Modeling Financial Instability

Toichiro Asada
Faculty of Economics, Chuo University
742-1 Higashinakano, Hachioji, Tokyo 192-0393, Japan
E-mail : asada@tamacc.chuo-u.ac.jp

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Abstract

In this paper, we reconsider Minsky's 'financial instability hypothesis' from the point of view of mathematical macrodynamic modeling. We start from a simple prototype small scale model of private debt and income with fixed prices. This system is similar to the Lotka-Volterra predator-prey system, in which private debt plays the role of predator and income plays the role of prey. Then, we extend the model step by step by introducing variable prices, inflation expectation, public debt and budget equation of the consolidated government. We also study the effect of macroeconomic stabilization policies by means of monetary and fiscal policies.

Key words : Minky's financial instability hypothesis, Lotka-Volterra system, private debt, income, public debt, inflation expectation, stabilization policy

1. Introduction

In contrast to the mainstream (Neoclassical) teaching that the capitalist economy is inherently stable, a representative American Post Keynesian economist Hyman Minsky(1919 – 1996) proposed the 'financial instability hypothesis', which means that the financially-dominated capitalist economy is inherently unstable (cf. Minsky 1975, 1982, 1986). It seems that the credibility of Minsky's hypothesis has been rapidly increasing in recent times, since we experienced the Japanese long term deflationary depression in the 1990s and the 2000s after the bubbly prosperity in the 1980s, the Latin American and Asian currency crises in the 1980s and the 1990s, and the global financial crises in USA and Europe that were initiated by the so called 'subprime mortgage crisis' of the US economy in 2008 after the prosperous period of the US economy in the 1990s and the 2000s. It looks as if the world economy in recent 30 years has been faithfully tracing Minsky's scenario of financial instability hypothesis.

In this paper, we shall try to present some formal mathematical models that are inspired by Minsky's idea. This paper is organized as follows. In section 2, we restate Minsky's 'financial instability hypothesis' by referring to Minsky's own writings. In section 3, we interpret Asada's(2001) very simple fixed price dynamic model that is expressed by a Lotka-Volterra type two-dimensional system of nonlinear differential equations, which can reflacts Minsky's perspective of financially-driven business cycle that is called 'Minsky cycle'. In section 4, we interpret the extended three-dimensional model by introducing price flexibility following Asada's(2004) procedure, and show that the *increase* of price flexibility tends to *destabilize* rather than stabilize the macroeconomic system in this model contrary to the teaching of the mainstream macroeconomics.

Minsky's 'financial instability hypothesis' means that 'the financially-driven capitalist economy is inherently unstable'. But, according to Minsky(1986), the stability/ instability of the system is by no means independent of the macroeconomic policies of the government and the central bank. That is to say, the appropriate policy mix of the fiscal and monetary policies by the government and the central bank can contribute to 'stabilize an unstable economy' if we quote from the title of Minsky's(1986) book. In sections 5 and 6, we present the further extended higher dimensional models that can study the effects of monetary and fiscal stabilization policies analytically. These models are designed to contribute to give the definite answer to the question concerning what kind of policy mix is appropriate or inappropriate from the point of view of macroeconomic stabilization. We also argue that the analytical conclusions that are derived from these models are quite consistent with the experience of the Japanese

economy under the serious deflationary depression in the 1990s and the 2000s, which is called the 'lost twenty years'. Section 7 is devoted to some concluding remarks.

2. Minsky's financial instability hypothesis restated

Minsky(1975, 1982, 1986) asserts that the process of endogenous business cycles inevitably entails the endogenous changes of the form of investment financing such that 'Hedge finance → Speculative finance → Ponzi finance'. Minsky interprets the meanings of these forms of financing as follows.

"If realized and expected income cash flows are sufficient to meet all the payment commitments on the outstanding liabilities of a unit, then the unit will be hedge financing. However, the balance-sheet cash flows from a unit can be larger than the expected income receipts so that the only way they can be met is by rolling over or even increasing debt; units that roll over debt are engaged in speculative finance and those that increase debt are engaged in Ponzi finance." (Minsky 1986, p. 203)

Under the depression process, economic agents become quite pessimistic so that they refrain from invest expenditure, and the repayment of the existing debt and hedge finance dominate in such an environment. As the amount of debt decreases, investment activities become more vigorous, and more bold speculative finance becomes dominant. At the last stage of the prosperity, economic agents become excessively optimistic and they become to be engaged in the Ponzi finance that heavily relies on large amount of borrowing. However, the default of a part of economic agents triggers off the financial crises and the economy rushes into the serious depression. Under the depression process, the hedge finance becomes dominant again. In this way, the waves of the 'pessimism' and the 'optimism' are repeated reciprocally. This is the essence of Minsky's 'financial instability hypothesis'. Minsky's hypothesis forms a striking contrast to the mainstream 'rational expectation hypothesis'. Let us quote from Minsky's writings again.[1]

