Induced Innovation, Endogenous Growth, and Income Distribution: a Model along Classical Lines*

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Abstract

This paper presents a classical micro-founded growth model with endogenous direction and size of technical change. In a standard induced innovation model firms freely adopt productivity improvements from an innovation possibilities frontier describing the trade-off between increasing capital or labor productivity. The shape of the innovation possibility frontier uniquely determines the steady state distribution of income. The model proposed allows firms to choose not only the direction but also the size of innovation by making innovation a costly activity requiring R&D investment. Comparative dynamics analysis shows that income distribution is sensitive to saving parameters and fiscal policy. In particular, an increase in the discount factor or in subsidy to R&D raises the labor share.

Keywords: Induced innovation, endogenous growth, direction of technical change, classical growth

JEL Classification: D33, O31, O33, O40.

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1 Introduction

Most modern growth theories focus on balanced growth - that is, paths along which the endogenous variables of an economy grow at constant, though not necessarily equal, rates, and where factors shares, the interest rate and the output-capital ratio are constant. Focusing on balanced (or steady state) growth is not only analytically convenient, but also empirically plausible. In fact, it is broadly consistent with the main stylized facts regarding growth and the distribution of income in industrialized economies, as first emphasized by Kaldor (1961) and later confirmed by others (see, for example, Romer 1989). Since Uzawa's (1961) seminal paper on balanced growth and innovation, it is universally accepted that steady state growth requires Harrod neutral technical change, which is, accordingly, often assumed.

Despite the popularity of the assumption, relatively little effort has been made to investigate the economic mechanism underlying the fact that technical change, in the long run, tends to raise labor productivity while leaving the output-capital ratio (or capital productivity) roughly constant. An antiquated, long submerged, but recently revived tradition has, however, addressed this issue. The idea that the direction along which technical change saves input requirements may be endogenous to the economic system traces back to at least Hicks (1932, pp. 124-5), who suggested that technical change would save factors becoming relatively more expensive. Fellner (1961), Kennedy (1964), Samuelson (1965), von Weizsacker (1966) and Drandrakis and Phelps (1966) formally developed this intuition, thereby forming the basis for the emerging theory of 'induced innovation' in the 1960s. In particular, Kennedy (1964) postulated that firms maximize the rate of cost reduction subject to an innovation possibility frontier, which represents the trade-off between growth rates of labor and capital productivity available to the firm. By so doing, he connected the choice of direction of technical change to factors shares: technical change resulted to be biased toward the factor of production whose share was increasing. In this scenario, unlike previous analyses on technical change, Harrod-neutrality emerged as a long run result from firms' maximizing behavior. The other proponents of the induced innovation literature obtained analogous results. Save for a few contributions (see Sha and Desai 1981, Skott 1981 and van der Ploeg 1987), however, the induced innovation tradition became largely dormant until its recent revival within different approaches to economic growth. Papers working within the neoclassical framework, both exogenous (Funk 2002) and endogenous (Acemoglu 2002, 2003, 2007), and within the classical-Marxian tradition (Dumenil-Levy, 2003; Foley, 2003; and Julius, 2005) follow the induced innovation hypothesis rather than restricting technology to purely labor augmenting technical change.

In addition to improving the generality of the analysis, allowing for the choice of direction of technical change has important consequences as to the determinants of steady state distribution of income. In fact, under the induced innovation hypothesis, factors shares are uniquely determined by the innovation technology, independently of saving parameters. As a consequence, income

distribution is also unaffected by policy action.

This paper presents a classical micro-founded growth model with endogenous direction and size of technical change. The productive structure of the economy is based on the endogenous growth models with perfect competition developed by Hellwig and Irmen (2001) and Irmen (2005). While in their analysis technical change merely improves labor productivity, I allow technical progress to augment both labor and capital productivity according to the induced innovation hypothesis. Contrary to the original formulation by Kennedy (1964) where the innovation possibility frontier is exogenous, and following Kamien and Schwartz (1969), I assume that the position of the frontier depends on the amounts of resources spent on R&D: direction and intensity of technical change are simultaneously determined as the profit maximizing choice of the firm. At the macroeconomic level, the model assumes a classical process of capital accumulation with exogenous labor supply as defined in Foley and Michl (1999), and a Goodwin's (1967) labor market where real wage growth responds positively to the employment ratio.

