

Seigniorage – where does it come from and who gets it?

An institutional perspective on currency creation.

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ABSTRACT

Modern economic Textbooks do not pay to much attention to the question of Seigniorage. If it is treated it is in the vast majority approached from a theoretical rather an institutional perspective. Instead of highlighting institutional differences between different monetary systems, a theoretical framework of a monetary system is presented. This may not be a disadvantage for “theory oriented” textbooks. This depends, however, on institutional change. The theoretical framework should reflect or be built upon the current institutional realities. In modern macroeconomic textbooks written by mainstream as well of heterodox scholars the monetary system is today more and more represented as a credit system. This change has not yet influenced the treatments of Seigniorage. Lacking behind a theoretical treatment of the revenue from Seigniorage for a credit system is problematic. It seduces to carry over insights which – as will be shown here – might turn out to be false under different institutional circumstances.

In this paper theoretical frameworks for the revenue from Seigniorage for different institutional settings will be developed. On this basis differences and similarities between the systems can be highlighted and assigning false properties can be avoided.

The two main result are, first: the Chicago-rule in its classical form does not hold for a credit system, and second: the inflationary pressure of fiat systems cannot be carried over to other institutional settings.

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Seigniorage is defined as the government's revenue from the creation of money, which is usually ensured by a monopoly. The next question then is, what is money and how is it supplied? Many definitions of money can be found in the literature. In modern textbooks money is defined by a record of characteristics, which is referred to as its functions. The most common functions are: means of exchange, store of value, unit of account, and means of deferred payments. Such definitions, as common as they may be, provide difficulties for the treatment of Seigniorage. "Money" issued by the government may fulfill all of these characteristics but it is almost never the only means for these functions. Looking at the subject from an institutional perspective makes it necessary to relate the theoretical characteristics to specific "objects"² of reality. To be able to restrict the analysis to those "objects" which relate to the government's Seigniorage a further characteristic shall be added. Regarding the revenue from Seigniorage the analysis is restricted to that part of the money stock which is issued by the legal authorities. This part of the monetary stock will be referred to as currency including not only coins but all kinds of money issued by the government, independent of their physical material. Thus, despite the fact that the government may issue paper, gold coins or anything else, if it is issued by the local authority it will be referred to as *currency*. Thus all currency is money, but not all money is currency. Seigniorage can therefore be defined as the revenue from the provision of the national currency. There have been different definitions and terms in the literature to restrict the analysis in similar manners. Keynes uses the expression of legal-tender money (see Keynes 1923, p. 44).

Seigniorage is usually distinguished into monetary, or gross Seigniorage (S), and fiscal Seigniorage (S^G), or net Seigniorage. Monetary Seigniorage is defined as the whole amount of Seigniorage generated. Fiscal Seigniorage is defined as the revenue after allowances for different sorts of cost from the provision of the currency have been made. The difference between the two depend (among other influences) on the costs generated by sustaining the monetary system, operational cost of the monetary authorities and gains or losses from foreign exchange trade. For a more detailed treatment of the institutional differences influencing the gap between gross and net Seigniorage see Klein and Neumann (1990) or Bofinger, Reischle, and Schächter (1996, p. 53 ff.). For a closed economy, with no direct lending to the government by the monetary authorities the difference between fiscal and monetary Seigniorage can roughly be reduced to the fixed operational cost of the monetary authorities (C).

$$(1.1) \quad S^G = S - C.$$

Seigniorage is one of the oldest economic questions addressed. It can be found for example in the writings of Nicholas Oresme writing in the 14th century or in the dispute between the Saxon princes of the 16th century. Oresme argues especially against the debasement of coins which is as well at the

² Knapp uses the expression "money things".

core of the dispute between the Albertians and Ernestinians of the Saxon controversy (for an overview see Schefold 2004).³

The modern treatment of Seigniorage can at least be traced back to Keynes, who stated that: „A government can live for a long time, even the German government or the Russian government, by printing paper money. That is to say, it can by this means secure the command over real resources, resources just as real as those obtained by taxation. The method is condemned, but its efficacy, up to a point, must be admitted. A government can live by this means when it can live by no other. It is the form of taxation which the public find hardest to evade and even the weakest government can enforce, when it can enforce nothing else” (Keynes 1923, p. 37).

Keynes assumes a institutional framework which shall here be referred to as a *fiat currency*. Such a fiat currency can be printed by the government at its free will and which is issued through government spending. Such a currency is therefore exogenously supplied to the economy. Treatments of Seigniorage based upon such a monetary system can be found in almost all publications on the topic. This includes the classical works on the subject such as Auernheimer (1974, p. 600), Bailey (1956, p. 102), Cagan (1956, p. 25 ff.), Calvo (1978, p. 506), Chamley (1985, p. 37), Fisher (1982, p. 301), Friedman (1948 and 1971, p. 846), Hawtrey (1930, p. 210 f.), Keynes (1923, p. 37), Klein and Neumann (1990, p. 209), Mankiw (1987, p. 327), McCallum (1983), and Niehans (1980, p. 141), as well as more recent publications on the topic such as Abel, Bernanke, and Croushore (2011, p. 598), Blanchard and Fisher (1989, p. 179 and p. 514), Bofinger, Reischle, and Schächter (1996, p. 48), Borchert (2003, p. 50), Cecchetti (2008, p. 474), Heijdra and Pleog (2002 p. 348 ff.), Issing (2011, p. 256), Jarchow (1993, p. 314), Ljungqvist, L. and T. J. Sargent (2004, p. 861, and p. 918), Romer (2001, p. 510), Tobin (1986, p. 11), Trehan and Walsh (1990, p. 98), Walsh (2003, p. 137), or Woodford (1995)).

There are, of course, slight differences among these authors. Some new classical authors determine inflation via the public deficit, others determine the public deficit via the rate of interest. Some authors – for instance see Auernheimer (1974), Calvo (1978), Chamley (1985) and Woodford (1995) – work in continuous time frameworks while most authors prefer to model in discrete time. Abel, Bernanke, and Croushore (2011, p. 598) analyze the Seigniorage for a stationary economy and therefore suggest that Seigniorage is limited to inflationary gains. This is similar to the treatment of Friedman (1948) and Bailey (1956). Others, such as Burton and Lombra (2006), Cecchetti (2008), Fisher (1982), or Mishkin (2007) take a international perspective, arguing that using a foreign currency is expensive in terms of forgone Seigniorage revenue. Klein and Neumann (1990) highlight the institutional differences between countries and their impact on gap between monetary and fiscal Seigniorage. The theoretical core of the authors mentioned above is however

3 Further authors to incorporate here – Azpilcueta 1556, Ortiz 1558, Serra – Lessius 1572, Hornigk – Justi – James Stuarts – John Law – Smith 1776 – Ricardo 1824 – Mill – Marshall

almost identical and there is no substantial difference to be found in their analysis, except for the limited scope of some works (e.g. restricting to a stationary economy).

A second group of authors define Seigniorage for an exogenously supplied fiat currency as the opportunity cost, or avoided interest payments for the government. While the former approach focused on flows of currency, this definition focuses on the stock of currency. If the government finances expenditure by issuing non-interest bearing currency instead of issuing interest bearing bonds it avoids at least partly interest payments to the public. This reduces the financing cost of the government which can also be interpreted as Seigniorage. Such a definition can be found in the already mentioned Burton and Lombra (2006, p. 651) as well as in Bofinger, Reischle, and Schächter (1996, p. 48), Klein and Neumann (1990, p. 209), Mishkin (2007, p. 386), Phelps (1973, p. 68) or Riese (1986, p. 125).

From a theoretical perspective these approaches are similar and can be related to each other and under some assumptions even become the same. Empirically, however, they lead to different estimates of Seigniorage. Their biggest similarity can be found in the institutional underpinnings. The authors of both approaches assume what has been labeled a fiat currency, even though some do not explicitly refer to the institutional settings (but implicitly assume them).

However, there are other monetary regimes possible, and some authors – even though loosely – mention the existence or possibility of different institutional settings, such as the revenue from Seigniorage from issuing metal coins. However, none of these authors develops an analytical framework similar to the one for a fiat currency.

The historical example is a monetary system based on what shall be called a *commodity currency*. The currency issued consists of physical objects which are produced and labeled by the government. A simple example is a gold or a metal standard. In such a system the Seigniorage is earned as a fee in the mints. If e.g. gold is brought to the government's mint some minted coins are kept by the government. Further developed governments sold the monopoly for exploiting the silver or gold mines. Here the fee on individuals is replaced by the fee charged for the monopoly. The latter is mentioned by Blankart (2006, p. 388), Bofinger, Reischle, and Schächter (1996, p. 48), Bordo (1986, p. 340), Illing (1997, p. 53), Klein (1982, p. 167), and Richter (1990, p. 321) although none of them provides an analytical framework for such a monetary system.

A third monetary system which is treated in the literature is based on what we shall call an endogenous *credit currency*. A credit currency may be paper based or commodity based. However, the currency it is not issued through government spending, but by lending to the public. Depending on the supply mechanism used by the monetary authorities Seigniorage is earned in form of interest payments from commercial banks or in terms of acquiring interest bearing commercial bonds in exchange for non-interest bearing currency. Such a definition can be found in publications of the

Bundesbank (e.g. Bundesbank 2010, p. 70). To my knowledge no contemporary author analyzing Seigniorage mentions such a monetary system. This is more than surprising as authors who proclaim a credit based monetary system such as Woodford (2003) skip the question of the Seigniorage or if they treat it return to an exogenously supplied fiat currency as in Woodford (1995). Therefore there is no analytical framework or any reference to the creation of Seigniorage for a credit currency.

The institutional frameworks are not found very frequently in the literature, but if studied closely there are hints. For instance Ricardo (1824, p. 10) distinguishes a credit currency (which is issued through lending and he calls “paper money”) from a fiat currency (which is issued by printing and spending it and he calls “forced government paper”). Schefold (2004, p. 68) compares the hyperinflation of fiat currency systems to the medieval debasements, and Reinhart and Rogoff (2009, p. xxxiii) loosely add the government defaults. The purpose of this chapter is to repeat the standard framework used in the literature to analyze the laws governing the Seigniorage revenue from supplying an exogenous fiat currency for a closed economy, in which the the central bank is not allowed to lend directly to the government.⁴ After that similar and comparable frameworks for a commodity based currency and a credit based currency shall be developed and be compared to each other. The aim will be to identify similar patterns for the different institutional frameworks and to show if there are differences between the different currencies.

A last note on the applied method seems to be necessary. The monetary systems, which have been defined above, as “ideal types” in the sense of Max Weber's *Idealtypus*. Thus the three distinguished systems are idealized and somehow extreme cases which are hardly found in reality. In reality monetary systems are usually a mixture of the three ideal types of monetary systems. In the Euro zone for example we have a paper based credit currency, supplied through central bank lending. However, governments still possess – though limited – the right of coinage. Most recently the European Central Bank started the purchase of government bonds (even though restricted to the secondary market). It seems that the modern monetary system combines elements of all three ideal types. However, it will be argued, that there is always a dominant system which rules the behavior of the whole system (see chapter 4).