"Financing is often based on an assumption 'that the existing state of affairs will continue indefinitely'(GT p. 152), but of course this assumption proves false. During a boom the existing state is the boom with its accompanying capital gain and asset revaluations. During both a debt-deflation and a stagnant recession the same

[1] The abbreviation 'GT' in this quotation means Keynes'(1936) 'general theory'.

conventional assumption of the present always ruling is made; the guiding wisdom is that debts are to be avoided, for debts lead to disaster. As a recovery approaches full employment the current generation of economic soothsayers will proclaim that the business cycle has been banished from the land and a new era of permanent prosperity has been inaugurated. … But in truth neither the boom, nor the debt-deflation, nor the stagnation, and certainly not a recovery of full-employment growth can continue indefinitely. Each state nurtures forces that lead to its own destruction." (Minsky 1975, p. 128)

Such a typical scenario of the process of financially-driven business cycles that was presented by Minsky is called 'Minsky cycle'.

3. Minsky cycle as a transformed Lotka-Volterra system

Minsky himself did not formulate formal mathematical models that reflect his basic idea on the 'financial instability hypothesis'. However, a lot of mathematical models that were inspired by Minsky's idea have been produced up to now since the seminal paper by Taylor and O'Connell (1985) was published.[2] In this section, we shall summarize the essence of the model that was presented by Asada(2001) as a typical example of such mathematical models. This approach interpret the 'Minsky cycle' as a transformed Lotka-Volterra system of mathematical biology, which is based on the dynamic interaction of predator and prey.[3]

The reduced form of a system of equations that was presented by Asada(2001) consists of the following two-dimensional system of nonlinear differential equations.

(i) $\dot{d} = f_1(d, y)$

(ii) $\dot{y} = f_2(d, y; \alpha)$; $\alpha > 0$ (1)

where a dot over the symbol is time derivative, and the meanings of the symbols are as follows.

$d = D/K =$ private debt-capital ratio. $y = Y/K =$ income-capital ratio. $D =$ stock of

[2] Following examples are some of such works. Arena and Raybaut(1998), Asada(2001, 2004, 2006), Bhaduri(2011), Charles(2008a, 2008b), Charpe, Chiarella, Flaschel and Semmler(2011), Delli Gatti and Gallegati(1995), Foley(1987), Jarsulic(1990), Keen(2000), Kuroki(1994), Pally(1996) and papers in Semmler(ed.)(1989). Nasica(2000) provides an excellent survey of the related topics.

[3] See Gandolfo(2009) chap. 23 as for the exposition of the Lotka-Volterra system. Goodwin(1967) is very famous contribution as a quite interesting application of this system to macrodynamic economic theory.

the real debt of the private firms. $K =$ real capital stock that is owned by the private firms. $Y =$ real national income. $\alpha =$ parameter that reflects the adjustment speed of the disequilibrium in the goods market. Variable y is used as a surrogate variable that reflects the degree of the utilization of capital stock and the rate of employment (1 – rate of unemployment) in the labor market. In this simplified version of the model, the fixed price is assumed, and the government's economic activity and international transaction are abstracted from.[4]

Furthermore, each partial derivative in this model becomes as follows because of some economic reasons (cf. Asada 2001).

$$f_{11} = \partial f_1 / \partial d < 0,\ \ f_{12} = \partial f_1 / \partial y > 0,\ \ f_{21} = \partial f_2 / \partial d < 0,$$
$$f_{22} = \partial f_2 / \partial y > 0,\ \ \underset{(-)}{f_{11}}\underset{(+)}{f_{22}} - \underset{(+)}{f_{12}}\underset{(-)}{f_{21}} > 0 \qquad (2)$$

Eq. (1) (i) can be derived from the fact that the firms' investment expenditure that exceeds the firms' internal finance must be debt-financed.[5] Eq. (1) (ii) expresses the 'quantity adjustment' process in the goods market disequilibrium, which implies that the rate of utilization of the capital stock fluctuates according as the excess demand in the goods market per capital stock is positive or negative. In both equations (1) (i) and (1) (ii), the following investment function is incorporated.

$$g = g(y, \rho - \pi^e, d)\ ;\ g_y = \partial g / \partial y > 0,\ \ g_{\rho - \pi^e} = \partial g / \partial(\rho - \pi^e) < 0,$$
$$g_d = \partial g / \partial d < 0 \qquad (3)$$

where $g = \dot{K} / K =$ rate of investment (rate of capital accumulation), $\rho =$ nominal rate of interest, $\pi^e =$ expected rate of price inflation, and $\rho - \pi^e =$ expected real rate of interest. Asada(2001) derived this type of investment function with debt effect from the expected profit maximization behavior of the firms by using the 'principle of increasing risk' due to Kalecki(1937, 1971), which means that firms' risk increases as the investment expenditure increases.

