(Preliminary version)

REAL EXCHANGE RATE AND SHORT-TERM FLUCTUATIONS IN AN EXPORT-LED AND BALANCE-OF-PAYMENTS CONSTRAINED GROWTH MODEL

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Abstract

This work attempts to contribute to theoretical and empirical literature on growth and development within the Keynesian-Kaldorian school of thought by providing an analytical framework that seeks to reconciliate both the export-led and the balance-of-payments constrained growth models into a further general mathematical specification. Such a model enables us to analyse much more broadly some important issues concerning the impact of a currency depreciation on growth as well as the convergence process of the current growth rate after an undervaluation towards the long-term growth rate.

Keywords: export-led growth, balance-of-payments constraint, exchange rate.

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I.   INTRODUCTION

In this work we develop a theoretical formal model that can describe the impact of a currency undervaluation on short-term growth from an export-led and balance-of-payments constrained growth models approach. When it comes to the phenomenon of growth in open economies, within the Keynesian-Kaldorian tradition, we may highlight basically two theoretical models, namely, the Export-Led Growth model (EL) developed by Dixon & Thirlwall (1975) and the Thirlwall's (1979) Balance-of-Payment Constrained Growth (BPC) model. As for the Dixon & Thirlwall's (1975) EL growth model, it is known the exchange rate plays no role in determining the growth paths of a country as in the short as in the long term. With respect to the Thirlwall's (1979) BPC growth model, according to its equations, it seems to be theoretically possible for a policymaker to keep the growth rate above the long-term growth rate indefinitely without causing any balance-of-payments crisis only by constantly depreciating the real exchange rate at a certain positive rate of change. However, in practice, such a possibility does not seem to be (politically) feasible, as it would probably lead to an exceedingly high inflation in the home country and eventually even damage growth. Thirlwall's (1979) model simply assumes the purchase power parity (PPP) holds in the long run but does not show formally and explicitly how the real exchange rate corrects its misalignments over time.

Regarding the relation between exchange rate and growth, under a Keynesian-Kaleckian-Kaldorian perspective, we can point out some theoretical works tackling this issue (Blecker, 1989, 1999; Bhaduri & Marglin, 1990; Bresser-Pereira & Nakano, 2003; Barbosa-Filho, 2006; Gala, 2007; Porcile & Lima, 2010; Razmi, Rapetti and Skott, 2012; Lima & Porcile, 2012). The relationship between exchange rate and growth is undoubtedly important. However, such a relationship is widely disregarded in the balance-of-payments constrained growth model approach. The only works with some developments in this model are Barbosa-Filho (2006) and Porcile & Lima (2010). Barbosa-Filho (2006) states an undervaluation improves exports competitiveness, accelerates technological progress and modifies the country's specialization pattern through changes in the income elasticities of demand for exports and imports ratio, therefore boosting long-term growth. Porcile & Lima (2010) proposed a model wherein the real exchange rate actually plays an important role in a balance-of-payments constrained growth model. In their model the level of employment and the elasticity of labour supply can prevent the currency appreciation, thereby impacting directly on income distribution and growth in developing countries.

However, the literature has been ignoring the workings of the exchange rate within the BPC growth model. This paper contributes to the literature by developing a growth model wherein the real exchange rate (RER) presents an inner dynamics of self-adjustment towards PPP in the long run. It means we will include an exchange rate pass-through mechanism into the home country's domestic prices and allow this domestic price specification for open economies into the canonical Thirlwall's (1979) BPC growth model in order to analyse how the exchange rate affects growth in the short term and how both the RER as well as the growth rate adjust themselves to their respective long-term values. In our model, the effect of a currency depreciation on short-term growth is described by a non-linear equation which means such an impact depends on several initial conditions, thereby increasing considerably the complexity of any analysis on RER policies in the short run.

II THE KEYNESIAN-KALDORIAN GROWTH MODELS IN OPEN ECONOMIES

II. 1 The Export-Led Growth Model (Dixon & Thirlwall, 1975)

The EL growth model follows the Kaldorian tradition wherein the main autonomous demand component for a country is the foreign demand for its exports. "According to Kaldor, regional growth is fundamentally determined by the growth of demand for exports, to which the rate of growth of investment and consumption adjust" (Dixon & Thirlwall, 1975, p. 491). Therefore, we can write:

$$y_t = \gamma(x_t) \qquad (1)$$

where y_t is the rate of growth of output in time t; x_t is the rate of growth of exports in time t; γ is the constant elasticity of output growth with respect to export growth.

Then the authors define the multiplicative export demand function:

$$X_t = P_{dt}^{\rho} P_{ft}^{\delta} Z_t^{\varepsilon} \qquad \rho < 0, \delta > 0, \varepsilon > 0 \qquad (2)$$

X_t is the quantity of exports; P_{dt} is the domestic price; P_{ft} is the foreign price; Z_t is the level of the foreign income; ρ is the price elasticity of demand for exports; δ is the cross elasticity of demand for exports; ε is the income elasticity of demand for exports. In rate of changes:

$$x_t = \rho p_{dt} + \delta p_{ft} + \varepsilon z_t \qquad (3)$$

The lower case letters stand for the rate of change of the variables. The mark-up pricing equation:

$$P_{dt} = (W_t/R_t)T_t \qquad (4)$$

W_t is the level of money wage; $R_t = Y_t/L_t$ is the average product of labour (L_t); T_t is $1 + \%$ mark-up on unit labour cost. Taking the rate of change from (4):

$$p_{dt} = w_t - r_t + \tau_t \qquad (5)$$

The Verdoorn's Law goes as the following equation:

$$r_t = a + \lambda y_t \qquad \lambda > 0 \qquad (6)$$

where a is rate of autonomous productivity growth; λ is the Verdoorn coefficient. Substituting equations (3), (5) and (6) in (1) we have:

$$y_t = \frac{\gamma\big[\rho(w_t - a + \tau_t) + \delta p_{ft} + \varepsilon z_t\big]}{1 + \gamma\rho\lambda} \qquad (7)$$

Given $\rho < 0$, it is easy to see the positive relationship between the Verdoorn coefficient and the rate of growth of output. According to the authors "the dependence of the productivity growth on the growth rate *per si* is not sufficient to cause differences in regional growth rates unless the Verdoorn coefficient varies between regions" (Dixon & Thirlwall, 1975, p. 493).