Comparative dynamics analysis shows that steady state productivity growth, income shares and employment ratio are sensitive to saving parameters and fiscal policy. In particular, an increase in the discount factor and a subsidy to R&D raise the labor share, productivity growth and employment. These results, as we discuss below in more detail, conflict with those obtained by Acemoglu (2003) in his integration of endogenous growth and induced innovation hypothesis.

The rest of the paper is organized as follows. Section 2 presents the model; Section 3 provides the intuition of the main result, and discusses the assumptions and the relation to the literature of the paper; Section 4 concludes.

2 The model

2.1 Production

We consider a one sector growth model where output, the numerarie, can be used both for consumption and investment. Final production is carried out by a fixed population of competitive firms. Production requires labor and productive capacity as inputs. Final good manufactured at t can be used both to increase productive capacity, and to improve technology at $t+1$. Output of firm i at $t+1$ is given by

$$y_{i,t+1} = \min\left\{b_{i,t+1}x_{i,t+1}, a_{i,t+1}n_{i,t+1}\right\}, \tag{1}$$

where x is productive capacity, n is the measure of workers employed and b and a represent, respectively, firms' capital and labor productivity. At time t, firms are endowed with some capacity x_t, and have access to the economy-wide productivity levels A_t and B_t. They invest to increase capacity and to improve technology. Improvements in technology are private knowledge for the period

when they are introduced; they become freely available to all firms after it. Innovation technology is described by

$$\begin{aligned} b_{i,t+1} &= B_t[1 + h(M_{i,t}/y_{i,t})\mu_{i,t}] \\ a_{i,t+1} &= A_t[1 + h(M_{i,t}/y_{i,t})g(\mu_{i,t})]. \end{aligned} \tag{2}$$

This representations of innovation technology combines endogenous growth and induced innovation. The function $g(\mu)$ is the original Kennedy's innovation possibility frontier and it satisfies $g'(.)$, $g''(.) < 0$. It represents the trade-off between augmenting capacity productivity (μ) or labor productivity ($g(\mu)$). Contrary to its original formulation, the position of the frontier is not exogenous, but it is determined by the firm's amount of resources invested to improve technology, i.e. R&D investment (M). The function $h(.)$ determines the improvement in productivity along a given direction of technical change and satisfies the Inada regularity conditions. The innovation possibility frontier, like its original version, is time-invariant - i.e. pursuing technological change along a specific direction does not change the productivity of innovating along that direction[1]. For notational convenience we posit $M/y \equiv m$, the reason for considering m instead of M as the argument of $h(.)$ will be discussed in the next section. Installing productive capacity x_{t+1} requires $K(x_{t+1}, x_t) = x_t\Gamma(x_{t+1}/x_t)$ units of final good at time t. The cost function of capacity satisfies $\Gamma(1) = \Gamma'(1) = 0$, and $\Gamma'(.), \Gamma''(.) > 0 \ \forall x_{t+1}/x_t \neq 0$.

Since production emerges only at period $t+1$, firms have to borrow at time t to pay for the cost of investment in capacity and innovation; they pay the interest rate i_t upon the amount of output borrowed.

At time t, firm i chooses $x_{i,t+1}$, $n_{i,t+1}$, $\mu_{i,t}$, and $M_{i,t}$ to maximize the profit function

$$\Pi_{i,t} = \frac{1}{1+i_t}\left[y_{i,t+1} - w^e_{t+1}n_{i,t+1} - (1+i_t)M_{i,t} - (1+i_t)x_{i,t}\Gamma(x_{i,t+1}/x_{i,t})\right],$$

where w^e is the expected wage rate. Current profit maximizing plans depend on the expected level of the wage rate in the next period as the current choices of capacity and technical change in fact fix the amount of labor firms will decide to hire in the next period. Given the Leontief technology, profit maximization implies $n_{i,t+1} = b_{i,t+1}x_{i,t+1}/a_{i,t+1}$. The first order condition with respect to x_{t+1}, μ_t, and M_t are