The question whether a monetary system is dominated by one or another ideal type is an empirical question, and faces other difficulties than those we shall dwell on here. To judge this empirical question seems to be very difficult as it is one of the most controversial questions in economics. It is clear that in an ideal type fiat currency as defined here currency supply is exogenous and in a paper based credit currency supply is endogenous. Apparently it is much more difficult to reach an agreement on the question whether the currency supply of an empirical monetary regime is

⁴ Gains and losses from trading foreign currencies and interest earnings from the government do not appear in the Seigniorage equation, as they sometime do in the literature.

endogenous or exogenous. Authors proclaiming an exogenous supply assume that the government or the central bank controls the stock of currency and thereby influences the level of prices. Authors proclaiming an endogenous supply assume that the government or the central bank controls the price of the currency (usually the interest rate at which currency is issued through lending). The amount of currency the public wishes to hold follows from individual calculations after this price is given. An overview of the different positions can be found in Schefold (2002). This question has never been fully settled even though it can be traced back at least to the Bullion-Anti-Bullion controversy of the 19th century. For a detailed overview of this dispute see Arnon (2011). As difficult as this question seems it is not much of a riddle here. It is sidestepped by focusing on the ideal types.

1 Seigniorage from supplying a fiat currency

As mentioned above there are two concepts, a flow and a stock approach. The flow based concept is sometimes referred to as the monetary Seigniorage, even though this is a little confusing for it uses the term monetary which is additionally used to distinguish monetary and fiscal Seigniorage. The stock approach is usually referred to as the opportunity cost approach, wherein Seigniorage is earned when interest bearing bonds can be substituted for non-interest bearing currency. We will deal mostly with the monetary concept in this chapter which is the most widely used concept. The “standard” framework can be found in very similar ways in Bailey (1956, p. 102), Blanchard (2009, p. 519), Blanchard and Fischer (1989, p. 179 ff. and p. 519 ff.), Bofinger, Reischle, and Schächter (1996, p. 58 ff.), Friedman (1948 and 1971, p. 846), Illing (1997, p. 57 ff.), Issing (2011, p. 256 ff.), Jarchow (1993, p. 308 ff.), Klein and Neumann (1990, p. 211 ff.), Ljungqvist and Sargent (2004, p. 918 f.), Mankiw (1987, p. 327), Romer (2001, p. 510 ff.), Trehan and Walsh (1990, p. 100 ff.), and Walsh (2003, p. 135 ff.).

The monetary concept pictures a institutional setting wherein a government prints its notes to pay for wages, salaries and goods. The Seigniorage (S^F) in nominal terms (S^FP) is therefore given by the amount of freshly printed currency (\dot{B}_t). The most able and lengthy treatment of such a policy can be found in Keynes (1923). The printing press increases the stock of currency and may affect the level of prices. The Seigniorage is therefore usually not measured in nominal terms but corrected for the new level of prices (P). Therefore the Seigniorage is given by the flow of printed currency divided by the level of prices.

$$(1.2) \quad S_t^F = \dot{B}_t/P_t, \text{ where } \dot{B}_t = B_t - B_{t-1}$$

It can be rewritten as:

$$(1.3) \quad S_t^F = \dot{B}_t/B_t \cdot B_t/P_t = g_{B,t} \cdot B_t/P_t, \text{ with } g_{B,t} = \dot{B}_t/B_t.$$

If we introduce the quantity equation ($B=kPY$) we can replace B/P and receive

$$(1.4) \quad S_t^F = g_{B,t} k_t Y_t.^5$$

As a share of national income we get

$$(1.5) \quad S_t^F/Y_t = g_{B,t} k_t$$

Thus the government can acquire the more resources the more currency it prints. Even thou it is recognized empirically that during a hyperinflation the legal tender is not completely abolished (see Keynes 1923⁶) it is usually assumed that households are able to substitute the national currency by alternative means. In other words the demand for currency might shrink substantially. All authors mentioned assume in one or the other way a negative relation between the currency supply (esp. inflation) and the demand for currency. A usual way to do so in the literature is to replace kY with a demand function for "real balances". A variety of demand functions can be found in the literature. Usually they are referred to as money demand functions instead of currency demand functions.⁷ The common feature of these approaches is that the demand depends positively on national income and negatively on inflation (flow approach) or the rate of interest (stock approach). Therefore Seigniorage (or Seigniorage as a share of national income) is maximized.

$$(1.6) \quad \max \{S_t^F/Y_t\} \text{ for } d\{S_t^F/Y_t\}/d\pi_t = \delta g_{B,t}/\delta\pi_t k_t(\pi_t) + g_{B,t} \delta k_t(\pi_t)/\delta\pi_t = 0.$$

To derive equation (1.6) we have to relate g_B and π . This can be done using the quantity equation in growth terms.

$$(1.7) \quad g_{B,t} = \pi_t + g_{k,t} + g_{Y,t}$$

Using this equation the first derivative is:

$$(1.8) \quad k_t(\pi_t) + \delta g_{k,t}/\delta\pi_t k_t(\pi_t) + \delta g_{Y,t}/\delta\pi_t k_t(\pi_t) + (\pi_t + g_{k,t} + g_{Y,t}) \delta k_t(\pi_t)/\delta\pi_t = 0.$$

To receive a popular result three things have to be assumed. First, a growth in currency does not induce growth, and therefore $\delta g_{Y,t}/\delta\pi_t = 0$. Second, the change in the demand for currency does not depend on inflation, and therefore $\delta g_{k,t}/\delta\pi_t = 0$.

$$(1.9) \quad k_t(\pi_t) + \pi_t \delta k_t(\pi_t)/\delta\pi_t + (g_{k,t} + g_{Y,t}) \delta k_t(\pi_t)/\delta\pi_t = 0.$$

$$(1.10) \quad \delta k_t(\pi_t)/\delta\pi_t \pi_t/k_t(\pi_t) = -1 - (g_{k,t} + g_{Y,t})/k_t(\pi_t) \delta k_t(\pi_t)/\delta\pi_t.$$

For $(g_{k,t}+g_{Y,t})$ equal to zero, the optimizing level of inflation is reached if the elasticity of demand with respect to inflation is minus one. This is the usual result for an optimizing monopolist. If $(g_{k,t}+g_{Y,t})$ greater zero, the optimum is moved to lower levels of inflation. With the usual definition of the elasticity (η) we can rewrite equation (1.10).

5 kPY is usually understood as "money" demand. Thus it is assumed, that the "money" demand of the public leads to a equal demand in currency. In other words we assume that "money" is 100% currency, thus a narrow banking system (see Goodhard 1995) or 100% reserve system (see Fisher 1935). Fractional reserve banking will be dealt with in the next chapter.

6 "Moreover, the conveniencies of using money in daily life are so great that the public are prepared, rather than forgo them, to pay the inflationary tax, provided it is not raised to a prohibitive level" (Keynes 1923, p. 43).

7 Referring to money instead of currency is highly missleading. All authors treating the revenue from the supply of legal tender assume in that specific chapter that the overall supply of money equals the supply of legal tender by the government. Thus it is assumed (in a stark contrast to other chapters, especially to those treating the money multiplier) that all money is legal tender, or in our words fiat currency. We will come back to this muddle in the next chapter.

$$(1.11) \quad \eta_{k,\pi} = -1 - (g_{k,t} + g_{Y,t})/k_t(\pi_t) \delta k_t(\pi_t)/\delta \pi_t.$$

Usually a third assumption about the demand function is made. The most widely used function is of the so called Cagan-type, an exponential function which depends negatively on inflation.⁸ Here a Cagan-type function of the following form will be used:

$$(1.12) \quad k_t(\pi_t) = e^{-\hat{k}\pi}.$$

For this function the first derivative equals the function itself times a constant.

$$(1.13) \quad -\hat{k} k_t(\pi_t) = \delta k_t(\pi_t)/\delta \pi_t$$

We can therefore rewrite equation (1.8) as

$$(1.14) \quad (\hat{k} - g_{B,t}) k_t(\pi_t) = (\hat{k} - \pi_t - g_k - g_{Y,t}) k_t(\pi_t) = 0, \text{ with } k_t(\pi_t) > 0.$$

The result of this equation was made famous by Friedman (e.g. 1971). Under the assumptions made we receive an optimal rate of currency growth and an optimal rate of inflation. While the rate of currency growth is a constant regarding other factors, the optimal rate of inflation varies with different rates of income growth or changes in liquidity demand.

$$(1.15) \quad \pi_t^* = \hat{k} + g_{k,t} + g_{Y,t}, \text{ or } g_{B,t}^* = \hat{k}.$$

The maximum Seigniorage is then given by:

$$(1.16) \quad \{S_t^F/Y_t\}^* = \hat{k} e^{-\hat{k}(\hat{k}+g_k+g_Y)}$$

Such insights are hidden if the analysis is restricted to a stationary economy with a given and fixed currency demand such as presented in Abel, Bernanke, and Croushore (2011), Bailey (1956) or others. The later analysis suggest that the Seigniorage is only generated for a positive level of inflation. This is not true and becomes obvious if revenue from Seigniorage is decomposed in two terms. Such a decomposition is for instance done by Auernheimer (1974, p. 605).

$$(1.17) \quad S_t^F = (\pi + g_k + g_Y) k_t Y_t, \text{ or}$$

$$(1.18) \quad S_t^F = \pi k_t Y_t + (g_k + g_Y) k_t Y_t.$$

The first part of the right hand side of equation (1.18) is due to inflation. The second part is due to changes in the demand for currency. Therefore Seigniorage can be positive with no inflation. If the increase in the currency supply is matched by a growth in currency demand due to an increase in liquidity preference (g_k) or due to economic growth (g_Y) a positive non-inflationary Seigniorage is possible. At some point however an increase in the supply of currency through government expenditure will cause an inflationary pressure. It is however not certain when this point is reached, because the government spending might at least influence the growth rate. The Chicago school usually assumes that government spending does not affect the growth rate (see Calvo 1978, p.505).

⁸ The specific form of the function used in the literature varies and could fill a whole chapter on its own. However, the result regarding the Seigniorage does not depend strongly on the specific functions which can be found in the literature. A common form is an exponential function which is sometimes referred to as cagan-type (early examples are Cagan 1956, Bailey 1956, or Auernheimer 1974). The difference regarding the shape of the curve is mostly restricted to its right hand end. Either it intersects the abscissa or it adapts asymptotically to it. For an overview see Bofinger, Reischle, and Schächter (1996, p. 471 ff.).