To analyze the stability conditions the authors introduce one-lag period variables in the right-hand-side of the equation (3):

$$x_t = \rho p_{dt-1} + \delta p_{ft-1} + \varepsilon z_{t-1} \qquad (8)$$

Substituting (8) in (1) and solving the first order difference equation, we have the general solution:

$$y_t = A(-\gamma\rho\lambda)^t + \frac{\gamma\big[\rho(w_{t-1} - a + \tau_{t-1}) + \delta p_{ft-1} + \varepsilon z_{t-1}\big]}{1 + \gamma\rho\lambda} \qquad (9)$$

where A is the initial condition.

Thus, the path of y_t, as $t \longrightarrow \infty$, depends on the coefficient $\gamma\rho\lambda$. If $|\gamma\rho\lambda| > 1$, y_t will achieve a "cumulative divergence from equilibrium". Nevertheless, the authors claim that taking realistic parameters, y_t will tend to an equilibrium level. This result must indicate a constant difference in growth between different regions according to each region's equilibrium growth rate.

II. 2   The Balance-of-Payment Constrained Growth Model (Thirlwall, 1979)

As the economy grows the need for imports increases alike. It means that there is always the possibility for a country to achieve a rate of growth of output inconsistent with the sustainability conditions of current account in the long run. Accordingly, there is no guarantee *a priori* that the equilibrium growth rate established by the previous model is to be consistent with the balance of payments equilibrium for an indefinite period of time. This subsection presents the BPC growth model based on McCombie & Thirlwall (1994).

The expression of the balance-of-payments equilibrium on current account is:

$$P_d X = P_f M E \tag{10}$$

where M is the quantity of imports; E is the exchange rate.

Taking the rate of change from (10):

$$p_d + x = p_f + m + e \tag{11}$$

Using a multiplicative import demand function and redefining the price elasticities in the export demand function for ease of exposition, we have:

$$M = \left(\frac{P_f E}{P_d}\right)^{\psi} Y^{\pi} \qquad \psi < 0, \pi > 0 \tag{12}$$

$$X = \left(\frac{P_d}{P_f E}\right)^{\eta} Z^{\varepsilon} \qquad \eta < 0, \varepsilon > 0 \tag{13}$$

where ψ is the price elasticity of demand for imports; π is the income elasticity of demand for imports. In rates of change:

$$m = \psi(p_f + e - p_d) + \pi y \tag{14}$$

$$x = \eta(p_d - p_f - e) + \varepsilon z \qquad (15)$$

Substituting (14) and (15) in (11) and rearranging the terms we have the rate of growth consistent with balance-of-payments equilibrium:

$$y = \frac{\varepsilon z + (1 + \eta + \psi)(p_d - p_f - e)}{\pi} \qquad (16)$$

If PPP holds in the long run, equation (16) reduces to:

$$y = \frac{\varepsilon z}{\pi} = \frac{x}{\pi} \qquad (17)$$

This result states that the balance-of-payments equilibrium growth rate is determined by the growth rate of the foreign demand and by income elasticities of demand for export and import ratio. Equation (17) states the so-called Thirlwall's Law.

As it can be noticed, comparing equations (7) from EL growth model and (17) from the BPC growth model we can see that the foreign income plays an important role in both theoretical frameworks. As for the parameters, still comparing these models, in the EL growth one we have the Verdoorn coefficient, whereas, in the BPC growth one we can find the income elasticity of demand for imports. In spite of these parameters represent important concerns in the Keynesian-Kaldorian tradition both cannot be found at the same time in any of these views. In practical terms, picking up one model means picking up one parameter at the expense of the other. Hence, reconciling these two views in one further general model can be considered a very important problem for post-Keynesian theorists of growth.

II. 3   The Export-Led and Balance-of-Payment Constrained Growth Model (Thirlwall & Dixon, 1979)

Thirlwall and Dixon (1979) attempted to develop an EL growth model based on Kaldorian principals and taking into account the external constraint to growth. Putting equations (5) and (6) into (16), we have:

$$y = \frac{\varepsilon z + (1 + \eta + \psi)(w - a + \tau - p_f - e)}{\pi + \lambda(1 + \eta + \psi)} \qquad (18)$$

The equation above represents the growth rate of the Export-Led and Balance-of-Payment Constrained growth model (ELBPC). According to this model, there are basically two ways of suppressing the real exchange rate effect in the long run. Firstly, as suggested by Thirlwall & Dixon (1979), assume the Marshall-Lerner condition does not hold in the very long run (i.e. $|\eta + \psi| = 0$). This case is known as the "elasticity pessimism". If it does happen, it is easy to see in equation (18) that the basic Thirlwall's Law holds. The problem with this assumption is, however, that Marshall-Lerner does not hold in the short run but it does in the long run. Actually that is the very explanation for the occurrence of the J-curve. Secondly, assume that PPP holds in the long run. If it does, equations (14) and (15) turn themselves into $m = \pi y$ and $x = \varepsilon z$. Furthermore, taking the condition required for the balance-of-payments equilibrium in (11) we have $y = x/\pi$. As an special situation, also pointed out by the authors, of the second case where PPP holds, assume the Verdoorn coefficient is equal to zero ($\lambda = 0$).

Then, it gives:

$$p_d = w - a + \tau \qquad\qquad (5^*)$$

$$r = a \qquad\qquad (6^*)$$

Then (16) turns into:

$$\pi y = \varepsilon z + (1 + \eta + \psi)(w - a + \tau - p_f - e) \qquad (16^*)$$

Substituting (5^*) into (16^*) it gives us (16) back. Assuming that PPP holds it leads us $y = x/\pi$ once again.