$$B_t\left(1 + h(m_{i,t})\mu_{i,t}\right)\left(1 - \frac{w^e_{t+1}}{A_t[1 + h(m_{i,t})g(\mu_{i,t})]}\right) = (1+i_t)\Gamma'(x_{i,t+1}/x_{i,t}) \tag{3}$$

$$\frac{w^e_{t+1}\left[\left(1 + h(m_{i,t})g(\mu_{i,t})\right) - g'(\mu_{i,t})(1 + h(m_{i,t})\mu_{i,t})\right]}{A_t[1 + h(m_{i,t})g(\mu_{i,t})]^2} = 1 \tag{4}$$

[1] The absence of path-dependence in the productivity of innovation technology has been first criticized by Nordhaus (1973). Skott (1981) and more recently Acemoglu (2003) develop induced innovation growth models with history-dependent innovation technology.

$$\frac{x_{i,t+1}}{x_{i,t}} h'(m_{i,t}) \left[\mu_{i,t} - \frac{w^e_{t+1}}{A_t} \frac{\mu_{i,t} - g(\mu_{i,t})}{[1 + h(m_{i,t}) g(\mu_{i,t})]^2} \right] = (1 + i_t). \tag{5}$$

Notice that $w_t/A_t \equiv \omega_t$ is labor income share and, accordingly, $w^e_{t+1}/A_{t+1} \equiv \omega^e_{t+1}$ is the labor share expected at $t+1$ [2]. ω^e_{t+1} becomes the key variable in choosing innovation direction. Inspection of condition (4) allows us to state the following

Lemma 1 *For a given level of R&D investment, an expected increase in the labor share biases the direction of technical change toward labor productivity augmentation.*
Proof. *See Appendix A* ■

The result of the original induced innovation literature is confirmed in our analysis where the position of the frontier is endogenous.

2.2 Society

There are three classes in society. Workers supply labor inelastically and earn the wage rate w, which they fully spend on current consumption. Entrepreneurs organize production and earn firms' profits; profits are consumed at the end of the production period. Capitalist households earn the interest rate i upon their wealth and rank consumption patterns according to the functional

$$\sum_{t=1}^{\infty} \beta^t \ln c_t. \tag{6}$$

Given initial wealth W_0, they choose the sequence $\{c_t, W_t\}_{t=1}^{\infty}$ which maximize (6) subject to the intertemporal budget constraint

$$c_t + W_t \leq (1 + i_{t-1}) W_{t-1}, \tag{7}$$

and to

$$c_t \geq 0, \; W_t \geq 0. \tag{8}$$

Proposition 2 *Given the initial wealth W_0 and the sequence of interest rates $\{i_t\}_{t=0}^{\infty}$, households choose the sequences of consumption and wealth that satisfies*

$$c_t = (1 - \beta)(1 + i_{t-1}) W_{t-1} \tag{9}$$

$$W_t = \beta (1 + i_{t-1}) W_{t-1} \tag{10}$$

[2] It could be argued that the labor productivity level considered to calculate the expectation on tomorrow's labor share should be today's productivity A_t. The reasoning would be that the innovating firm has a one period monopoly over the technology it develops and, at the same time, firms do not take into account the possibility that their own behavior might be adopted by the rest of firms population. Both definitions of ω^e_{t+1} support our following Lemma 1. The distinction is irrelevant for the rest of the analysis.

for $t = 1, 2, ...$

The proof of this proposition is standard. We provide it in Appendix B as the timing of our problem is slightly different from the usual one.

Also note that the households first order condition imply the usual Euler equation for consumption growth:

$$c_{t+1} = \beta(1 + i_t)c_t. \tag{11}$$

2.3 The Aggregate Economy

We assume a representative firm in order to move to the macroeconomic equilibrium. Solving for the equilibrium of the economic system at a certain point in time requires that we specify the equilibrium conditions for the interest rate and the wage rate, together with the way expectations on the wage rate are determined.

The role of the interest rate is to clear the goods market. It will assume the value at which the economy's saving is fully invested either in innovation or in capacity:

$$Y_t = w_t B_t x_t / A_t + (1-\beta)(1+i_{t-1})W_{t-1} + \Pi_{t-1}/(1+i_{t-1}) + M_t + x_t \Gamma(x_{t+1}/x_t). \tag{12}$$

Current production is used either as workers', households' or entrepreneur's consumption or as investment in innovation and capacity.

