credit crisis of 2008 and what it means*. New York: Public Affairs.

Szemberg, Michael, Lall Ramrattan, Aron A. Gottesman [eds.] (2006), *Samuelsonian Economics in the Twenty-first Century*, New York: Oxford University Press