Lastly, still considering the second case where PPP must hold so we can reach the Thirlwall's Law in the long run, we bid another solution. On rearranging equation (18), we have:

$$[\pi + \lambda(1 + \eta + \psi)]y = \varepsilon z + (1 + \eta + \psi)(w - a + \tau - p_f - e) \quad (18^*)$$

Adding $-(1 + \eta + \psi)\lambda y$ in both sides of (18^*) and substituting equations (5) and (6), it takes us back to the classical equation (16). Afterward, assuming that PPP holds, we finally find the Thirlwall's Law again. This solution may be considered more general if compared to the other ones aforementioned, for, through it, we can get to the Thirlwall's Law using an

inner mechanism of the model without any special constraint on the parameters, as demonstrated.

Provided those remarks, we must say Dixon & Thirlwall (1979) had a very good attempt of reconciling the EL and the BPC growth models. However, if we want to analyse the impact of RER on growth, domestic price must be a function of nominal exchange rate and foreign price alike. If it is not, the mechanism of the exchange rate pass-through cannot operate in the domestic prices, assuming a constant rate of change of the money wage, and hence the PPP will not be achieved in the long run.

II. 4 The Export-Led and Balance-of-Payment Constrained Growth Model Revisited

Let us assume that there are only a large foreign country and a small home country which faces a balance-of-payments constraint and that both produces and exports only one sort of consumption good with imperfect substitution between the foreign and the home exports. We assume as well that the home country imports intermediate inputs from the foreign country. In practical terms it means we will split the imports in two different categories, namely, consumption good imports (M^C) and imported intermediate inputs (M^K). By doing so, we have now an extended trade account:

$$P_d X = E\left(P_f M^c + P_f^k M^k\right) \qquad (19)$$

where P_f remains as the consumption good imports price index and P_f^k is the imported inputs price index. In rate of change:

$$p_d + x = e + \theta_c\left(p_f + m^c\right) + \theta_k\left(p_f^k + m^k\right) \qquad (20)$$

where $\theta_c = EP_f M^c / P_d X$ and $\theta_k = EP_f^k M^k / P_d X$.

The import demand functions are:

$$M^c = j\left(\frac{EP_f}{P_d}\right)^\psi Y^{\pi_c} \qquad (21)$$

$$M^k = \mu Y \qquad (22)$$

where j and μ are constants. In rates of change:

$$m^c = \psi\left(e + p_f - p_d\right) + \pi_c y \qquad (23)$$

$$m^k = \pi_k y \tag{24}$$

where $\pi_k = 1$ since the ratio Y/M^k is constant in equation (22) and consequently M^k does not depend on the real exchange rate.

Now we will try to extend the BPC growth model by making domestic prices a function of exchange rate and foreign prices. To do so, at first we must disaggregate the costs within the domestic price in two parts:

$$P_d = T(P^E + P^W) \tag{25}$$

where P^E is the unit cost of imported intermediate inputs and P^W is the unit labour cost.

Here we assume that P^E is determined by foreign market where PPP holds. Since the home country produces only one sort of good, one possibility to extend the model is to allow imported intermediate inputs into the production process, as follows:

$$P^E = P_f E \mu \tag{26}$$

Where, once again, $\mu = M/Y$ is constant over time.

As for P^W, we have:

$$P^W = W/R \tag{27}$$

Putting (26) and (27) into (25), we have:

$$P_d = T\left(\frac{W}{R} + P_f E \mu\right) \tag{28}$$

In rate of change:

$$p_d = \tau + \varphi(w - r) + (1 - \varphi)(p_f + e) \tag{29}$$

where $\varphi = P^W/(P^E + P^W)$. If growth rate of mark-up tends to zero due to the international competition and if growth rates of labour productivity and real wage equalize in the long run it is easy to prove that PPP holds. Making $w - r = p_d$ and $\tau = 0$ and substituting into (29) we have $p_d = p_f + e$. Moreover, we must notice that in the long run φ remains constant, than PPP holds (see appendix I).

Substituting then (29), (24), (23), (15) and (6) into (20) and, for the sake of the exposition, assuming $p_f = p_f^k$ and $\pi = \theta_{ct}(\pi_c) + \theta_{kt}(1)$, we reach the revisited model:

$$y = \frac{\varepsilon z + (1 + \eta + \theta_c \psi)\big[\tau + \varphi(w - a - p_f - e)\big]}{\pi + \varphi\lambda(1 + \eta + \theta_c \psi)} \tag{30}$$

Now we must make some remarks on this specification. At first, comparing with equation (18), it is theoretically possible to add one more way of suppressing the long run effects of real exchange rate to the two former ones aforementioned by making $\varphi = 0$, given $\tau = 0$. In this case it is easy to see that the equation (30) turns into the Thirlwall's Law. The problem in this case is that $\varphi = 0$ means, in practical terms, to rule out the unit labour cost from the costs of production and hence from the price index as well, provided that this parameter stands for the weight of labour over the overall cost. So henceforth we will assume $\varphi_t \in (0,1)$.

Accordingly, in order to have Thirlwall's Law we must rearrange equation (30):

$$[\pi + \varphi\lambda(1 + \eta + \theta_c \psi)]y$$
$$= \varepsilon z + (1 + \eta + \theta_c \psi)\big[\tau + \varphi(w - a - p_f - e)\big] \tag{30*}$$

Subtracting $\varphi_t \lambda(1 + \eta + \theta_{ct}\psi)y_t$ from both sides of equation (30*) and considering $\tau_t = 0$, $p^W = w - r$ and $p^E = p_f + e$ in the long run (see appendix I), we obtain:

$$\pi y = \varepsilon z + (1 + \eta + \theta_c \psi)[\bar{\varphi}(p^W - p^E)] \tag{30**}$$

As demonstrated in appendix I, if φ is constant over time, then $p_t^W = p_t^E$ and, consequently, (30**) yields us the Thirlwall's Law. For alternative proofs on how to obtain the Thirlwall's Law from the revisited model, please see appendix II.