Since labor demand at a certain point in time is determined by the previous period investment in capacity and technical change there is no guarantee that the labor market will clear. Accordingly: $L_t \geqslant B_t x_t / A_t$, where L_t is labor supply at time t. Let us assume that labor market tightness influences the expectation on next period wage rate, and let such tightness be measured by the employment rate $v_t \equiv n_t / L_t = B_t x_t / (A_t L_t)$. Then, expectations on the wage rate can be modeled according to

$$w^e_{t+1} = f(w_t, v_t) \tag{13}$$

with $f_{w_t}, f_{v_t} > 0$.

Finally, we impose that in equilibrium expectations are correct

$$w^e_{t+1} = w_{t+1}. \tag{14}$$

For any initial condition $\{A_0, B_0, x_0, W_0\}$ we have eight conditions (3, 4, 5, 9, 10, 12, 13, 14) to determine a sequence of temporary equilibrium values for eight endogenous variables $(x_{t+1}, \mu_t, M_t, c_t, W_t, i_t, w_{t+1}, w^e_{t+1})$.

Notice the different nature of the prices of labor and capital. The interest rate adjusts instantaneously to clear the market of loanable funds, while the wage rate is determined by its past history and by the disequilibrium on the labor market in the previous period.

2.3.1 Steady State

Let us start by defining γ_j as the steady state growth factor of variable j. In a steady state, capital productivity, factors income shares and the interest rate need be constant. This implies $\mu = 0$, $\gamma_w = \gamma_A$, and, using (2), $\gamma_A = 1+h(m)g(0)$. Morevoer, from the production function we obtain $\gamma_y = \gamma_x = \gamma_A$; and, from the constraint on output uses, (9) and (10), we have $\gamma_y = \gamma_M = \gamma_W = \gamma_c = \beta(1+i)$.

The first order conditions for the firm can be rewritten as

$$B(1-\omega) = (1+i)\Gamma'(1+h(m)g(0)) \tag{3 bis}$$

$$\omega = \frac{1+h(m)g(0)}{1+h(m)g(0)-g'(0)} \tag{4 bis}$$

$$h'(m)\omega g(0) = 1+i. \tag{5 bis}$$

Let us now derive the steady state equilibrium condition on the goods market, normalized by output. First, notice that households' wealth at $t-1$ was completly lent to entrepreneur for investment, so that $W_{t-1} = M_{t-1} + K(x_t, x_{t-1})$; next, use the fact that in steady state both investment uses grow at the constant factor $\beta(1+i)$. (12) becomes $Y_t = \omega B x_t + (1+i)(M_{t-1} + x_{t-1}\Gamma(x_t/x_{t-1})) + \Pi_{t-1}/(1+i)$. Finally, divide both sides by Bx_t and rearrange, to find

$$1 = \omega + \frac{[m + \Gamma(1+h(m)g(0))/B]}{\beta} + \frac{\pi}{\beta(1+i)^2}, \tag{12 bis}$$

where π is the steady state profit share.

The four bis conditions plus the Euler equation for consumption determine the steady state values for B, ω, i, m, π. The steady state employment ratio can then be derived by defining the function $F(v_t) \equiv f(w_t, v_t)/w_t = w_{t+1}/w_t$, and by setting

$$F(v) = \gamma_A = 1 + h(m)g(0). \tag{15}$$

Since $g'(0) < 0$, inspection of (4 bis) shows that the steady state value of the labor share is economically meaningful, being positive and smaller than one. Moreover, since it is a positive function of m ($d\omega/dm = -g'(0)h'(m)g(0)/(1+h(m)g(0)-g'(0))^2 > 0$), any factor affecting the output share of R&D investment will change the steady state distribution of income.

2.3.2 Comparative dynamics and policy

We now aim at addressing the effects of changes in preferences and policy instruments on steady state distribution, growth and employment.