In the simplified model in this section, the fixed price is assumed so that we have $\pi^e = 0$. Furthermore, in this section, we assume that the central bank always acts to keep the nominal rate of interest ρ to be constant, which means the lack of the active monetary policy. Therefore, π^e and ρ do not enter into the system of equations (1) as variables.

In this model, the change of the private debt-capital ratio $\dot{d}$ becomes an *increasing*

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[4] Later, we shall interpret the extended models that can treat the price movement and macroeconomic stabilization policies by the government including the central bank.
[5] It is assumed that the firms are debtors and the capitalists are creditors, and the issues of new shares are abstracted from.

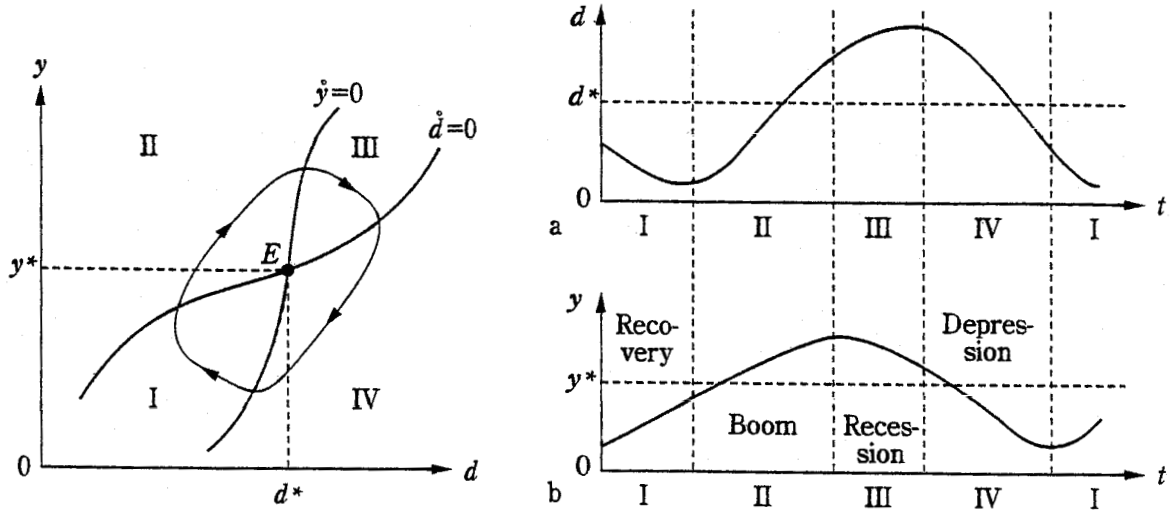


Figure1. Emergence of Minsky cycle
(Source : Asada 2001, P.81)