Now we finish the exposition of the ELBPCR growth model. As we have seen, this model allows us to reconciliate both EL and BPC growth models in a further general mathematical specification than the Thirlwall & Dixon's (1979). The following section dedicates to the developments concerning the role played by the RER in the ELBPCR model.

III   THE IMPACT OF AN UNDERVALUATION ON SHORT-TERM GROWTH, ACCORDING TO THE REVISITED MODEL

Let us now present the effect of an undervaluation on short-term growth according to the BPC and ELBPC growth model. In this section we will assume the current growth rate is balance-of-payments constrained over time. Since the current growth rate might differ from

the balance-of-payments constrained one (and usually it does) in the short-term, here we will make such an assumption for the sake of the exposition once we are interested actually in describing how the balance-of-payments constrained growth rate evolves over time. Therefore, as for the BPC model, we obtain such an effect by differentiating the equation (16) with respect to the exchange rate as follows:

$$dy = -\frac{(1 + \eta + \psi)}{\pi} de \qquad (31)$$

As for the ELBPC model, we must differentiate equation (18) with respect to the exchange rate:

$$dy = -\frac{(1 + \eta + \psi)}{\pi + \lambda(1 + \eta + \psi)} de \qquad (32)$$

As we can see, given the Marshall-Lerner condition and additionally assuming $\pi > |\lambda(1 + \eta + \psi)|$ in equation (32), according to the BPC and ELBPC growth models a depreciation yields a positive and constant, albeit different from each other, impact on the growth rate.

As for the ELBPCR model developed here, knowing the effect of an undervaluation on short-term growth becomes a quite complex task. If we want to account for the overall impact of an undervaluation on growth according to the equation (30) we must consider, not only the direct effects of the exchange rate but also its indirect effects via φ and θ_c. Since $\varphi = P^W/(P^E + P^W)$, we can say that a currency depreciation reduces φ. As for the impact of an undervaluation on θ_c, since $\theta_c = EP_f M^c / P_d X$, the net result might be ambiguous. Here we will assume without loss of generality (WLG) the increase in E caused by an undervaluation exceeds the expected decrease in $(P_f M^c / P_d X)$, thereby raising θ_c. Therefore, in order to analyse the impact of an undervaluation on growth we must take into account the partial effect, not only of e, but also of φ and θ_c on short-term growth. In other words, we must take the total differential of equation (30) and consider the remaining variables growing at constant rates as follows:

$$dy = \frac{\partial y}{\partial \theta_c} d\theta_c + \frac{\partial y}{\partial \varphi} d\varphi + \frac{\partial y}{\partial e} de \qquad (33)$$

Let us analyse equation (33). Firstly, as aforementioned, e and φ are inversely related whereas e and θ_c are positively related, which means if $de > 0$ WLG, then $d\varphi < 0$ and

$d\theta_c > 0$. Secondly, we must analyse the sign of the partial derivatives. Let us start with $\partial y / \partial e$:

$$\frac{\partial y}{\partial e} = -\frac{\varphi(1 + \eta + \theta_c\psi)}{\pi + \varphi\lambda(1 + \eta + \theta_c\psi)} > 0 \qquad (34)$$

assuming $\pi > |\varphi\lambda(1 + \eta + \theta_c\psi)|$, we have $\partial y / \partial e > 0$. Now let us check the sign of $\partial y / \partial \varphi$. Since we are interested in knowing if such a partial derivative is either positive or negative we will only consider its numerator for the sake of the formal exposition. Whether we assume $\partial y / \partial \varphi > 0$ WLG, and then we add $-(1 + \eta + \theta_c\psi)[\pi + \varphi\lambda(1 + \eta + \theta_c\psi)]\lambda y$ on both sides of the inequality, we come to the following condition:

$$(p^E - p^W) > 0 \qquad (35)$$

that is, we have $p^E > p^W \Leftrightarrow \partial y / \partial \varphi > 0$ (see appendix III). Such a condition tells us that if in the initial moment the unit cost of the imported intermediate inputs is growing at a higher rate than the unit labour cost, then the partial effect of a reduction in φ will generate a reduction in y. Thus, if the condition (35) is fulfilled, then the partial effect of a depreciation on growth, via φ, can be described as follows: if $p^E > p^W$ holds (and consequently $\partial y / \partial \varphi > 0$), then a currency depreciation reduces φ ($d\varphi < 0$), yielding a negative partial effect on growth. The last partial derivative to be analyse is $\partial y / \partial \theta_c$. Once again, in order to know if such a partial derivative is either positive or negative we will only consider its numerator. By assuming $\partial y / \partial \theta_c > 0$ WLG and adding $-\psi\varphi[\pi + \varphi\lambda\alpha]\lambda y$ on both sides of the inequality as in the appendix IV, we have the following initial condition, given $= 0$:

$$(p^E - p^W) < 0 \qquad (36)$$

then we have $p^E < p^W \Leftrightarrow \partial y / \partial \theta_c > 0$ (see appendix IV). Therefore, assuming the initial growth rate equals the Thirlwall's Law growth rate, if right after an undervaluation we have $p^E > p^W$ the condition (36) is not fulfilled and $\partial y / \partial \theta_c < 0$. This means that after a depreciation we will find $d\theta_c > 0$ and $\partial y / \partial \theta_c < 0$, yielding a negative partial effect on growth. However, we reckon the partial effect of an undervaluation on θ_c, that is, $(\partial y / \partial \theta_c)d\theta_c$ can be neglected. Since $\theta_c = EP_f M^c / P_d X$, even though we are assuming the increase in E caused by an undervaluation exceeds the expected decrease in $(P_f M^c / P_d X)$, we must bear in mind that after an undervaluation E and $(P_f M^c / P_d X)$ move in opposite directions in the short term, yielding a small variation in θ_c, that is, $d\theta_c \cong 0$. As for the partial

differential ($\partial y / \partial \theta_c$), since this term is multiplied by a rather small number as in the numerator as in the denominator, we can also argue such a partial differential is neglectable.