Let us start with the role of discounting. From the Euler equation and $\gamma_y = \gamma_A$ we obtain the interest rate as a function of the R&D investment m: $(1+i) = (1+h(m)g(0))/\beta$. By substituting this value and $\omega = (1+h(m)g(0))/(1+h(m)g(0)-g'(0))$ into (5 bis) and by rearranging, we find

$$\frac{h'(m)g(0)}{1+h(m)g(0)-g'(0)} = \frac{1}{\beta}. \tag{16}$$

We are now in a position to state

Proposition 3 *the steady state productivity growth, the labor share and the employment ratio are higher, the higher is β.*

Proof. See Appendix C. ■

A higher β means that households are less impatient with respect to consumption. They will be willing to lend more, thus reducing the interest rate and raising the incentive to invest. The increase in R&D investment raises permantly productivity growth, which, in turn, feeds into the labor share and the employment rate.

Turning now to fiscal policy, let α be the tax/subsidy rate on investment in R&D. α will represent a tax if $\alpha \leq 0$, and a subsidy in case $\alpha \geq 0$. Subsidies on investment are financed through lump sum taxes on capitalist households; analogously, in case of a positive tax rate, taxes on investment are transferred to capitalists. The fiscal budget remains unaffected. The model (3-5 bis, 12 bis) becomes

$$B(1-\omega) = (1+i)\Gamma'(1+h(m)g(0)) \tag{3 ter}$$

$$\omega = \frac{1+h(m)g(0)}{1+h(m)g(0)-g'(0)} \tag{4 ter}$$

$$h'(m)\omega g(0) = (1-\alpha)(1+i) \tag{5 ter}$$

$$1 = \omega + \frac{1}{\beta}\left[m + \Gamma(1+h(m)g(0))/B\right] + \frac{\pi}{\beta(1+i)^2}. \tag{12 ter}$$

Notice that equation (12 ter) is not directly affected by the policy action: a change in the cost of R&D investment (αM) cancels out with the change in households wealth, which would be either consumed or invested.

Substituting for ω from (4 ter) and for i from the Euler equation into (5 ter), we obtain

$$\frac{h'(m)g(0)}{1+h(m)g(0)-g'(0)} = \frac{(1-\alpha)}{\beta},$$

whose inspection provides

Proposition 4 *the steady state productivity growth, the labor share and the employment ratio are higher, the higher (the lower) the subsidy (the tax) to R&D investment.*

We omit the proof as it is analogous to proof of Proposition 3, when differentiating m with respect to α rather than to β.

The subsidy to R&D lowers the marginal cost of innovation, thus raising the steady state value of m.

3 Discussion

3.1 Intuition of the main result

The endogeneity of steady state income distribution is the main implication of the paper. We shall try to provide an intuition for this result by comparing it to the original 1960s version of the induced innovation hypothesis. In the original version (see for example Drandrakis and Phelps 1966, p.829), firms' optimal direction of technical change is obtained at the tangency point between the (exogenous) innovation possibility frontier and an isocost-reduction line depending on factors' shares. In our notation, μ_t satisfies

$$g'(\mu_t) = -\frac{1-\omega_t}{\omega_t},$$

which shows that the slope of the frontier equals the ratio of unit factors cost. Since steady state requires Harrod-neutrality, the steady state distribtion is uniquely determined by the slope of the innovation possibility frontier at $\mu = 0$: $\omega = 1/(1 - g'(0))$. In our framework, on the contrary, (4) implies

$$g'(\mu_t) = -\frac{1+h(m_t)g(\mu_t)}{1+h(m_t)\mu_t}\frac{1-\omega^e_{t+1}}{\omega^e_{t+1}}.$$

The ratio of unit factors cost need be adjusted for productivity growth in each factors. Unless $g(\mu_t) = \mu_t$, i.e. technical change is Hicks-neutral, the size of R&D investment does affect the 'adjusted' unit factors cost ratio. This is what occurs in steady state where $g'(0) = -\left[1 + h(m)g(0)\right](1-\omega)/\omega$. Income distribution is not fixed by the tangency condition because a change, say an increase, in the labor share can be offset by an increase in labor productivity growth, which reduces the unit labor cost.