For an economy outside of full employment this is however questionable. Other demand functions lead to similar results. If we apply a linear demand function the optimal currency growth is determined by the characteristics of the demand function. The rate of inflation depends on the rate of income growth and the change in liquidity demand.

$$(1.19) \quad k_t = k^{\max} - \hat{k} g_{B,t}, \text{ with } k^{\max} > 0, \hat{k} > 0.$$

Using the same assumptions made above, the first order condition is:

$$(1.20) \quad k_t(\pi_t) - \hat{k} g_{B,t} = k^{\max} - 2\hat{k} g_{B,t} = 0.$$

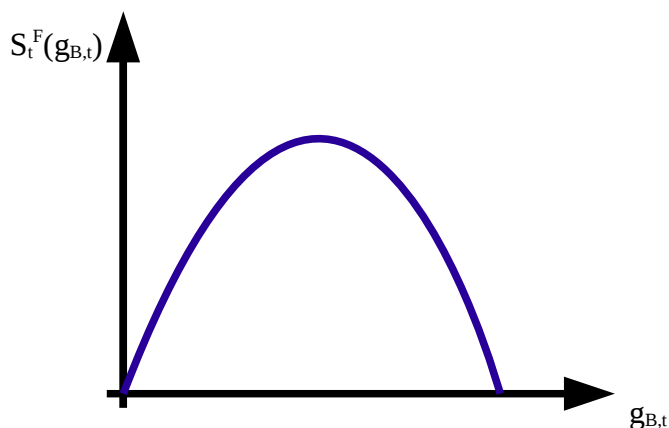
$$(1.21) \quad g_{B,t}^* = k^{\max}/2\hat{k}, \text{ or } \pi_t^* = k^{\max}/2\hat{k} - g_{k,t} - g_{Y,t}.$$

The maximum Seigniorage is then received for π_t^* .

$$(1.22) \quad \{S_t^F/Y_t\}^* = 0,25\hat{k}k^{\max 2}$$

In any case the government faces a trade-off. By printing currency the government increases its revenue from Seigniorage while – at the same time – it reduces the demand for its currency by causing inflation. The government does however not control the level of inflation directly. Depending on the demand function, respectively its elasticity, the Seigniorage may have different shapes. For a linear currency demand the revenue curve from Seigniorage is quadratic. Exponential currency demand functions lead to similar revenue curves, except for approaching the abscissa asymptotically. Even though usually presented this way, the curves are not limited to positive values of currency growth. In other words, shrinking the volume of currency has to be financed by means of other taxes. The following graph is based on the linear demand function (1.19).

Graph 1.1: Seigniorage for fiat currency.



Reference: Graph based on Bofinger, Reischle, and Schächter (1996, p. 70).

If the government seeks to finance some part of its debt by printing currency and an inflation is caused by such finance operations two sorts of inflation are usually distinguished, anticipated and unanticipated inflation. The trade-off described above is the government's trade-off for anticipated inflation. Unanticipated inflation implies that the government announces a different inflation and

currency growth then it actually pursues. This “cheating” or surprising of the countries citizens temporarily increases the return for the government. The additional revenue depends on the level of inflation from which the maneuver is started. The government will increase its revenue due to the additional printing of notes, but it will loose part of its real value as the level of prices rises. The return can be estimated if currency demand is hold constant for a higher rate of currency growth.

$$(1.23) \quad S_t^{F_i} = g_{B,t}' k_t(.) Y_t$$

The additional Seigniorage as a share of national income is given by the return from cheating minus the honest return.

$$(1.24) \quad \Delta S^F/Y_t = (S_t^{F_i} - S_t^F)/Y_t = g_{B,t}' k_t(.) - g_{B,t} k_t(.) = (g_{B,t}' - g_{B,t}) k_t(.)$$

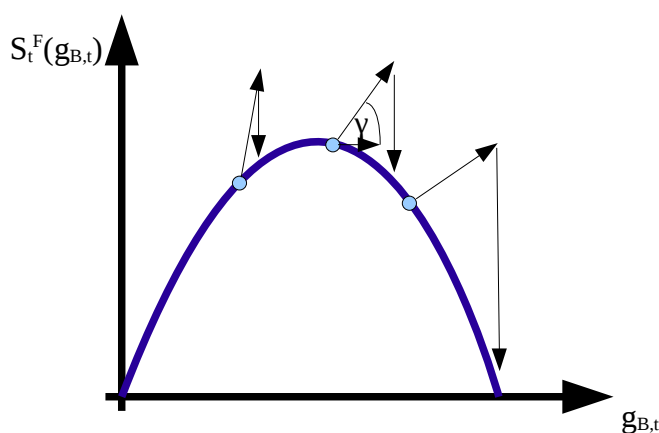
The slope of this disturbance can be expressed by the angle γ .

$$(1.25) \quad \gamma = \text{acttan}((\Delta S^M/Y_t)/(g_{B,t}' - g_{B,t})) = \arctan(k_t(.))$$

If $k_t(.)$ depends negatively on $g_{B,t}$ (which it does monotonically for the assumed demand functions) that arctan has a negative slope. Therefore we can conclude that γ depends positively on $g_{B,t}$. Thus the higher the currency growth we depart from, the higher the inflation, or $dy/dg_{B,t} > 0$. This generates a feedback effect, which leads to a continually shrinking revenue from Seigniorage once the maximum revenue growth rate is exceeded. We will come back to this effect later (see chapter 5).

The following graph is based on the linear demand function (1.19). It shows how unanticipated inflation first increases the revenue from Seigniorage (upward arrows), and how the next periods Seigniorage is reduced for the higher expected levels of inflation (downward arrows). The higher the actual level of anticipated inflation, the worse the government has to cheat its citizens, to generate the same additional revenue. The reason is the increasing angle γ .

Graph 1.2: Dishonest Seigniorage for fiat currency.



Reference: Illustration based on Bofinger, Reischle, and Schächter (1996, p. 70).

The opportunity cost approach differs from the monetary approach. If a government is able to pay

goods and services by printing non-interest bearing fiat currency instead of issuing interest bearing bonds the government saves the interest payments. For an exogenously supplied fiat currency the opportunity cost approach derives the Seigniorage (S^{Fopp}) from the interest which would have been charged on government bonds (i^{Dc}) times the stock of currency which was issued by government spending in the past. Thus it derives the Seigniorage from stocks instead of flows. Among the authors mentioned above Phelps has written the classical contributions on this subject. The Seigniorage is given by the stock of currency (B_t) divided by the price level (P) times the rate of interest on government debt.

$$(1.26) \quad S_t^{Fopp} = i^{Dc} B_t/P_t$$

If the rate of interest on government bonds (i^{Dc}) equals the rate of currency growth (g_B) both approaches are the same. The same analogy can be reached if the rate of interest paid on government bonds is split in a real and a nominal part (as supposed by the simple Fisher equation):

$$(1.27) \quad S_t^{Fopp} = (\pi_t + r_t) B_t/P_t = \pi_t B_t/P_t + r_t B_t/P_t$$

If “the real rate of interest” charged on government bonds equals the growth in currency demand the opportunity cost approach equals the monetary approach. If we assume ($g_k = 0$) there is a more familiar way to say this. In this case the “real rate” on government bonds has to be equal to the rate of economic growth, thus we have to be in a golden rule equilibrium. A problem of this approach is to determine the correct interest rate. Government bonds are issued with a maturity of up to 30 years, the interest charged on newly issued bonds will vary over time. Thus if the government does not roll over its whole debt every period the correct rate of interest will be given as a weighted average from history. Therefore, as has been highlighted by Klein and Neumann (1990) “most empirical research uses the concept of monetary seigniorage” (Klein and Neumann (1990), p. 211). The first order condition is analogue to the flow approach. If we assume that $k(\cdot)$ depends negatively on inflation, we get a similar first order condition as in equation (1.10).

$$(1.28) \quad k_t(\pi_t) + \delta r_t / \delta \pi_t k_t(\pi_t) + \pi_t \delta k_t(\pi_t) / \delta \pi_t + r_t \delta k_t(\pi_t) / \delta \pi_t = 0.$$

If we assume similar to the proceeding above that the real interest rate (rate of growth for the golden rule) is not affected by the rate of inflation we get:

$$(1.29) \quad \delta k_t(\pi_t) / \delta \pi_t \pi_t / k_t(\pi_t) = -1 - r_t \delta k_t(\pi_t) / \delta \pi_t, \text{ or}$$

$$(1.30) \quad \eta_{k,\pi} = -1 - r_t \delta k_t(\pi_t) / \delta \pi_t$$

The optimal rate of interest is determined by the elasticity of demand, and the optimal rate of inflation varies. It is reduced by a positive rate of “real” interest and vice versa.

2 Seigniorage from supplying a commodity based currency.

To my knowledge there is no analytical treatment in the literature trying to generalize in the same manner as above the returns from Seigniorage for a commodity currency. Commodity based

monetary systems are mentioned in more historically oriented books, like Carsons (1963), an in purely economical works, such as Blankart (2006), Bofinger, Reischle, and Schächter (1996), Bordo (1986), Illing (1997), Klein (1982), or Richter (1990), but neither of them derives an theoretical framework.

For a commodity currency the Seigniorage (S^C) is earned as a fee in the governments mints. If gold is brought into the mint a fraction of the minted coins is kept by the government (τ). Another way to collect the Seigniorage is to sell the monopoly to exploit a countries mines. In both cases the government has to define the metal content of a coin (e). If (Γ) ounces of gold are brought to the mint ($e\Gamma$) coins will be minted from that and the government keeps a part as Seigniorage ($\tau e\Gamma$). The supply of currency (B) is then determined by the amount of gold brought to the mint (Γ). Citizens are free to bring their gold to the mint or keep it at home. The demand for currency from the public therefore depends the tax charged for the minting process. The Seigniorage is then given by the newly issued coins divided by the level of prices.

$$(1.31) \quad S_t^C = \tau_t e_t \dot{\Gamma}_t / P_t = \tau_t \dot{B}_t / P_t.$$

With the transformation already applied in equation (1.3) we get

$$(1.32) \quad S_t^C = \tau_t g_{B,t} B_t / P_t$$

as in equation (1.4) we can replace B/P with kY .

$$(1.33) \quad S_t^C = \tau_t g_{B,t} k_t Y_t$$

Maximizing the Seigniorage as a share of national income similar to equation (1.6) the fist order condition is given by

$$(1.34) \quad g_{B,t} k_t(\cdot) + \tau_t \delta g_{B,t} / \delta \tau_t k_t(\cdot) + \tau_t g_{B,t} \delta k_t(\cdot) / \delta \tau_t = 0.$$

The first term measures the revenue from the tax, the second term measures the effect of taxation on the reduction in the change in currency demand, and the third term measures the effect of taxation on liquidity preference. This can be rearranged in a similar way to the results of chapter 1 .

$$(1.35) \quad \delta k_t(\cdot) / \delta \tau_t \tau_t / k_t(\cdot) = -1 - \delta g_{B,t} / \delta \tau_t \tau_t / g_{B,t}, \text{ or}$$

$$(1.36) \quad \eta_{k,\tau} = -1 - \eta_{g,\tau}$$

If changes in currency demand do not depend on the tax, the optimum is given, similar to equation (1.10), by minus one. The usual result for a revenue maximizing monopolist. If changes of currency demand depend negatively on the tax (which is most reasonable) the optimal tax is reduced.