Therefore, from a stage of initial equilibrium, wherein both PPP and the Thirlwall's Law hold, given an undervaluation, the equation (33) has on its right-hand side a positive and two negative terms.

$$dy = \underbrace{\underbrace{\frac{\partial y}{\partial \theta_c}}_{-} \underbrace{d\theta_c}_{+}}_{-} + \underbrace{\underbrace{\frac{\partial y}{\partial \varphi}}_{+} \underbrace{d\varphi}_{-}}_{-} + \underbrace{\underbrace{\frac{\partial y}{\partial e}}_{+} \underbrace{de}_{+}}_{+} \qquad (33^*)$$

Analysing equation (33^*) and regarding ($\partial y / \partial \theta_c$)$d\theta_c$ is neglectable, in economic terms, what happens is the depreciation has basically two effects. On the one hand, it raises the foreign demand for domestic goods, hence boosting exports. On the other hand, an undervaluation increases the unit cost of imported intermediate inputs, thus harming the price-competitiveness of the domestic goods. The main difference between the revisited model and the previous ones is that in the ELBPCR model the impact of a depreciation on growth is not constant anymore over time. Moreover, the analysis of the role played by the RER on these growth models becomes much more complex in the revisited one due to the existing non-linearity in its equations constituting the partial differentials in (33). In the next section we will try to show through computational simulation the effects of an undervaluation on short-term growth.

IV   REAL EXCHANGE RATE AND GROWTH IN THE REVISITED MODEL: A COMPUTATIONAL SIMULATIONS ANALYSIS

As shown in the previous section, the impact of a depreciation on short-term growth according to the revisited model, unlike the BPC and ELBPC models, is non-linear throughout time, hence increasing meaningfully the complexity of any analysis concerning the utilization of the exchange rate as a macroeconomic policy tool. Therefore, we must run a series of simulations in order to understand how the dynamics between exchange rate and growth goes over time. To do so, we arbitrarily set some fixed values for parameters and rates of change of certain variables regarded as exogenous as well as some initial values for endogenous variables in level, according to the specification of the models presented in this work. The chosen parameters are the following ones:

TABLE I

Parameters				
ε	π	η	ψ	λ
1.1	3.6	-0.65	-0.8	0.5

The rates of change of the exogenous variables are:

TABLE II

Exogenous variables – rates of change					
z	w	p_f	r	p_d	$\tau = a$
5	8	2.5	1.5	5	0

The initial values of the variables in level are:

TABLE III

| Variáveis em nível – valores iniciais				
W_0	$R_0 = AY_0^{\lambda}$	P_{f0}	E_0	φ_0
2	1	1	1	0.6667

where $A = 1$ and $Y_0 = 1$.

Let us analyse now the impact of a depreciation on short-term growth according to the revisited model. In order to do so, we will assume the initial rate of change of the nominal exchange rate is 4.8% each period of time (weekly, monthly, quarterly, yearly, etc.) and from a given moment onwards the monetary authority of the home country decides to raise the rate of change of the nominal exchange rate from 4.8% to 15% each period in order to boost growth. Figure I below shows the simulation according to the revisited model over 50 periods.

In the first two periods of the simulation PPP holds and the current growth rate equals the Thirlwall's Law growth rate. In the third period we introduce an exogenous shock in the system. We increase the rate of change of nominal exchange rate from 4.8% up to 15% per period. According to the BPC and ELBPC canonical models, a permanent increase in the nominal exchange rate yields a permanent increase in the growth rate, as thoroughly shown in the partial differentials (31) and (32). With respect to the ELBPCR model developed here, a permanent increase in the nominal exchange rate leads to a higher RER in the short-run.

Nonetheless, due to its exchange rate pass-through mechanism, changes in the nominal exchange rate inexorably find their way to domestic prices therefore correcting any possible RER misalignments over time. As for the ELBPCR growth rate, according to the simulation pictured in Figure I, we can see that after the sharp increase due to the undervaluation the ELBPCR growth rate converges asymptotically towards the Thirlwall's Law. It happens because the effect of a depreciation is twofold, assuming the partial effect $(\partial y/\partial\theta_c)d\theta_c$ is neglectable. On the one hand, a depreciation improves domestic goods price-competition by boosting foreign demand for domestic goods, thereby relaxing the home country external constraint. On the other hand, it makes imported intermediate inputs more costly, hence harming the domestic goods price-competition. Accordingly, right after an undervaluation, the home country growth rate will steadily fall until the rise in the production costs of the domestic goods completely offsets its gains from trade and the ELBPC growth rate is tantamount to the Thirlwall's Law.