3.2 Relation to the literature

This paper is related to different streams of literature. In recent years, some contributions have provided microfoundations of endogenous growth models under perfectly competitive conditions. The endogenous determination of the rate of technical change in models with perfect competition has been traditionally problematic. When factors of productions are paid according to their marginal productivities, and constant returns to scale in labor and capital are assumed, the whole product is just sufficient to pay their remuneration, and nothing is left to reward the cost of introducing an innovation. Hellwig and Irmen (2001), and Irmen (2005) have overcome this impasse by assuming strict convexity in investment cost of capacity: increasing marginal costs of capacity provide the equilibrium inframarginal rents necessary to pay for the cost of innovation. Convex investment cost in capacity and innovation also guarantee that firm's production

plans be bounded. While they assume that technical change merely improves labor productivity, I have introduced the induced innovation hypothesis into their microeconomic framework: technical progress can augment both labor and capital productivity. To the purpose, I have followed Kamien and Schwartz (1969), who modified Kennedy's innovation possibility frontier by making its position dependent on the amount of resources spent on R&D.

At the onset of the induced innovation literature, some contributions investigated the joint determination of direction and rate of technical progress, but none of them developed a full-fledged growth model for a market economy. Nordhaus (1967) studied the case of a benevolent planner who maximizes an intertemporal measure of society's consumption; Kamien and Schwartz (1969) solved the microeconomic problem of a profit maximizing firm; von Weizsäcker (1966) determined the conditions under which the growth rate of consumption is maximized in a two sector market economy. More recently, Funk (2002) produced a micro-founded growth model with induced innovation, perfect competition and convexity of investment cost; however, he assumed an exogenous innovation possibility frontier.

Similarly to Hellwig and Irmen (2001), Bester and Petrakis (2003) and Irmen (2005) I postulate Leontief production function. Contrary to those models, even though at a given point in time substitutability between capital and labor is zero, firms can substitute one factor of production to the other by directing innovation towards either of them. From an empirical standpoint this assumption seems to be, at least qualitatively, reasonable given the evidence that the short-run elasticity of substitution is much lower than the long-run one (see Caballero and Hammour, 1998, pp. 7-8). From a more theoretical point of view, it does not conflict with Jones' (2005) result that a long-run Cobb-Douglas production function can be obtained as the envelope of short-run production functions with much more limited substitutability; nor with Acemoglu's (2003) take that capital and labor are complements (though not strictly) and technical change is labor augmenting[3]. Combining a fixed-coefficient production function with the choice of direction of technical change provides a possible micro-foundation for Kaldor's (1957) suggestion to abolish the distinction between movements along the production function due to capital deepening, and shifts of the production function due to technical change: changes in factors proportion require technical progress[4]. A similar idea has been developed by Foley and Michl (1999) and Michl (1999) through the concept of *fossil production function*. They show that a pattern resembling a smooth neoclassical production function with capital deepening can be obtained as the trace of successive adoptions of labor saving

[3] The 'impossibility theorem', a generally accepted result in production theory due to Diamond and MacFadden (see Diamond, MacFadden and Rodriguez 1978), claims that it is impossible to simultaneously estimate the elasticity of substitution and the bias of technical change. The usual way out of this impasse consists in assuming some restriction on the structure of technical change. This view has been recently challenged by Leon-Ledesma, Adam and Willman (2010).

[4] von Weizsäcker (1966, p. 245) comments: 'Substitution takes time and it can therefore not strictly be distinguished from technical progress (an assumption which also underlies Kaldor's technical progress function)'.

and capital using technical change. My model differs from their approach as firms do not face an exogenous labor-saving capital-using evolution of technology, but they choose both the size and the direction of innovation.

Several papers (see Shah and Desai, 1981; van der Ploeg, 1987; Thompson, 1995; Foley, 2003; and Julius, 2005 among the others) have investigated the implication of integrating a Goodwin-like labor market with the endogenous direction of technical change in growth models. They have shown that the cyclical behavior of the Goodwin model, once coupled with the innovation possibility frontier, collapses to a stable steady state with constant factor shares, capital productivity and growth rate of labor productivity. The steady state is fully determined by the innovation technology. This is not the case in my model as the innovation possibility frontier is endogenous; accordingly, preferences and policy affect the long run outcomes of the economy.