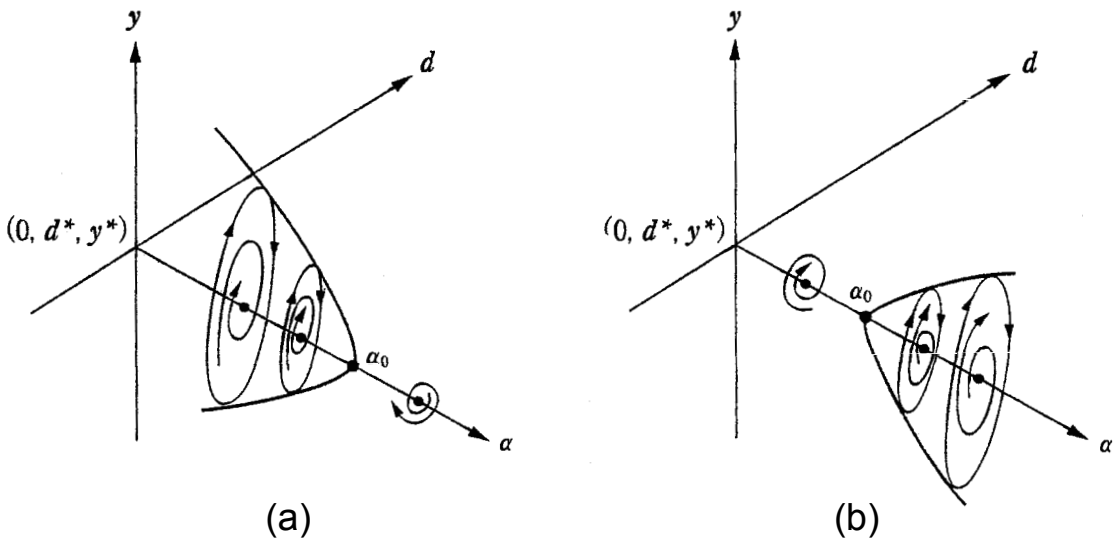


Figure2. Subcritical Hopf bifurcation(a)
and supercritical Hopf
bifurcation(b)
(Source : Asada 2001, p. 83)

function of the income-capital ratio y ($f_{12} > 0$), because the increase of y induces the increase of the debt financing through the rise of the investment expenditure g. On the other hand, the change of the income-capital ratio $\dot{y}$ becomes a *decreasing* function of the debt-capital ratio d ($f_{21} < 0$), because the increase of d induces the decrease of the investment expenditure, which contributes to the reduction of the excess demand in the goods market through the decrease of the effective demand. These characteristics imply that we can consider the system of equations (1) as a transformed 'Lotka-Volterra system', in which the variable d plays the role of predator and the variable y plays the role of prey.

Asada(2001) proved that the cyclical fluctuations occur and in particular, closed orbits around the equilibrium point exist at some range of the parameter value α by means of the Hopf bifurcation theorem.[6] Figure 1 illustrates a typical closed orbit of income and private debt that is produced in this system and time trajectories of two variables corresponding to four phases of the closed orbit. Cyclical fluctuation of income and private debt in Figure 1 can be considered as a typical example of 'Minsky cycle'.

Figure 2 illustrates two types of bifurcation that can emerge in this system. Figure 2 (a) is an example of 'subcritical' Hopf bifurcation. In this case, the closed orbit exists in the range of the parameter value α at which the equilibrium point is locally stable and the closed orbit itself is unstable. Figure 2 (b) is an example of 'supercritical' Hopf bifurcation. In this case, the closed orbit exists in the range of the parameter value α at which the equilibrium point is locally unstable and the closed orbit itself is stable.

Both of the above mentioned two types of Hopf bifurcation are economically meaningful. The 'subcritical' case corresponds to the 'corridor stability' in the sense of Leijonfufvud(1973), which means that this system is immune from small shocks that can contain the initial position inside the 'corridor'(stable region), but it is vulnerable to large shocks that leave the initial condition outside the 'corridor'.[7] In the 'supercritical' case, the time trajectory of the variables that starts from the initial condition other than equilibrium point converges to the limit cycle, and the cyclical fluctuations persist indefinitely.

4. An extension to the model with flexible prices

In this section, we introduce the price flexibility into the model in the previous section following Asada(2004)'s procedure. We can formulate this extended model by means of

[6] See Gandolfo(2009) chap. 24 as for the exposition of the Hopf bifurcation theorem.
[7] See also Asada(2010) as for the interpretation of the 'corridor stability'.

the following three dimensional system of nonlinear differential equations.

(i) $\dot{d} = f_1(d, y, \pi^e; \varepsilon)$; $\varepsilon > 0$

(ii) $\dot{y} = f_2(d, y, \pi^e; \alpha)$; $\alpha > 0$

(iii) $\dot{\pi}^e = f_3(y; \gamma, \varepsilon)$; $\gamma > 0$ (4)

where the meanings of the symbols d, y, and α are the same as those in the previous section, and the meanings of other symbols are as follows. $\pi^e =$ expected rate of price inflation. $\varepsilon =$ parameter that reflects the price adjustment speed. $\gamma =$ parameter that reflects the adjustment speed of price expectation. Furthermore, the following properties of the partial derivatives as well as a set of inequalities (2) in the previous section are assumed.

$$f_{13} = \partial f_1 / \partial \pi^e > 0, \quad f_{23} = \partial f_2 / \partial \pi^e > 0, \quad f_{32} = \partial f_3 / \partial y > 0 \tag{5}$$

We can derive Eq. (4) (iii) as follows. In this extended model, the rate of price inflation π becomes an endogenous variable that fluctuates according as the following standard type of the expectations-augmented price Phillips curve.