For a Cagan-type demand function of (1.12) we get the following first order condition:

$$(1.37) \quad (\hat{k} g_{B,t} + \tau_t \hat{k} \delta g_{B,t} / \delta \tau_t - \tau_t g_{B,t}) k_t(\cdot) = 0.$$

Rearranging the equation and solving the first part gives us

$$(1.38) \quad \hat{k} = \tau_t - \hat{k} \eta \{ \delta g_{B,t} / \delta \tau_t \} \tau_t / g_{B,t}$$

$$(1.39) \quad \tau_t^* = \hat{k} (1 + \eta_{g,\tau}).$$

For a Cagan-type demand function the optimal rate of taxation (τ^*) is given by the elasticity of

demand in currency. The maximal Seigniorage as a share of national income, is then given as:

$$(1.40) \quad \{S_t^C/Y_t\}^* = \hat{k}(1+\eta_{g,t}) g_{B,t} e^{-\hat{k}^2(1+\eta_{g,t})}$$

The same result is obtained for a linear demand function similar to (1.19).

$$(1.41) \quad k_t(\tau) = k^{\max} - \hat{k}\tau, \text{ with } k^{\max} > 0, \hat{k} > 0.$$

Analogue to equation (1.33) the first order condition is:

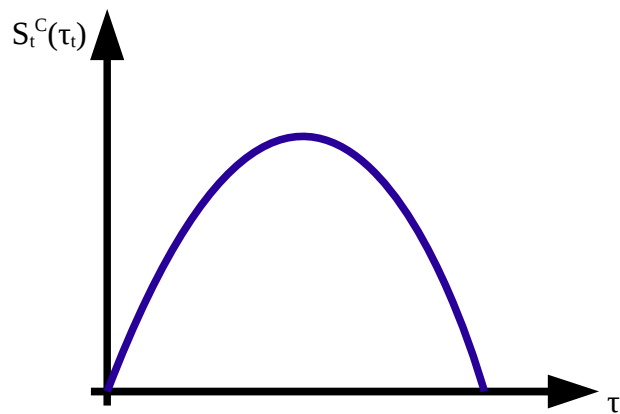
$$(1.42) \quad g_{B,t} k_t(\cdot) + \tau_t \delta g_{B,t} / \delta \tau_t k_t(\cdot) - \hat{k} \tau_t g_{B,t} = 0.$$

Which provides the optimum tax rate

$$(1.43) \quad \tau_t^* = k^{\max} / 2\hat{k} (1+\eta_{g,t}) / (1+0,5\eta_{g,t}).$$

The result is similar to the fiat currency. However the currency growth is not controlled by the government in this case. Therefore its elasticity regarding the tax enters into the optimization of the Seigniorage. The government faces a similar trade-off. By increasing the tax on gold deliveries the share the government receives increases, but at the same time less gold will be brought to the mint. The following graph is based on the linear demand function of equation (1.41). It shows the revenue from Seigniorage for different tax rates.

Graph 2.1: Seigniorage for a commodity currency.



Reference: Illustration based on own research.

Besides τ the government can vary e , the gold content of new coins. The gold content of the coins determines the level of prices. A change in the gold content (\dot{e}) would therefore equal a change in the level of prices (π). The gold content of already issued coins, however, cannot be changed ex post. Thus if the government decides to alter the gold content coins of different gold content, an of different value, will circulate at a time. To understand the possible impact of this, we will first consider the two most extreme cases. Old coins might circulate at their former value and new coins at a different value according to their gold content. If this is so, the level of prices will not be alter in terms of old coins, and neither the government or the public will gain or loose anything from such changes. However, if the government is able to enforce that old and new coins circulate at the same value – although limited by Grasham's law and the import and export of coin – the distribution

of the tax burden, the nominal returns from Seigniorage and the level of prices will be affected. In the commodity currency system the Seigniorage is usually paid by those bring gold to the mint. In other words by those who demand fresh currency. Debasing the currency then shifts the tax burden from the latter to the ones who brought gold to the mint in the past. The level of prices depends, for a commodity currency system, on the value of the gold coins. The value of the coins is determined by their production cost. From the perspective of a citizen these production cost are the value of the gold plus Seigniorage.

$$(1.44) \quad P_t = e_t(1+\tau_t)$$

We can use this to rewrite the equation for the Seigniorage (1.31) as:

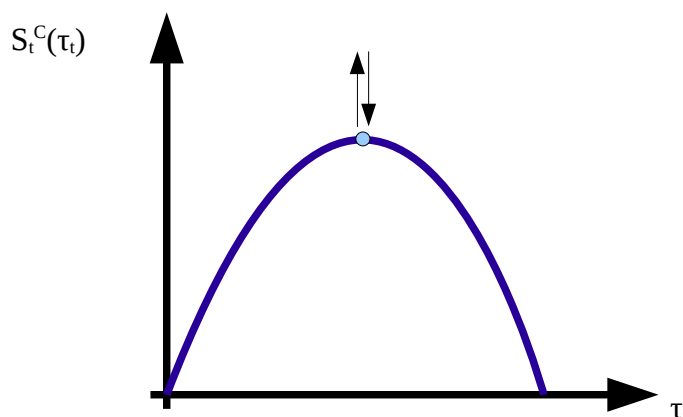
$$(1.45) \quad S_t^C = \tau_t / (1+\tau_t) \dot{G}_t$$

The first derivative with respect to the price level is then given by:

$$(1.46) \quad \delta S_t^C / \delta P_t = \tau_t / (1+\tau_t) \delta \dot{G}_t / \delta P_t$$

In other words if all circulating coins are replaced by new coins and the level of prices has fully adjusts to the change, the Seigniorage will be of the same height as before. The reason is that “real” currency demand does not depend on the level of prices. In the new equilibrium the government will receive the same amount of Seigniorage as before. If τ is constant an increase in e will – in the long run – increase nominal Seigniorage as much as the level of prices. In the transition period the revenue from Seigniorage will be increased. If the government debases its coins, and the level of prices is still determined by the old coins, the government increases its returns immediately after the debasement. From now on the old coins will be extinguished by exportation, re-minting and by tax payments.

Graph 2.2: Dishonest Seigniorage for a commodity currency.



Reference: Illustration based on own research.

The graph above shows the revenue from dishonest Seigniorage for the linear demand function of equation (1.41). Debasements do not alter the tax, therefore the increase in Seigniorage is pictured by a straight upward arrow. The adaption to the new level of prices, and thereby returning to the old

level of Seigniorage, takes time (even though presented as a single downward arrow). Starting from the old level of prices the government can buy above the value of the freshly issued coins. Tax payments however will be delivered mostly in old, heavy coins as most coins in circulation are still of good quality. By issuing lighter coins, the recycling of heavy coins through taxation, the import and export of coins, and Gresham's law in general, the old coins will be extinct from circulation and more and more coins will be new ones until the price level adapted to the new gold content of the coins. The inflation ends at this point and the Seigniorage will be back at its old level, shrinking the more the price level adapts to its new level. Until then the government receives an additional Seigniorage revenue. After the level of prices adjusted to the changed gold content of the coins the nominal Seigniorage will have increased, its real value however will be back to its old level.⁹

3 Seigniorage from supplying a credit currency.

A credit based currency is usually supplied as credit by a government's monetary authorities, e.g. a central bank. Similar to the commodity currency the authorities determine the price of the currency and accommodate the public demand for this price. In this case Seigniorage stems from the interest payments of those demanding the credit from the monetary authorities. This definition fits to the description of Seigniorage by the Bundesbank (2010, p. 70). Woodford (2003) who is one of the modern authors who conduct analysis upon a credit based monetary system skips questions regarding currency supply or Seigniorage. This suggests that he assumes that there is no monopoly or any issuing cost of currency. However, if a credit currency is supplied by a monopolist, the monopolist will be able to acquire a return for providing the currency supply. Thus if currency is supplied by the monetary authorities they receive the gross revenue (monetary Seigniorage) and transfers the net revenue (fiscal Seigniorage), after allowance for cost to run the monetary system are made, to the government (the treasury). This is similar to the opportunity cost approach for a fiat currency, which is applied in Bofinger, Reischle, and Schächter (1996, p. 50 ff.), or Klein and Neumann (1990, p. 210). To keep the analysis comparable to those above, we shall keep the assumption of a closed economy. A further assumption is necessary to be able to analyze a pure credit currency. If the government is allowed to borrow directly from the monetary authorities, without the commitment to repay the loans the system becomes a fiat currency system, in which the government uses the "printing press". We shall therefore exclude direct loans to the government by the central bank.¹⁰ For a closed economy and no lending to the government the central bank budget consists of interest payments (i^{CB}) for loans to the private sector (B) minus maintenance cost of the monetary system. The monetary Seigniorage (S^K) is then given as the interest payments on the stock

⁹ This is, of course, only true if the price level fully adjusts to a change in the gold content of coins, a constant minting fee τ , and no discrimination of coins depending on their gold content.

¹⁰ This assumption fits to the euro zone where direct central bank lending to governments is forbidden.

of credit granted divided by the level of prices.

$$(1.47) \quad S_t^K = i_t^{CB} B_t/P_t$$

This looks very familiar to the opportunity cost approach of the supply of a fiat currency. However, in the former case the interest rate for the fiat currency system was endogenous and the currency supply was exogenous. In the case of a credit currency the central bank controls the interest rate charged and the demand for currency is endogenous. Another difference is the rate of interest. The rate of interest on government bonds used in the opportunity cost approach is replaced by the rate of interest charged by the central bank. The demand for currency (B) can again be substituted by kPY using the quantity equation.

$$(1.48) \quad S_t^K = i_t^{CB} k_t Y_t.$$

Similar to the negative relationship between currency demand and inflation for the fiat currency and the negative relationship between currency demand and the minting-tax we can introduce a negative relationship between currency demand and the interest rate charged on currency. The first order condition for the maximization of Seigniorage as a share of national income with respect to the rate of interest charged on central bank loans is received similar to the proceedings above.

$$(1.49) \quad k_t(\cdot) + i_t^{CB} \delta k_t(\cdot) / \delta i_t^{CB} = 0.$$

Rearranging this equation leads to:

$$(1.50) \quad \delta k_t(\cdot) / \delta i_t^{CB} i_t^{CB} / k_t(\cdot) = -1, \text{ or}$$

$$(1.51) \quad \eta_{k,i^{CB}} = -1.$$

We received the similar result to chapter 1 and 2 . The monopolist maximizes his income with respect to the elasticity of demand for currency.

For a Cagan-type demand function similar to (1.12) we get the revenue maximizing rate of interest by rearranging the first order condition.

$$(1.52) \quad i_t^{CB*} = 1/\bar{k}.$$

The maximal Seigniorage as a share of national income, is then given as:

$$(1.53) \quad \{S_t^K/Y_t\}^* = 1/\bar{k}e.$$

The same result can again be obtained for a linear demand function similar to (1.19).

$$(1.54) \quad k_t(i_t^{CB}) = k^{\max} - \bar{k}i_t^{CB}, \text{ with } k^{\max} > 0, \bar{k} > 0.$$

The Seigniorage maximizing rate of interest can be received by rearranging the first order condition similar to (1.33).

$$(1.55) \quad i_t^{CB*} = k^{\max}/2\bar{k}.$$

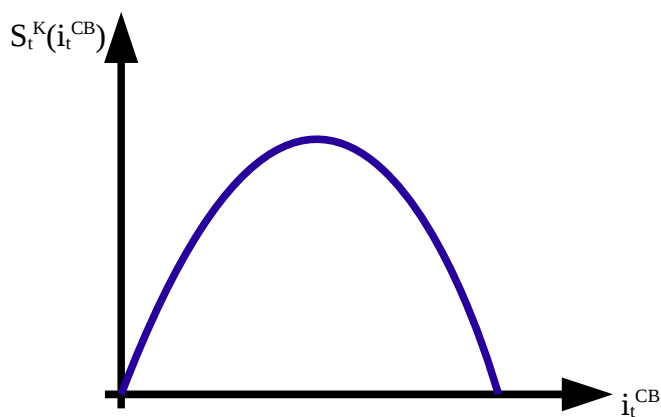
Plugging the revenue maximizing interest rate into the equation for the Seigniorage determines its maximum.

$$(1.56) \quad \{S_t^K/Y_t\}^* = 0,25 \bar{k} k^{\max 2}.$$

There is a revenue maximizing central bank rate which leads to a maximum Seigniorage. These

results parallel the findings for the other monetary systems. The government faces a trade-off. The higher the rate of interest the higher the gross return but the lower the demand for currency. The graph following is based on the linear demand function of equation (1.54). It pictures the revenue from Seigniorage for alternatives bank rates.