Figure I – RER and growth in the revisited model
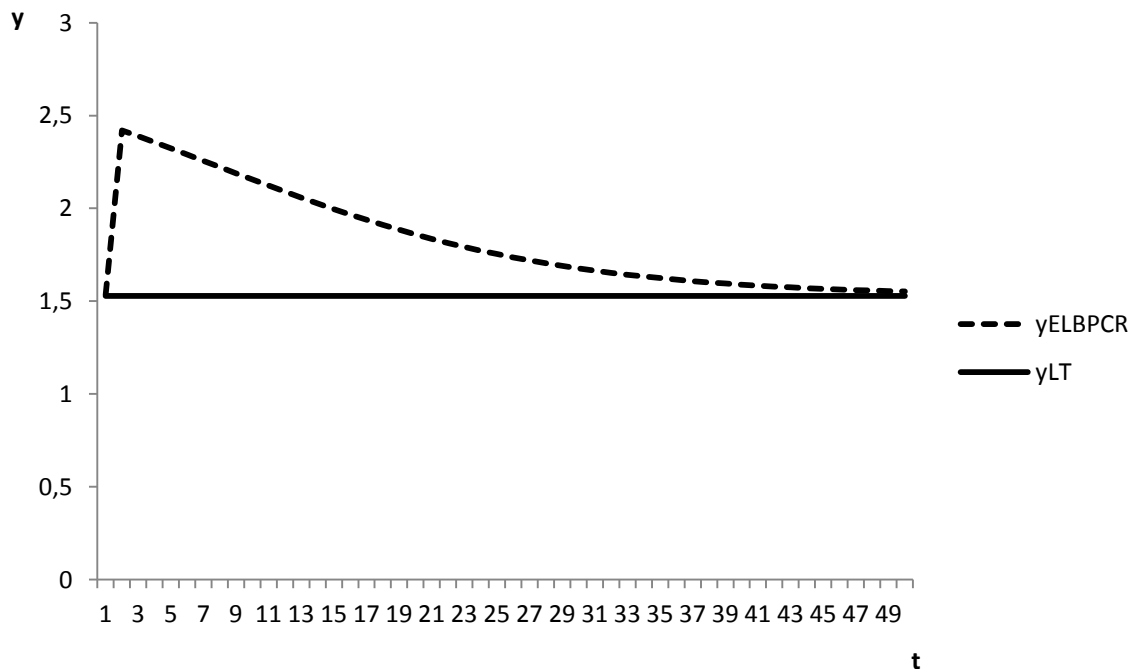
Let us show in Figure II what happens if the monetary authority raises up the rate of change of the money exchange rate even more, being now from 4.8% to 20% per period. If we comparing Figures I and II, the main conclusion is quite straightforward. The higher the undervaluation, the higher will be the peak attained by the growth rate in the first moment and consequently the higher will be the speed of convergence of the growth rate towards the Thrilwall's Law.

Figure II – RER and growth in the revisited model
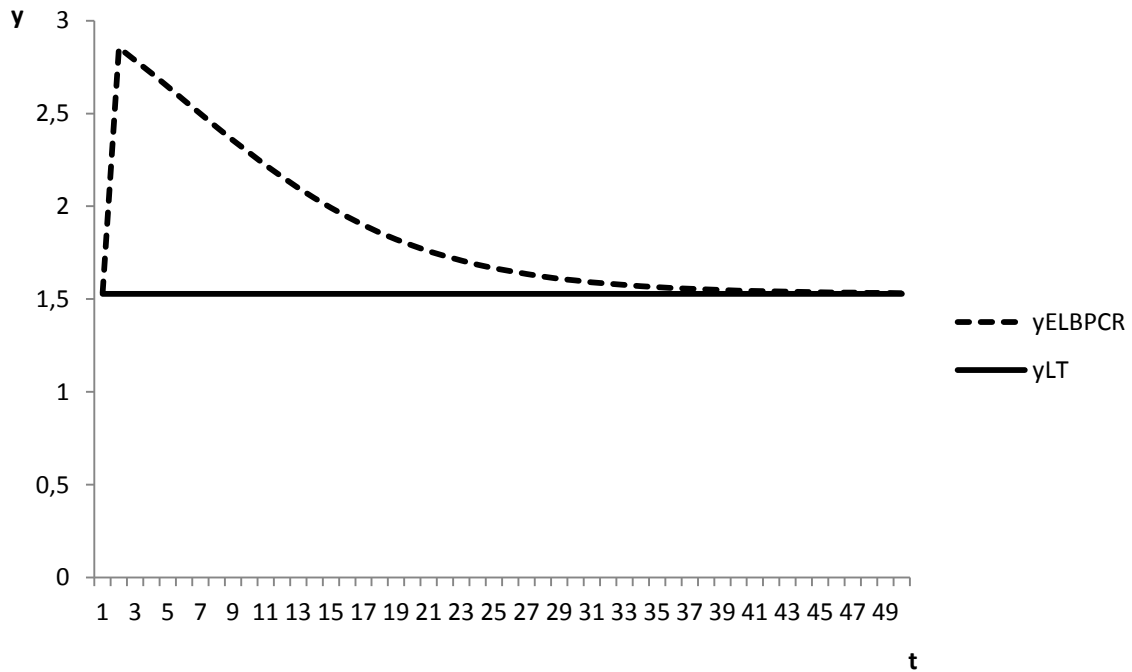
Hitherto, we have only been considering the case wherein the monetary authority raises the rate of change of the nominal exchange rate in a given moment and then depreciate the currency each period at the same constant rate over time. Let us now consider a possible scenario wherein the monetary authority decides to constantly depreciate the currency at increasing rates of change period by period. In our last simulation we came up with the idea of setting up an hypothetical scenario wherein the initial rate of change of the money exchange rate is 0.02% and then the monetary authority raises this rate of change also in 0.02% per period for over than 600 periods. The sequence of the rates of change of the money exchange rate would approximately be: $e = \{t_1 = 0.02, t_2 = 0.04, t_3 = 0.06, \dots, t_{600} = 12, \dots\}$.[1] We will try to show in Figure III that such a scenario can bring important informations for a better understanding of the model and its possibilities.

Figure III pictures a growth rate of the revisited model that is quite peculiar. Initially the growth rate presents a decreasing trajectory. In other words, an undervaluation is harming growth, inspite of the assumption that the Marshall-Lerner conditions holds. This is a striking conclusion. In more generic terms, it means if the initial rate of change of the money exchange rate is small enough an increase in such a rate will yield a lower growth rate, even if the Marshall-Lerner condition holds. In order to get rid of such a plight the monetary authority must keep on depreciating the currency at higher rates period by period until the rate

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[1] Let there is a bijection $e: I_i \to \mathbb{R}^+$, wherein $I_i = \{p \in \mathbb{N}; p \leq i\}$, which associates each natural number i to a real number e_i, called the *i-th* term of the sequence, such that $e_i = ie_0$.

of change of the money exchange rate is large enough. In economic terms it means for sufficiently small rates of change of money exchange rate, an increase in such a rate does not generate sufficiently large gains from trade that could overcompensate the increase in the domestic production cost caused by more expensive imported intermediate inputs, thus harming the domestic goods price-competition and, consequently, short-term growth. Intuitively, we may say that in order to evaluate the impact of a currency depreciation on short-term growth the monetary authority must take into account the distribution between the labour cost and the imported intermediate inputs within the productive structure of the domestic firms. Whether, for instance, the domestic firms present a high imported intermediate inputs share on overall production cost, then a sufficiently small depreciation might have a downside effect on short-term growth.