Acemoglu (2003) produced a model of endogenous size and direction of technical change by generalizing a Schumpeterian growth model with horizontal innovation to encompass both labor- and capital- augmenting technical change. He finds that, along a balanced growth path, changes in fiscal policy have no effects on income distribution, whereas changes in the discount factor will only have second order effects. Assumptions on innovation technology explain why his findings conflict with our results. While in Acemoglu the R&D sector employs scientist as inputs, we have assumed that innovation is produced by means of foregone output. Scientists (i.e. labor) are a scarce factor; output, on the contrary is an accumulable factor, which explains the permanent effects of changes in the allocation of resource on growth and distribution.

3.3 Assumptions

In the previous subsection we have discussed the role of convexity in the related literature. Our specification of innovation and capacity investment technology requires some further elaboration. In our setting, cost convexity - that is to say $h''(.) < 0$ and $\Gamma''(.) > 0$ - is necessary to assure that the profit function be strictly concave so that the first order conditions (3-5) are sufficient and necessary for a maximum. As usual in the literature on investment, the assumption of convexity can be justified through the idea of adjustment costs. Moreover, we have assumed zero marginal cost at the origin - $h'(0) = \infty$ and $\Gamma'(1) = 0$ - to assure positive investment and productivity growth. Finally, we have expressed the output of investing either in innovation (b_{t+1}/B_t, a_{t+1}/A_t) or in capacity (x_{t+1}/x_t) as growth factors; as a consequence, in order to keep the unit cost of increasing productivity and capacity constant, and to make balanced growth possible, we have used R&D investment share of output ($m \equiv M/y$) as input in the innovation production function, and we have scaled the cost of capacity to its initial level (x_t).

We have postulated a fixed population of competitive firms. The assumption is equivalent to the existence of a limited entrepreneurial supply. Accordingly, as it is customary in general equilibrium analysis (see McKenzie, 1959), equilibrium extra-profits can be conceived of as rents rewarding the scarce factor

entrepreneurship.

Firms are assumed to maximize instantaneous profits, that is, they adopt a myopic maximization strategy. The assumption is consistent with the entrepreneurs' consumption behavior. It also seems reasonable as regards R&D investment as innovations are private knowledge only for the current period, after which they are freely revealed to all firms. Since the level of technology which will be available after the end of the current period cannot be known in advance by any firms, they will not plan the following periods innovation strategy.

4 Conclusions

At the beginning of our discussion, we have claimed that empirical evidence broadly supports the focus on balanced growth. A relatively recent literature has formally investigated the issue. Noting that balanced growth implies the stability of output, investment and consumption growth rates over time, a series of papers originating with King et al. (1991) considered the existence of a cointegrated vector for investment and consumption ratios as evidence of balanced growth. Except for the US, these analyses (see Serletis and Krichel, 1995; and Harvey et al., 2003) have typically rejected the steady state hypothesis. Further econometric investigation (Clemente et al., 1999; and Attfield and Temple, 2010), however, has vindicated balanced growth analysis by introducing the possibility of structural breaks in the long run relation between investment and consumption ratios. The underlying economic interpretation of this approach is that changes in some of the parameters determining the steady state may move an economy to a different growth path[5]. On the distributional side, it appears that the transition to different steady states in most OECD countries has been associated with a declining labor share (see Checchi and Garcia-Peňalosa 2008, 2010). The induced innovation literature typically predicts that functional distribution should be stable unless changes in innovation technology occur; this paper shows, on the contrary, that the distribution of income can be affected by fiscal policy and saving behavior even within the induced innovation framework.