Graph 3.1: Seigniorage for a credit currency.

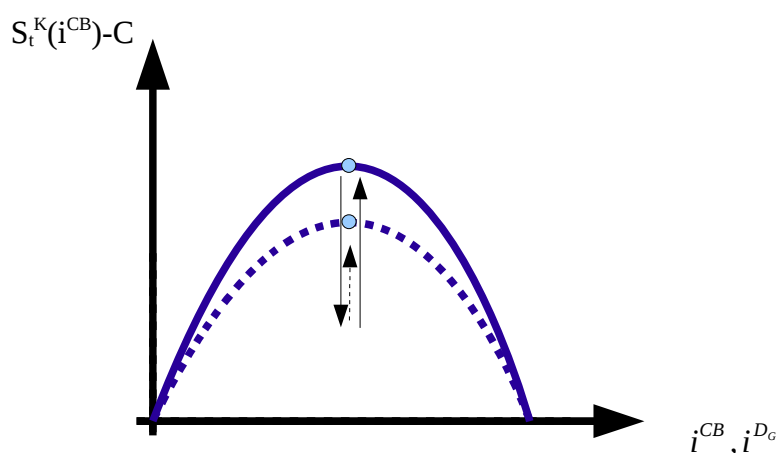


Reference: Illustration based on own research.

There is as well an analogy to the dishonest inflation we have found in the last two chapters. Instead of issuing additional paper (fiat currency), or to devalue the whole stock (commodity currency), in a credit currency system the government has to default on its debt. A – not perfectly serious – way to describe this process would be that of “wizard currency”, which disappears after the government paid its expenses. Depending on who gets paid with the “wizard currency” bears the loss.

If the government defaults on central bank loans, or on bonds owned by the central bank, the government will not be able to raise its revenue by defaulting. The revenue gain from Seigniorage to the treasury equals a loss to the central bank. After a reduction in the central banks return the revenue returns to its former height (graph 3.2). If the monetary authorities increase the rate of interest charged for government debt to cover future losses (thus increase their risk premiums) the government might “lose” some part of its fiscal Seigniorage (indicated by the dotted line in graph 3.2). Besides this redistribution of the revenue from Seigniorage, at all times, the monetary Seigniorage is not altered. Every increase in the fiscal Seigniorage of the government will only reduce the central banks share of the monetary Seigniorage, and the whole enterprise will be found to be a zero sum game. The graph below is based on the linear demand function present in equation (1.54). It draws the part of fiscal Seigniorage which the central bank transfers to the treasury. The part of the fiscal Seigniorage generated is not included. The later is the difference between the old optimum (blue point on the straight line) and the amount by which the central bank's revenue is reduced due to the loss on the government debt.

Graph 3.2: Dishonest Seigniorage for a credit currency.



Reference: Illustration based on own research.

If the loss caused by the default is too big the revenue from Seigniorage may even be negative. If the interest rate is at the Seigniorage maximizing level, raising the interest rates to increase the central banks returns is not an option. The central bank may absorb a part of the loss, if there were some reserves accumulated in the past. However, if there are no reserves, or the reserves are depleted, the government is forced to raise funds by ordinary taxation and inject equity into the central bank to avoid its default. The government is not forced to do so. If the central bank defaults, it would be “as if” the government would be printing money. In the latter case the monetary system would be altered. The currency supply would be exogenously determined by the government's spendings, and therefore the economy would be drifting in a fiat currency system. This is a question of mixed monetary systems, which are no pure ideal-types. This point will be dealt with in chapter 4 .

The other possibility would be defaulting on bonds which are held by private lenders. This is however possible in all monetary systems and will be addressed in chapter 5 .

The possibility of default is not limited to the government. In a credit currency system there is the additional possibility, which has not been addressed so far, of a default on central bank loans by private lenders. This possibility will be examined with respect to the governments Seigniorage in chapters 6 and 7 .

4 Mixed monetary systems.

So far the analysis has been restricted to “ideal types” of monetary systems in the sense of Max Weber's *Idealtypus*. Three systems have been distinguished which are somehow extreme cases and hardly found in reality. In reality monetary systems are usually a mixture of the distinguished ideal types. The possible historical examples are manifold, paper based currency systems which are to some extent backed by gold, or credit currency systems in which the government is allowed to print

currency. However, different mixed systems, combining features from different ideal type monetary systems, will – in respect to the revenue from Seigniorage – be determined by the predominant type. Due to the high number of possible combinations the argumentation will be restricted to two cases.

Case 1 – A commodity currency with additional fiat currency: Imagine a government issuing additional paper currency in a commodity currency system. The paper currency has a fixed exchange rate regarding the commodity currency. If the government owns the reserves to back the paper currency the system will behave as if it would be a pure commodity currency system. If the government however issues paper beyond its reserves two things may happen. If the government can be forced to convert one into the other the government will gain Seigniorage by increased issuance of paper and lose the additional Seigniorage due to reserve drains. Before reserve drains start to strengthen the government may generate some additional (dishonest) Seigniorage similar to issuing debased coins. Alternatively the government may abolish the convertibility. In this case a fiat currency system is established. Thus we find two monetary systems at a time each governed by its own laws.

Case 2 – A credit currency with additional fiat currency: Assume a government issuing paper currency in a credit currency system. There are two possible ways to do so. The government may literally print currency, or borrow it from the central bank and default on these loans. Either way the government issues paper currency and increases its returns from Seigniorage, e.g. to buy goods and services or to repay privately owned debt. At the same time it alters the (private) demand to borrow currency from the central bank by supplying non borrowed currency. For every unit of currency issued the government receives Seigniorage and loses future interest payments on the central bank's loans. As long as the government issues less currency than is demanded the system behaves like a credit commodity system, even though some part of future Seigniorage has already been capitalized. If it is discounted correctly the present value of the revenue from Seigniorage is, however, not altered until the government issues more currency than would have been demanded in form of loans. In case the central bank absorbs additional currency by paying interest on surplus reserves the government would lose the gained revenue from Seigniorage in form of interest payments. This can be seen as another form of government debt. The government spends a certain sum which the central bank has to pay in terms of interest payments. If discounted correctly the present value of future payment will be equal to the gained Seigniorage. In case the central bank does not absorb the additionally supplied currency the central bank will lose control of the interest rate. In other words the fiat currency system will dominate. The government controls the stock of currency exogenously and the interest rate is determined by the market.

Thus if the government prints paper currency to repay its debt, it enters into a mixed monetary system. This has already been described by Ricardo. He highlighted that if the government issues

the currency through a central bank (which he refers to as paper money) it is supposed to stick to good banking principles. Therefore he concluded that a national central bank should be independent of the government to avoid an unfortunate return to a fiat based system (in his words a forced government paper).

„It is said that Government could not be safely entrusted with the power of issuing paper money; that it would most certainly abuse it; and that, on any occasion when it was pressed for money to carry on a war, it would cease to pay coin, on demand, for its notes; and from that moment the currency would become a forced Government paper. There would, I confess, be great danger of this, if Government—that is to say, the Ministers—were themselves to be entrusted with the power of issuing paper money. But I propose to place this trust in the hands of Commissioners, not removable from their official situation but by a vote of one or both Houses of Parliament“ (Ricardo 1824, p. 10 f.).

In other words, if monetary systems are mixed there is no fundamental change in the laws governing the Seigniorage. A mixed monetary system can be understood employing the insights of the independent ideal types.

5 Government debt and Seigniorage.

A government realizes the dishonest Seigniorage in a credit currency system by defaulting on its debt. We have discussed the possibilities of a government defaulting on debt owned by the central bank in chapter 3 and 4 . Now consider privately owned government debt. The government can issue debt by taking up loans from private banks or by issuing government bonds. Therefore monetary policy (including Seigniorage) is always tied to fiscal policy. There is a trade-off between covering government expenses by revenue from Seigniorage or by issuing debt. In his “Plan for the establishment of a national bank” (Ricardo 1824) he argued that instead of borrowing currency at interest the government should instead issue the currency.

“It is evident, therefore, that if the Government itself were to be the sole issuer of paper money, instead of borrowing it of the Bank, the only difference would be with respect to the interest:—the Bank would no longer receive interest, and the Government would no longer pay it: but all other classes in the community would be exactly in the same position in which they now stand. It is evident, too, that there would be just as much money in circulation; for it could make no difference, in that respect, whether the 16 millions of paper money now circulating in London, were issued by Government, or by a banking corporation” (Ricardo 1824, p. 2 f.).

For this chapter the difference between bank loans and government bonds shall be neglected, and all debt be treated alike. Governments issue bonds at certain points in time and therefore at one point in time bonds of different duration can be found. The budget equation of the government in a

closed economy is then given by the government's earnings from taxes (T), the revenue from Seigniorage (S^G) and the revenue from newly issued debt (\dot{D}_G) minus the expenses for goods and service (G) and interest payments on the accumulated debt ($r^{DG}D_G$).¹¹

$$(1.57) \quad G_t + \int \{r_t^{DG} D_{G,t}\} = T_t + S_t^G + \dot{D}_{G,t} \text{ or}$$

$$(1.58) \quad S_t^G - \int \{r_t^{DG} D_{G,t}\} = G_t - T_t - \dot{D}_{G,t}.$$

Similar equations can be found in Klein and Neumann (1990, p. 210), Bofinger, Reischle and Schächter (1996, p. 53), or Phelps (1973, p. 71). The government can increase its spending by issuing new bonds (\dot{D}_t) or by increasing the revenue from Seigniorage. However the government has to take into account that maximizing the revenue from Seigniorage might effect the interest payments on past debt. The government therefore maximizes the right hand side of the equation.

$$(1.59) \quad \max\{G_t - T_t - \dot{D}_{G,t}\} \text{ or } \max\{S_t^G - \int \{r_t^{DG} D_{G,t}\}\}$$

This can be related to the monetary Seigniorage according to equation (1.1).

$$(1.60) \quad S_t = S_t^G + C - \int \{r_t^{DG} D_{G,t}\}$$

By adding the last term the first order conditions of the last chapters are altered. The old optima still maximize the governments revenue from Seigniorage, but they do not take the interest payments on government debt into account. If this is done the first order conditions are slightly altered.