However, when the rate of change of money exchange rate is large enough the growth rate starts increasing, reach its peak way up high and inevitably converges again at diminishing rates towards Thirlwall's Law in the long term.

Figure III – RER and growth in the revisited model
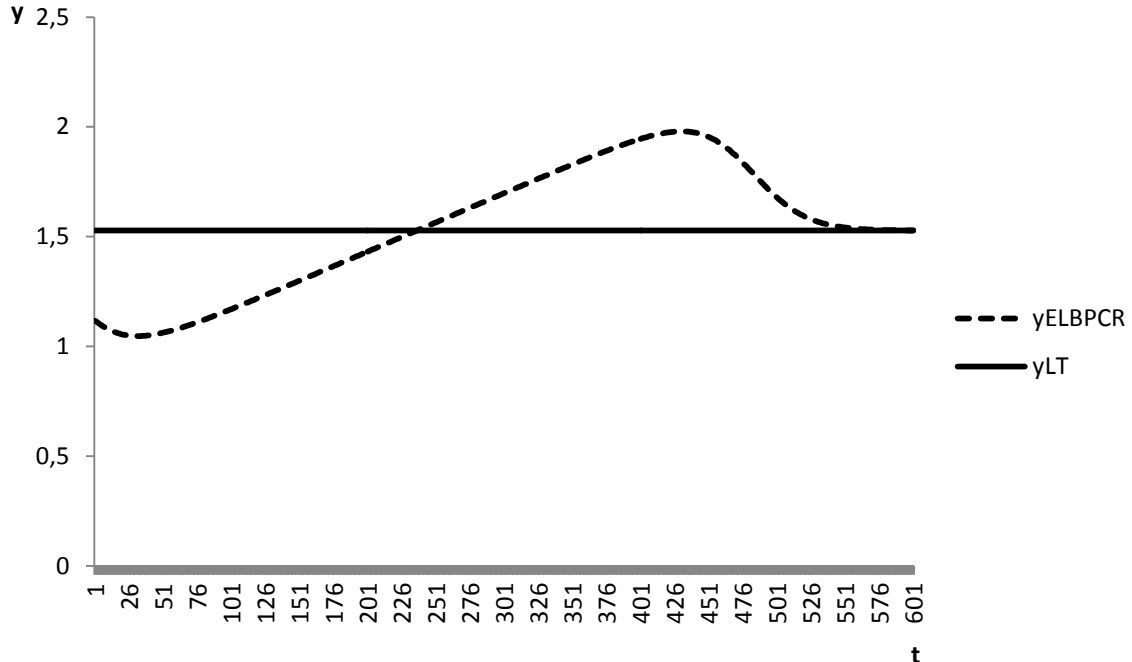
CONCLUDING REMARKS

The present work sought to contribute to the Keynesian-Kaldorian literature on growth and development by providing an analytical framework able to reconciliate both the export-

led and the balance-of-payments constrained growth models into a further general mathematical specification.

This revisited growth model developed here includes an inner mechanism of RER adjustment towards PPP which draws our attention to important issues concerning the short-term impact of an undervaluation on growth, e.g. the initial conditions before the undervaluation and the magnitude of the undervaluation. Such a mechanism also enables us to thoroughly account for the convergence process of the short-term growth rate towards the Thirlwall's Law.

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Appendix I

Proposition: given $\tau = 0$, in the long term $\varphi = \bar{\varphi} \Rightarrow p_d = p_f + e$.

Proof:

$$\varphi = \frac{P^W}{P^E + P^W} \Rightarrow \frac{1}{\varphi} = 1 + \frac{P^E}{P^W}$$

If P^T/P^W is constant, then φ is constant as well. Therefore:

$$\frac{P^E}{P^W} = k \ \text{ or } \ ln\left(\frac{P^T}{P^{NT}}\right) = ln(k)$$

where $k > 0$ is a constant. Taking the differential with respect to time, we have:

$$p^E = p^W$$

Until now it is proven that $\varphi = \bar{\varphi} \Rightarrow p^E = p^W$. The next step is to prove the following:

$$p^E = p^W \Rightarrow p_d = p_f + e$$

Rewriting equation (23) and considering $\tau = 0$ in the long term, we obtain:

$$p_d = \varphi p^W + (1 - \varphi)p^E \qquad (*)$$

wherein $p^W = w - r$ and $p^E = p_f + e$, according to the equations (27) and (26).

This way, considering that $\varphi = \bar{\varphi} \Rightarrow p^E = p^W$ in $(*)$, we have:

$$p_d = p^{NT} = p^T = p_f + e, \text{ which had to be demonstrated.}$$

Appendix II

Proof I:

Provided the long-term assumptions $\varphi = \bar{\varphi}$ and $\tau = 0$, we have $p^W - p^E = 0$, as demonstrated in appendix I. Moreover, considering that in the long-term $p^W = w - r$ and $p^E = p_f + e$, by substituting these terms and equation (6) into $p^W - p^E = 0$, we have:

$$w - a - p_f - e = \lambda y \qquad (\blacksquare)$$

Substituting ($\blacksquare$) into (30) and taking into account that $\varphi = \bar{\varphi}$ and $\tau = 0$, we obtain:

$$y = \frac{\varepsilon z + (1 + \eta + \theta_c \psi)\bar{\varphi}\lambda y}{\pi + \lambda\bar{\varphi}(1 + \eta + \theta_c \psi)}$$

$$[\pi + \lambda\bar{\varphi}(1 + \eta + \theta_c\psi) - \lambda\bar{\varphi}(1 + \eta + \theta_c\psi)]y = \varepsilon z$$

$$y = \frac{\varepsilon z}{\pi}$$

Proof II:

Being $h = w - a - p_f - e$ and taking the total differential from equation (30), we have:

$$\frac{dy}{dt} = \frac{\partial y}{\partial z}\frac{dz}{dt} + \frac{\partial y}{\partial \tau}\frac{d\tau}{dt} + \frac{\partial y}{\partial \varphi}\frac{d\varphi}{dt} + \frac{\partial y}{\partial h}\frac{dh}{dt}$$

As in the long-term $d\tau/dt = d\varphi/dt = 0$, assuming that (■) holds, we have:

$$\dot{y} = \frac{\varepsilon\dot{z}}{\pi + \lambda\bar{\varphi}(1 + \eta + \theta_c\psi)} + \frac{\bar{\varphi}(1 + \eta + \theta_c\psi)\lambda\dot{y}}{\pi + \lambda\bar{\varphi}(1 + \eta + \theta_c\psi)}$$

wherein $\dot{h} = \lambda\dot{y}$. Rearranging the terms of this equation we have:

$$\dot{y} = \frac{\varepsilon\dot{z}}{\pi}$$

Appendix III

Proposition: $p^E > p^W \Leftrightarrow \partial y/\partial\varphi > 0$.

Proof:

Firstly, let us prove: $\partial y/\partial\varphi > 0 \Rightarrow p^E > p^W$.

$$\frac{\partial y}{\partial\varphi} = \frac{\alpha[\pi + \varphi\lambda\alpha]h - \lambda\alpha[\varepsilon z + \alpha\,(\tau + \varphi h)]}{[\pi + \varphi\lambda\alpha]^2} > 0$$

where $(1 + \eta + \theta_c\psi) = \alpha$ and $(w - a - p_f - e) = h$. We are interested in analyse the following condition: $\partial y/\partial\varphi > 0$. Thus, considering only the numerator of the differential above and adding $-\alpha[\pi + \varphi\lambda\alpha]\lambda y$ to both sides of the inequality and rearranging the terms, we have:

$$\alpha[\pi + \varphi\lambda\alpha](h - \lambda y) > \lambda\alpha[\varepsilon z + \alpha\,(\tau + \varphi h) - (\pi + \varphi\lambda\alpha)y]$$

Since $(h - \lambda y) = p^W - p^E$ and assuming $(\pi + \varphi\lambda\alpha) > 0$, we have:

$$p^W - p^E < \lambda(y - y)$$

$(p^E - p^W) > 0$, which had to be demonstrated.

We still have to prove the other way round, that is: $p^E > p^W \Rightarrow \partial y/\partial \varphi > 0$.

Firstly we have to rewrite the equation (30):

$$(\pi + \varphi\lambda\alpha)y = \varepsilon z + \alpha\,(\tau + \varphi h)$$

Adding $-\alpha\varphi\lambda y$ on both sides of the equality, we obtain:

$$y = \frac{\varepsilon z + \alpha\,[\tau + \varphi(p^W - p^E)]}{\pi}$$

Taking the differential:

$$\frac{\partial y}{\partial \varphi} = \frac{\alpha(p^W - p^E)}{\pi}$$

Therefore, if $p^E > p^W$, then $\partial y/\partial \varphi > 0$, which had to be demonstrated.

Appendix IV

Proposition: in the long term, $p^E < p^W \Leftrightarrow \partial y/\partial \theta_c > 0$.

<u>Proof:</u>

Firstly, we must prove: $\partial y/\partial \varphi > 0 \Rightarrow p^E < p^W$.

$$\frac{\partial y}{\partial \theta_c} = \frac{\psi[\tau + \varphi h][\pi + \varphi\lambda\alpha] - \lambda\varphi\psi[\varepsilon z + \alpha\,(\tau + \varphi h)]}{[\pi + \varphi\lambda\alpha]^2} > 0$$

where $(1 + \eta + \theta_c\psi) = \alpha$ and $\left(w - a - p_f - e\right) = h$. We have to analyse if $\partial y/\partial \varphi > 0$. Therefore, once again we will consider only the numerator of the differential above. Afterwards we will add $-\psi\varphi[\pi + \varphi\lambda\alpha]\lambda y$ to both sides of the inequality and rearrange the terms:

$$\psi[\pi + \varphi\lambda\alpha][\tau + \varphi(h - \lambda y)] > \psi\varphi\lambda[\varepsilon z + \alpha\,(\tau + \varphi h) - (\pi + \varphi\lambda\alpha)y]$$

Since $(h - \lambda y) = p^W - p^E$ and assuming $(\pi + \varphi\lambda\alpha) > 0$, we have:

$$\tau + \varphi(p^W - p^E) > \varphi\lambda(y - y)$$

$$\varphi(p^E - p^W) < \tau$$

Since $\varphi \in (0,1)$ and $\tau \to 0$ in the long term, then:

$(p^E - p^W) < 0$, which had to be demonstrated.

We still have to prove the other way round, that is: $p^E < p^W \Rightarrow \partial y/\partial \theta_c > 0$.

Rewriting the equation (30):

$$\big(\pi + \varphi\lambda(1 + \eta + \theta_c\psi)\big)y = \varepsilon z + (1 + \eta + \theta_c\psi)\,(\tau + \varphi h)$$

Adding $-(1 + \eta + \theta_c\psi)\varphi\lambda y$ on both sides of the equality, we obtain:

$$y = \frac{\varepsilon z + (1 + \eta + \theta_c\psi)\,[\tau + \varphi(p^W - p^E)]}{\pi}$$

Taking the differential:

$$\frac{\partial y}{\partial \theta_c} = \frac{\psi\varphi\,(p^W - p^E)}{\pi}$$

Therefore, if $p^E < p^W$, then $\partial y/\partial\theta_c > 0$, which had to be demonstrated.