5 Appendices

5.1 Appendix A

Rewrite (4) as $\omega^e_{t+1}\left[1+h(m_{i,t})g(\mu_{i,t})-g'(\mu_{i,t})(1+h(m_{i,t})\mu_{i,t})\right]=\big(1+h(m_{i,t})g(\mu_{i,t})\big)$. Dropping time and firms indexes, total differentiation with respect to ω^e and μ yields:

[5] For example, Kemper et al. (2011) show that an exogenous reduction in the growth rate of technical change may have caused Germany to move to a lower steady state investment ratio.

$$d\omega^e[1+h(m)g(\mu)-g'(\mu)(1+h(m)\mu)] = d\mu\left[\omega^e\left(g''(\mu)(1+h(m)\mu)\right)+h(m)g'(\mu)\right].$$

Since $g', g'' < 0$, it follows

$$\frac{d\mu}{d\omega^e} < 0 \text{ and } \frac{dg(\mu)}{d\omega^e} > 0 \ .$$

5.2 Appendix B

The problem is to choose $\{c_t \geq 0, W_t \geq 0\}_{t=1}^{\infty}$ to maximize $\sum_{t=1}^{\infty} \beta^t \ln c_t$ subject to $c_t + W_t \leq (1+i_{t-1})W_{t-1}$, $t = 1, 2, ..$ given W_0 and $\{i_t\}_{t=0}^{\infty}$.

The Lagrangian is

$L(\{c_t, W_t, \lambda_t\}_{t=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^t \ln c_t - \sum_{t=1}^{\infty} \lambda_t(c_t + W_t - (1+i_{t-1})W_{t-1}) =$
$= \sum_{t=1}^{\infty} \beta^t \ln c_t - \sum_{t=1}^{\infty} \lambda_t c_t - \sum_{t=1}^{\infty}(\lambda_t - \lambda_{t+1}(1+i_t))W_t - \lambda_1(1+i_0)W_0.$

First order conditions are

$$\frac{\partial L}{\partial c_t} = \frac{\beta^t}{c_t} - \lambda_t \leq 0 \ (= 0 \text{ if } c_t > 0) \tag{17}$$

$$\frac{\partial L}{\partial W_t} = -\lambda_t + \lambda_{t+1}(1+i_t) \leq 0 \ (= 0 \text{ if } W_t > 0) \tag{18}$$

$$\frac{\partial L}{\partial \lambda_t} = -(c_t + W_t - (1+i_{t-1})W_{t-1}) \geq 0 \ (= 0 \text{ if } \lambda_t > 0) \tag{19}$$

for $t = 1, 2, ...$

First off, (17) can be satisfied only if both λ_t and c_t are positive. Indeed, with logarithmic utility, maximizing agents will never have zero consumption at any time. Conditions (19) imply that $\sum_{t=1}^{\infty} \lambda_t(c_t + W_t - (1+i_{t-1})W_{t-1}) = 0$, or $\sum_{t=1}^{\infty} \lambda_t c_t = \sum_{t=1}^{\infty}(\lambda_t - \lambda_{t+1}(1+i_t))W_t + \lambda_1(1+i_0)W_0$.

Using (17) $\sum_{t=1}^{\infty} \lambda_t c_t = \sum_{t=1}^{\infty} \beta^t = \beta/(1-\beta)$. (18) implies $\sum_{t=1}^{\infty}(\lambda_t - \lambda_{t+1}(1+i_t))W_t = 0$, hence $\lambda_1(1+i_0)W_0 = \beta/(1-\beta)$. Since from (17) $\lambda_1 c_1 = \beta$, we finally obtain $c_1 = (1-\beta)(1+i_0)W_0$. Plugging c_1 into (19) at $t = 1$, $W_1 = \beta(1+i_0)W_0$. (17) and (18) together imply the Euler equation $c_{t+1} = \beta(1+i_t)c_t$, which, used recursively with c_1 and W_1, yields (9) and (10).

5.3 Appendix C

Total differentiation of (16) yields

$$\frac{dm}{d\beta} = -\frac{(1+h(m)g(0)-g'(0))^2}{\beta^2\left(h''(m)g(0)(1+h(m)g(0)-g'(0)) - (h'(m)g(0))^2)\right)} > 0.$$

The increase in m raises: $\gamma_A = 1 + h(m)g(0)$ as $h'(m)g(0) > 0$, $\omega = (1+h(m)g(0))/(1+h(m)g(0) - g'(0)$ since $d\omega/dm = -g'(0)h'(m)g(0)/(1+h(m)g(0) - g'(0))^2 > 0$, and v as $F(v) = \gamma_A = 1 + h(m)g(0)$ and $F'(v) > 0$.

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