The first order condition for fiat currency (1.8) has to be expanded by the first derivative of the interest payments as a share of national income with respect to inflation. The debt is measured in real terms. It does not depend on inflation therefore. If the national income does not depend on inflation (an assumption already discussed above) we get the following result.

$$(1.61) \quad k_t(\pi_t) + \delta g_{k,t}/\delta \pi_t k_t(\pi_t) + \delta g_{Y,t}/\delta \pi_t k_t(\pi_t) + (\pi_t + g_{k,t} + g_{Y,t}) \delta k_t(\pi_t)/\delta \pi_t - 1/Y_t \delta \int \{r_t^{DG} D_{G,t}\}/\delta \pi_t = 0$$

Compared to the old first order condition (1.11) a term reflecting the impact of inflation on debt payments enters.

$$(1.62) \quad \eta'_{k,\pi} = \eta_{k,\pi} + \delta \int \{r_t^{DG} D_{G,t}\}/\delta \pi_t 1/\{k_t(\pi_t) Y_t\}.$$

To simplify the result assume that the government financed at a constant rate of interest, in other words (i^{DG}) is constant. The sum of the debt is then independent of the rate of interest and can disentangle the two.

$$(1.63) \quad \eta'_{k,\pi} = \eta_{k,\pi} + \delta r_t^{DG}/\delta \pi_t \int D_{G,t}/\{k_t(\pi_t) Y_t\}.$$

The effect of the interest payments is stronger for higher levels of accumulated government debt as a share of the national income. This impact is multiplied with the derivative of the real rate of interest with respect to inflation. The question how inflation influences the real interest on government debt depends the honesty of the government. If the nominally fixed interest payment corresponds to the anticipated inflation the real rate does not vary with respect to changes in inflation. If, however, the government prints more currency and increases inflation above the

11 All variables are in "real" terms. To receive the nominal terms they have to be multiplied with the level of prices.

announced level the real interest on government debt will be reduced. The reason is a nominally fixed interest rate and an unanticipated rate of inflation. In such a case the unanticipated inflation lowers the burden of past debt and increases the current Seigniorage, but at the same time, it reduce the future revenue from Seigniorage and might thereby induce a spiral of decreasing government revenue, and it might increase the future interest payments on government debt (increasing the risk of future unanticipated inflation). A strong effect on the benefits from unanticipated inflation depends on the maturity at which the government borrows. The longer the maturity of government debt, the lower the amount which has to be rolled over currently. Thus a lag is introduced. Higher inflation will drive up prices but past interest rates are to some extent fixed. Thereby the government sacrifices part of the Seigniorage but saves on debt interest payments. Phelps (1973, p. 75) mentions this effect but ignored it in his approach. In this case the derivative of the interest on government debt with respect to inflation would be negative and suggest that the government can increase its returns by slightly inflating its debt.

It may be surprising that this result holds similar for the opportunity cost approach. The opportunity cost approach measures the Seigniorage as avoided interest payments on government debt, however, it does not take into account that changes in the interest rate influence the value of past government debt.

The first order condition for commodity currency (1.34) is altered similarly.

$$(1.64) \quad g_{B,t} k_t(\tau_t) + \tau_t \delta g_{B,t} / \delta \tau_t k_t(\tau_t) + \tau_t g_{B,t} \delta k_t(\tau_t) / \delta \tau_t - 1/Y_t \delta \{r_t^{D_G} D_{G,t}\} / \delta \tau_t = 0.$$

The revenue maximizing level of taxation is therefore changed.

$$(1.65) \quad \eta'_{k,\tau} = \eta_{k,\tau} + 1/\{g_{B,t} k_t(\tau_t) Y_t\} \delta \{r_t^{D_G} D_{G,t}\} / \delta \tau_t$$

To simplify the result assume that the government financed at a constant rate of interest, in other words (r^{D_G}) did not vary over time. The sum of the debt is then independent of the rate of interest.

$$(1.66) \quad \eta'_{k,\tau} = \eta_{k,\tau} + \int D_{G,t} / \{g_{B,t} k_t(\tau_t) Y_t\} \delta r_t^{D_G} / \delta \tau_t$$

Similar to the fiat currency the impact on the first order condition varies positively with the volume of accumulated government debt as a percentage of the national income. If the government affects the level of prices by increasing the tax on minting or by debasing its currency it is possible to lower the “real” value of the nominally fixed debt payments. The derivative of the rate of interest on government debt with respect to the Seigniorage would be negative. The government could increase its revenue by inflating its debt. In contrast to the fiat currency there is no risk of a self-perpetuation inflationary spiral (which we found in this chapter and already in chapter 2).

The first order condition for a credit currency (1.49) has to be extended similarly by adding the interest payments on government debt.

$$(1.67) \quad k_t(i_t^{CB}) + i_t^{CB} \delta k_t(i_t^{CB}) / \delta i_t^{CB} - 1/Y_t \delta \{r_t^{D_G} D_{G,t}\} / \delta i_t^{CB} = 0.$$

The optimum is therefore given

$$(1.68) \quad \eta'_{k,iCB} = \eta_{k,iCB} + 1/k_t Y_t \delta \{r_t^{D_G} D_{G,t}\} / \delta i_t^{CB}.$$

If the first order condition is again simplified by assuming the accumulated debt to be independent of the rate of interest.

$$(1.69) \quad \eta'_{k,\tau} = \eta_{k,\tau} + \int D_{G,t} / \{g_{B,t} k_t(\tau_t) Y_t\} \delta r_t^{D_G} / \delta i_t^{CB}$$

Similar to the fiat and commodity currency the revenue maximizing rate of interest is reduced if the real interest on past debt depends positively on it and vice versa. A link between the real rate on government debt and the interest rate exists only for unanticipated changes of the interest rate. A rise in the central bank interest rate will lower the value of government bonds, similar to the above findings. Like before, the effect is stronger for higher levels of accumulated government debt relative to national income.

To recapitulate: A government can be honest in its announcements for the next period, let's say a year. In this case the revenue from Seigniorage has been labeled "honest". In case of false announcements for the next period, the Seigniorage was labeled "dishonest". Besides this temporary honesty the government can additionally be honest about the long term announcements. A government may be honest in its yearly announcements, but however it may cheat on those who bought debt in the past expecting low rates of inflation for some years. This will be called the *intertemporal honesty*. Using these terms, the revenue maximizing rate of inflation, the revenue maximizing central bank rate, and the revenue maximizing minting tax are not affected by the existence of government debt, if a government restricts itself to revenues from intertemporal honest Seigniorage. Thus the government does not cheat in a period (e.g. it announce the actually pursued rate of interest for the next year), and it does not cheat over time (e.g. the interest rates which will be pursued of the next years). In case of intertemporal dishonest but temporarily honest Seigniorage (thus a government cheating on government debt) the revenue maximizing rate of inflation, the revenue maximizing minting tax, and the revenue maximizing central bank rate are increased, and the revenue from Seigniorage, it is still supposed to be called that way, is increased.

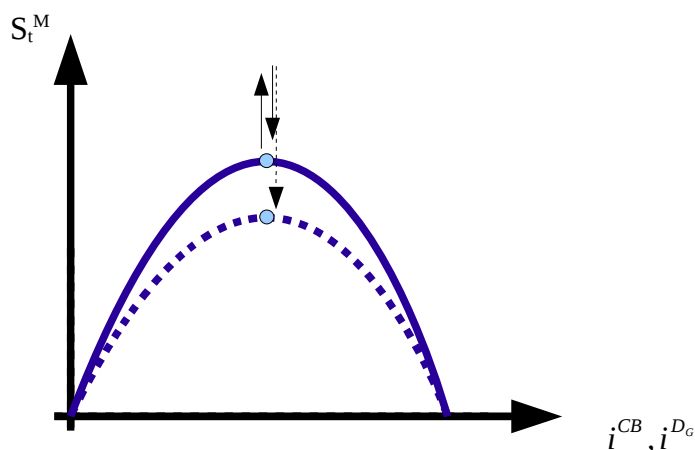
Besides this increase in Seigniorage the profitability of a debt inflation depends in all three monetary systems on the maturity of the issued government debt. If the government can inflate the debt and reduce the inflation and the future anticipated inflation before it is forced to roll over the debt, a gain can be achieved. On the contrary the shorter the maturity of a governments debt, the lower the gains. This is one of the most important reasons, why lending to governments in times of inflationary pressure usually gets more and more restricted to short maturities. The lenders reduce their expected losses in terms of unanticipated inflation.

So far we discussed the possibility of defaulting on government debt owned by the central bank (see chapter 3 and 4) and a inflating debt (in this chapter). It was shown that a government can not cheat on its debt, if the debt is held by its monetary authorities. For publicly owned debt, it has been

shown that the possibility of inflating the debt may induce higher revenue maximizing rates. The debt inflation had been labeled the intertemporal dishonest return from Seigniorage. There is also the possibility of temporal dishonest Seigniorage by defaulting on a fraction of maturing bonds in a certain period.

If the government defaults on some of its issued bonds the losses are primarily limited to the lenders. These lenders were paid with what has been called “wizard” currency, it disappeared some time after they received it. If the default is burdened solely upon the lenders the Seigniorage is temporarily increased with the lowest impact on inflation. After a one time boost of the Seigniorage revenue the economy may return to its former position (see graph 5.1).

Graph 5.1: Dishonest Seigniorage for a default on government debt (lender burden default).



Reference: Illustration based on own research.

However it is also possible that the trust of future lenders in the government's willingness to serve its debt is shaken. This could induce higher risk premium (even though the nominal interest rate is fixed), lower the demand for currency, and therefore alter the currency demand curve. This would lead to a shrunk revenue from Seigniorage (graph 5.1, dotted line).

6 Optimal Seigniorage.

So far the revenue from honest Seigniorage and the revenue from temporal and intertemporal dishonest Seigniorage has been analyzed. In these last chapters it was referred to the revenue maximizing rate of inflation, the revenue maximizing minting tax, and the revenue maximizing central bank rate. This chapter deals with the question which follows in the modern literature. After the laws of return have been studied, economist ask for the “legitimacy” or “optimality”. For medieval authors like Oresme it was somehow clear that a government should restrict itself to honest Seigniorage, and the latter should roughly be restricted to the minting cost, or in modern

terms, the maintenance cost of the financial system. This view still exists in the modern literature, even though today this question is embedded in welfare theory.

One of the most prominent economists who can be seen in Oresme's tradition is (the later) Milton Friedman. In earlier works Friedman argued for a constant rate of currency growth around 5%, which was an estimation of the optimal currency growth rate developed in chapter 1. Friedman's arguments are those developed in the same chapter. First, anticipated inflation is superior to unanticipated inflation due to the risk of an upward inflationary pressure. A government should therefore make its monetary policy transparent and stick to it, and second a 5% rule would roughly lead to a constant level of prices. This view is changed or advanced in his essay on the Optimum Quantity of Money (see Friedman 1969, p. 47 f.). Here Friedman challenged the goal of a constant level of prices. He argued that the conditions of an optimum would be the same for all goods, including money. Namely, marginal cost of production have to be equal to the price per unit. Assuming that the cost of production of a fiat currency are almost negligible Friedman concludes that the price of currency (which he identified with the nominal interest rate) should be zero. A view which Friedman kept in later publications and which is sometimes referred to as the "Chicago-rule" (See Issing 2011, p. 258).

"Our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero" (Friedman 1969, p. 34).

In other words Friedman pictures a world in which currency supply is adapted to currency demand, such that the nominal interest rate is zero. The rate of inflation is (according to the simple Fisher equation) the negative "real" rate of interest).

$$(1.70) \quad i^* = 0 = \pi^* + r, \pi^* = -r$$

This result can be translated in terms of money growth rates using the golden rule condition. The optimal rate of money growth should be zero, except for changes in liquidity preference.

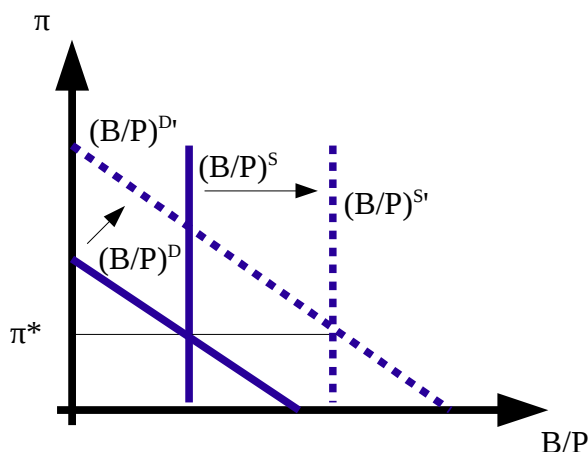
$$(1.71) \quad g_B^* = g_k, \pi^* = -g_Y.$$

To determine this equilibrium currency supply has to be adjusted to currency demand. Currency demand depends on an inflationary rise in currency supply in a twofold way. It rises because it depends positively on the level of prices and it is reduced because it depends negatively on the rate of price change. To formulate this interdependence difference equations would be necessary. This can be avoided using the (reasonable) assumption that $k(\cdot)$ depends on the rate of inflation but is independent of the level of prices. In this case the supply and demand equilibrium of currency is similar to the supply and demand equilibrium for the "real balances".¹² The real balances do not depend on the level of prices and therefore real currency demand $(B/P)^D$ is only dependent on the change of prices. The Chicago-rule demands that the real currency supply grows at a rate such that

¹² The term real balances is used for the money demand and supply divided by the level of prices.

the nominal rate of interest is zero. For an exogenous supply the currency growth rate is – as defined above – a constant (g_B). The demand depends negative on inflation. The following graph pictures such a currency supply which adapts to currency demand. For a given demand function ($(B/P)^D$ blue curve) the currency supply $(B/P)^S$ has to be such that it intersects the demand at the zero interest level. Supply is exogenous and therefore vertical (vertical blue curve). If the demand is altered (e.g. a shift in k or Y) to $(B/P)^{D'}$ (dotted line) the supply has to be adapted, here to $(B/P)^{S'}$ (vertical dotted blue curve), such that the interest rate does not rise permanently.

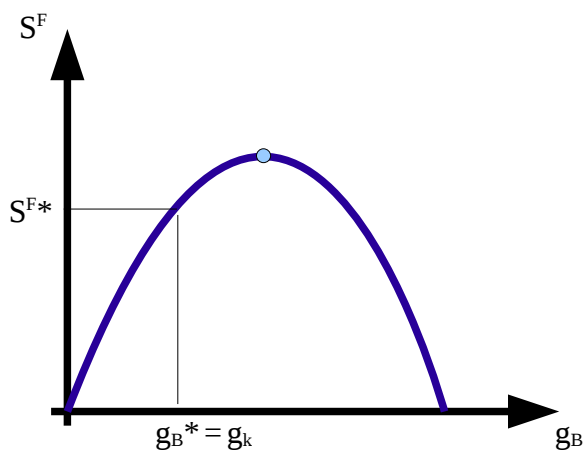
Graph 6.1: Money supply and money demand for a fiat currency



Reference: Illustration based on own research.

Combined with equation (1.4) we obtain the optimal revenue from Seigniorage. Even though the optimal level of inflation might be negative the revenue from Seigniorage can be positive. This result has been emphasized by Friedman (1971). It is hidden in analysis which are restricted to the stationary state.

Graph 6.2: Optimal seigniorage for a fiat currency regarding the Chicago-rule



Reference: Illustration based on own research.

This view is challenged by different authors who embed the analysis in a broader general equilibrium framework taking further influences into account.

Regarding the supply side James Tobin (1965) suggested to skip on of the assumption made. He introduced currency as a store of value into a general equilibrium framework (similar to that of Solow) and argued that a positive rate of inflation has positive effects on growth as inflation reduces the attractiveness of currency as a store of value. This validates the assumption that the rate of growth does not depend on the rate of inflation ($\delta g_{y,t}/\delta \pi_t = 0$). If this assumption is skipped the optimal rate of currency growth for a fiat currency depends positively on the impact of inflation on growth.

$$(1.72) \quad \eta''_{k,\pi} = \eta_{k,\pi} - \delta g_{y,t}/\delta \pi_t.$$

Therefore higher levels of inflation as suggested by the Chicago-rule might be optimal. The first order conditions for the commodity currency, see equation (1.36), and the credit currency, see equation (1.51), stay unaltered. The reason is, that a rising income would shift the currency demand function. This would increase the maximum revenue from Seigniorage but would – in both cases – not alter the first order condition.

Regarding the demand side Edmund Phelps criticized that the Chicago-rule neglects the impact of the revenue from Seigniorage on the government budget. The distortion from inflation has to be compared to that of other taxes. Thus in a world where taxes have to be raised and cannot be raised fully by non-distortionary lump-sum taxes Friedman's result does not apply. Instead, Seigniorage has to be judged in the optimal taxation tradition of Pigou and Ramsey. The optimal taxation approach compares the distortion caused by the burden connected to every tax. This highlights that a government has different sources of income. It can issue bonds, raise all sorts of taxes or issue its own currency. These different sources are therefore substitutes to each other. Phelps approach therefore tries to incorporate the effects of taxation on households demands for consumption and leisure. This is based on Ramsey (1927, p. 59), who argued that taxes should not alter the proportions in which commodities (which are rivals or complementary in consumption) are actually consumed. The distortion caused by Seigniorage is similar to that of all other taxes and therefore its height should depend on the burden of inflationary taxation compared to the burden of other taxes. In other words if currency demand is inelastic to inflation it would be a good source for government revenue and vice versa.

"This approach leads to the happy result that, as in the usual tax theory, the revenue from the inflation tax is simply the excess of the consumer's price over the producer's price (that is, price including tax less marginal cost) times the amount produced and purchased-just like the revenue from any other sort of tax" (Phelps 1973, p. 81 f.).

This approach to the optimal Seigniorage is sometimes referred to as the "revenue-smoothing

hypothesis". Which is derived from extensions of Phelps analysis by Mankiw (1987) and Barro (1979) (see Mankiw 1987, p. 327).

The approach of Tobin, Phelps and their followers is more prominent in modern textbooks. However, the question of the optimal Seigniorage has not been finally settled. Tobin's and Phelps' approach to broaden the analysis is convincing. It is highly probable that a higher Seigniorage does have further effects on utility and growth. To judge the benefits from a higher currency taxation requires to value the possible negative effects (e.g. utility losses), the positive effects (e.g. higher growth rates), and the opportunity costs (e.g. other sources of government revenue cause other utility losses). This task seems difficult and the possible approaches so manifold that until today there is no consensus on an welfare optimal Seigniorage. If, for instance, money enters into the production function Friedman's quest for a cost covering Seigniorage, independent of a general equilibrium framework, would be valid, even outside a world of lump-sum taxation. This applies also if currency is an intermediate good (see Walsh 2003, p. 138-39 or Issing 2011, p. 258).

Thus, as long as there is no consensus in these welfare-theoretical approaches on how currency should be seen (as final or intermediate good) and how it should be incorporated into the production and/or utility function the optimal level of Seigniorage cannot be finally determined.

7 Optimal Seigniorage revisited.

In the chapter above it was shown that there are good arguments in favor of a cost covering Seigniorage as well as there are arguments for a higher revenue from Seigniorage. A different way of saying this would be that all authors would agree that the revenue from Seigniorage should at least cover the cost caused by the provision of a currency. This includes old authors like Oresme and Friedman as well as the pioneers of modern welfare theory like Pigou or Ramsey.

Friedman suggests that the marginal cost of providing a fiat currency can be neglected. Even if not it seems most reasonable to assume that marginal cost are fairly small, and the cost of providing a fiat currency being for the greatest part fixed cost. This is however not true for a commodity currency or a credit currency. For a commodity money there are cost for the minting of every coin. A cost covering Seigniorage would have to be fairly positive. This argument is consensus among medieval authors like Oresme. The marginal cost for a paper based credit currency are not discussed at length with respect to Seigniorage. However it is well know from the corporate banking literature that every lending bank faces a risk. This risk stems from the borrowers default probability. For a fiat currency and a commodity currency the possibility of dishonest returns from Seigniorage are limited to the government. For a credit currency private borrowers can default on their loans. The Seigniorage (in terms of future interest payments and the debt) would for the most part be lost. Thus besides the fixed cost there is a risk of default connected to the creation of a credit currency.

This risk has to be incorporated into the marginal cost of a credit currency. If the Chicago-rule is applied to a credit currency, and the rate of interest set to zero, the private cost of acquiring currency and the social cost would differ. The credit risk would constitute an external effect. In the welfare theoretical tradition of Pigou and Ramsey taxes should be used to incorporate external effects. Ramsey's example was that the damage to streets caused by traffic should be burdened through taxes on cars and fuel. It was important to him that the external cost of traffic would be incorporated. However, this should be done without altering the usual proportions of consumption patterns. This is taken up by Phelps (1973) arguing that a positive price for currency would drive a "wedge" between the private marginal production cost of currency and the price actually charged. By the very same reasoning the cost covering Seigniorage for a credit currency has to be higher than for a fiat currency. The default risk of borrowers has to be incorporated into the price of lending by means of taxation, thus Seigniorage.

If there are fixed cost (C^{fix}) and a risk of default for the stock of money ($\Phi(B)$) the cost function for the provision of a credit currency ($C(B)$) would be given by

$$(1.73) \quad C(B) = C^{\text{fix}} + \Phi(B).$$

If the risk of a default on a unit of currency is the same for all issued units ($\Phi'(B) = \varphi$) and if the risk does not depend on the size of the stock of issued currency ($\Phi''(B) = 0$) the marginal cost are given by φ .

$$(1.74) \quad dC/dB = d\Phi(B)/dB = \Phi'(B) = \varphi.$$

This risk can be reduced if the government demands good security prior to issuing currency. This limits the risk, but it does not fully remove the risk the government faces. The minimum level of Seigniorage therefore depends on the default probability. Thus the results are altered for the credit based system. Friedman's claim of a zero interest rate is not true for a credit currency. The rate of interest to be charged needs to include the risk. If the physical cost of production per unit of currency are negligibly small the optimal central bank rate would be the default probability.

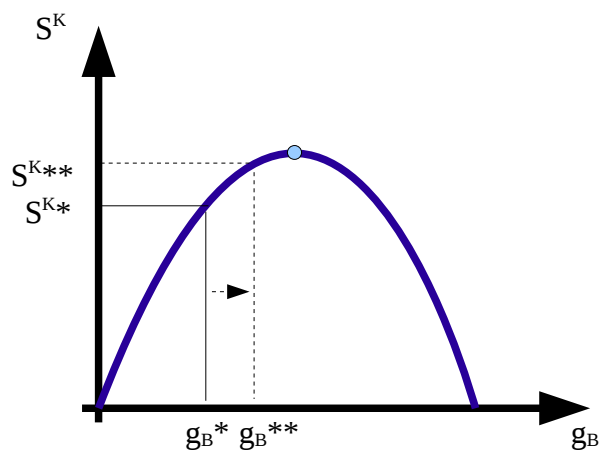
$$(1.75) \quad i^{\text{CB**}} = i^{\text{CB*}} + \varphi, \pi^{**} = \varphi - r$$

The optimal currency growth rate would be altered similarly. If the golden rule applies it would be given as the change in liquidity demand and the marginal default risk.

$$(1.76) \quad g_B^{**} = g_k + \varphi, \pi^{**} = \varphi - g_Y$$

Following these results the optimal (in the sense of cost covering) revenue from Seigniorage would be increased compared to the Chicago-rule.

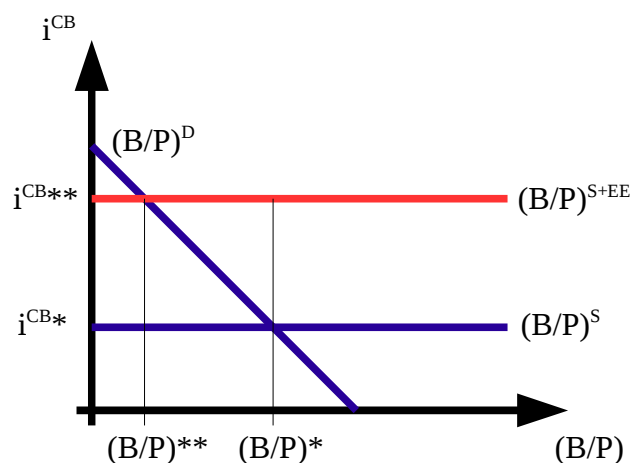
Graph 7.1: Cost covering Seigniorage for a credit currency including default risk.



Reference: Illustration based on own research.

The cost covering level of money growth and the central bank rate are therefore higher. This can be drawn in a supply and demand diagram for real balances similar to graph 6.1. The supply of real balances is now horizontal. The central bank offers as much currency as demanded against good collateral to established the desired rate of interest. The blue horizontal is the (private) marginal cost covering supply curve, $(B/P)^S$. The red horizontal is the supply curve derived for the social marginal cost, $(B/P)^{S+EE}$.

Graph 7.2: Cost covering privat and social money supply.



Reference: Illustration based on own research.

For an interest rate setting and currency accommodating central bank the interest rate covering the private cost (i^{CB*}) leads to a social loss if there is a positive default probability. The social welfare loss depends on the risk or probability of such losses (φ). Thus for a credit currency there will be no social optimum if the Seigniorage covers the private marginal cost of the currency supplying.

Instead the Seigniorage has to be higher, depending on the risk of a default on currency lending, or in other words a bank failure as it is usually a banking system which borrows currency in a monetary system built upon credit. The government or the monetary authorities should therefore at all times put a higher burden on borrowers than it is suggested by the (fiat currency) Chicago-rule. The aim of the European central bank of positive rate of inflation could be justified, if the central bank authorities judge the default risk to be greater than the rate of growth. A positive tax on borrowing indeed reduces borrowing for a given demand function, similar to those assumed here. This result would increase the social welfare as it incorporates the social cost into the price for private borrowing. This is the same social welfare gain from taxing pollution and for instance reducing the number of cars sold.

8 Summary

In every monetary system there is a direct income for the government if the monopoly of providing the currency is imposed. We discussed these returns for the individual systems as well as mixed systems in chapters 1 - 4 . For a commodity based currency this income is gained from the fee at the mints or the fees for the monopoly of exploiting the silver or gold mines. For a fiat currency this income is gained from the possibility of paying goods and services by printing currency. For the credit system this income is received in terms of interest payments for borrowing the currency from the monetary authority. This revenue cannot be increased unlimited. If the “cost” connected to a currency are increased it is to be assumed that the currency is going to be avoided to some extent. The citizens will reduce their demand for currency, at least to some extent.

A similarity between the sources of Seigniorage was their connection to the level of prices. The higher the fee on mints the higher the value of a coin and therefore the higher the level of prices. The more currency is printed the more the price level is raised, and lastly the higher the rate of interest in a credit system (in a long run perspective) the higher the growth of the level of prices.

The cost of Seigniorage is burdened upon different groups in the different monetary systems. If fiat currency is printed all owners of paper currency bear the cost due to the general rise in the price level. The tax on gold brought to the mint or the interest charged for supplying a credit currency is burdened upon those who demand additional currency.

Besides this “honest” source of Seigniorage there is a second way to increase the Seigniorage returns in all three monetary systems, the (temporal) dishonest Seigniorage. The commodity currency can be debased, the fiat currency can be inflated unanticipated, and the credit currency can be defaulted upon. This short run oriented boost of government revenue from Seigniorage has at all times been attacked heavily by some group of authors. The similarity between defaulting on government debt, unanticipated inflation and debasement has at latest been mentioned by Reinhart

and Rogoff, even though they do not treat this question extensively.¹³ Oresme attacks the debasement of coins as being theft while he admits that the government may charge a fee on coinage, thus even in the 14th century the two sources of Seigniorage were known and distinguished (see Schefold 2004, p. 86 ff.). This is true for modern authors as well, who refer to unanticipated inflation as fooling or surprising the nations citizens (for instance see Bailey 1956, p. 110, Bofinger, Reischle, and Schächter 1996, p. 69 f., Friedman 1969, p. 47 f.), and Issing 2011, p. 260 f.). The cost of such an dishonest boost of revenue from Seigniorage are paid by many while some may profit. If the commodity currency is debased and a differentiation of coins forbidden those who owned heavy, old coins bear the cost (which they may try to avoid of course). The same is true for fiat currency. Thus the cost are wide spread. In case of a default the group of people bearing the cost is – in contrast to the former two currency systems – at least on a first step limited to those who lend the currency, the monetary authorities. If this loss is taken by the central bank it reduces the return from Seigniorage. Defaulting on its currency does therefore not increase a governments returns from Seigniorage in a credit currency.

While a government can not increase its revenue from defaulting on its credit currency (without altering the dominant monetary system to a fiat currency system) it can default on other forms of debt. The gain from inflating the issued debt depend on the maturity of the issued debt and may under certain circumstances be beneficial. However it seems more difficult in a fiat currency system.

As shown in chapters 6 and 7 optimal levels of Seigniorage can be derived regarding cost covering approaches as well as welfare theoretical approaches (of Tobin or Phelps) for all three monetary systems. Even thou the question of the optimal Seigniorage is not finally answered in the literature a most peculiar insight was gained. The cost covering level of Seigniorage is not the same for all sorts of paper money. While Friedman's goal of a zero interest rate may apply for a fiat currency, it does not apply for a paper based credit currency. The cost covering Seigniorage for a credit currency has to cover for the default risk of borrowers. A positive central bank would be optimal then, if the default risk is greater then the rate of economic growth.

The second difference is the inflationary upward pressure for a fiat currency. A loss in the value of a fiat currency due to increased printing of notes, thus an increasing government deficit, affects all notes at a time. Inflation therefore spreads quickly and most easily. Sharing the burden among many may be judged as an advantage, however there is a great disadvantage too. Inflationary finance leads to a reduction in the regular tax revenue and a general reduction in currency demand. Thus it automatically increases the gap between taxes and expenditures, it lowers the future revenue from

13 “Early on across the world, as already noted, the main device for defaulting on government obligations was that of debasing the content of the coinage. Modern currency presses are just a technologically advanced and more efficient approach to achieving the same end” (Reinhart and Rogoff 2009, p. xxxiii).

Seigniorage for lower levels of currency demand and therefore increases the need for further inflationary pressure. This risk of acceleration has been mentioned in chapter 1 and chapter 5. This tendency has already been mentioned for instance by Keynes and Hawtrey.

Hawtrey (1930, p. 249 f.) states: „It is part of the inherent vice of inflationary finance that, once started, it is difficult to stop. The prices to be paid by the Government for the goods and services it needs rise almost immediately. The corresponding rise in the yield of the revenue occurs only after a considerable interval [...]. The deficit to be covered by borrowing is thus increased. [...] Thus, as soon as inflation has started, the deficit to be met by inflationary means is likely to become rapidly bigger. And deterioration will soon be frightfully accelerated by loss of confidence in the currency“ (Hawtrey 1930, p. 250). Therefore he concludes that: „Inflation is a deadly blight; once it has gained a hold, it will poison the whole economic system, and can only be eliminated, if at all, at the cost of exhausting efforts“ (Hawtrey 1930, p. 250).

Keynes differs between different inflationary phases and he is more optimistic about the possibility to avoid an accelerating pressure (see Keynes 1923, p. 45 ff.). If taken to the extreme however he agrees that an accelerating inflationary finance will: “In the last phase, when the use of the legal-tender money¹⁴ has been discarded for all purposes except trifling out-of-pocket expenditure, [inflationary taxation has] at last defeated itself” (Keynes 1923, p. 44).

The risk of an accelerating inflation seems to be a peculiar feature of fiat currency systems. Historical observations suggest that there were no such accelerating inflationary pressures in commodity currency systems despite that ongoing debasements are documented by many authors. Schefold (2004, p. 68) puts it this way: "An inflation involving paper money easily turns into hyperinflation and quickly escalates, making a currency reform unavoidable. Medieval rulers on the other hand, even entire dynasties, financed themselves for centuries by re-minting the currency. And yet the system neither collapsed, as with modern hyperinflation, nor was Seigniorage negligibly small, as with the stamping of gold under the gold standard."

The theoretical basis of this insight has been provided above. The laws governing the return from Seigniorage produce no accelerating effect for a commodity currency system. The same seems to hold true for the credit based system. Reinhart and Rogoff (2009, p. 32) argue that “[a]ll in all, debt intolerance¹⁵ need not be fatal to growth and macroeconomic stability, but it is still a serious impediment.” They do not downplay the possible negative impacts of a government default, but highlight that most banking crises are associated with much more severe economic downturns (see Reinhart and Rogoff 2009, p. 165) and that the difficulties of escaping a sovereign debt crises may

¹⁴ Legal-tender money is Keynes expression for what has been labeled currency here.

¹⁵ Debt intolerance is defined as the extreme duress caused by external debt levels typically involving a vicious cycle of loss in market confidence, spiralling interest rates on external government debt, and political resistance to repaying foreign creditors (see Reinhart and Rogoff 2009, p. 21).

be difficult, “[h]istory tells us, however, that graduation from recurrent banking and financial crises is much more elusive” (Reinhart and Rogoff 2009, p. 171). Thus a theoretical reasoning why inflationary spirals may take off has been provided above. Furthermore it has been shown that this risk is limited to a fiat currency and does not apply to a commodity currency or a credit currency.

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