$$\pi = \varepsilon(y - \bar{y}) + \pi^e \ ; \ \varepsilon > 0 \tag{6}$$

As for the price expectation formation, the following 'adaptive' expectation formation hypothesis (or the 'backward-looking' expectation formation hypothesis) is adopted.

$$\dot{\pi}^e = \gamma(\pi - \pi^e) \ ; \ \gamma > 0 \tag{7}$$

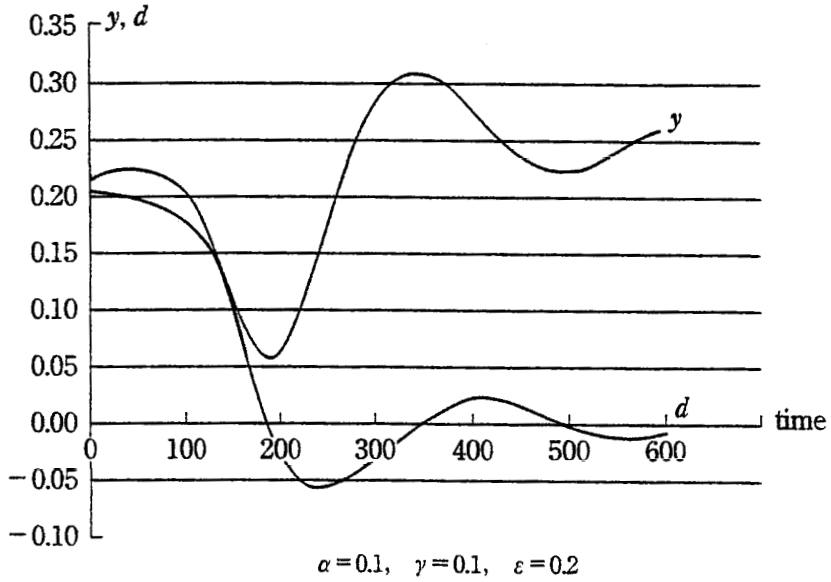
Under the absence of central bank's active commitment to inflation targeting like the Japanese economy under deflationary depression during the 1990s and the 2000s, the people who do not have any definite information are obliged to form their inflation expectation adaptively. This is the rationale of the hypothesis such as Eq. (7). Substituting Eq. (6) into Eq. (7), we have

$$\dot{\pi}^e = \varepsilon\gamma(y - \bar{y}) = f_3(y; \gamma, \varepsilon), \tag{8}$$

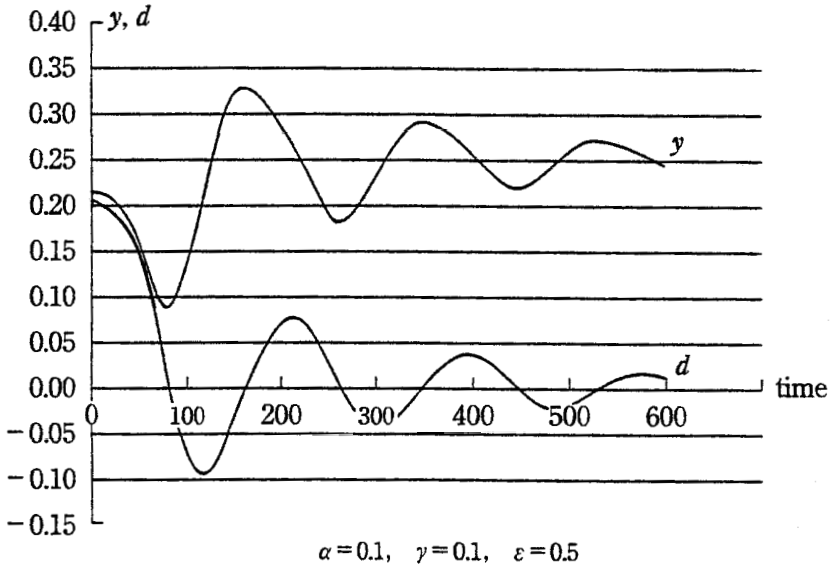
which is nothing but Eq. (4) (iii).

In this model, the nominal rate of interest ρ is still kept constant by the central bank's passive monetary policy. Nevertheless, the real rate of interest $\rho - \pi^e$ becomes a variable rather than constant through the variable expected rate of price inflation π^e, so that the variable π^e enters into equations (4) (i) and (4) (ii) through the investment function (3).

Both of the rates of change $\dot{d}$ and $\dot{y}$ become increasing functions of π^e (that is,



(a)



(b)

Figure3. Numerical illustration (1)
(Source : Asada 2004, p. 50)

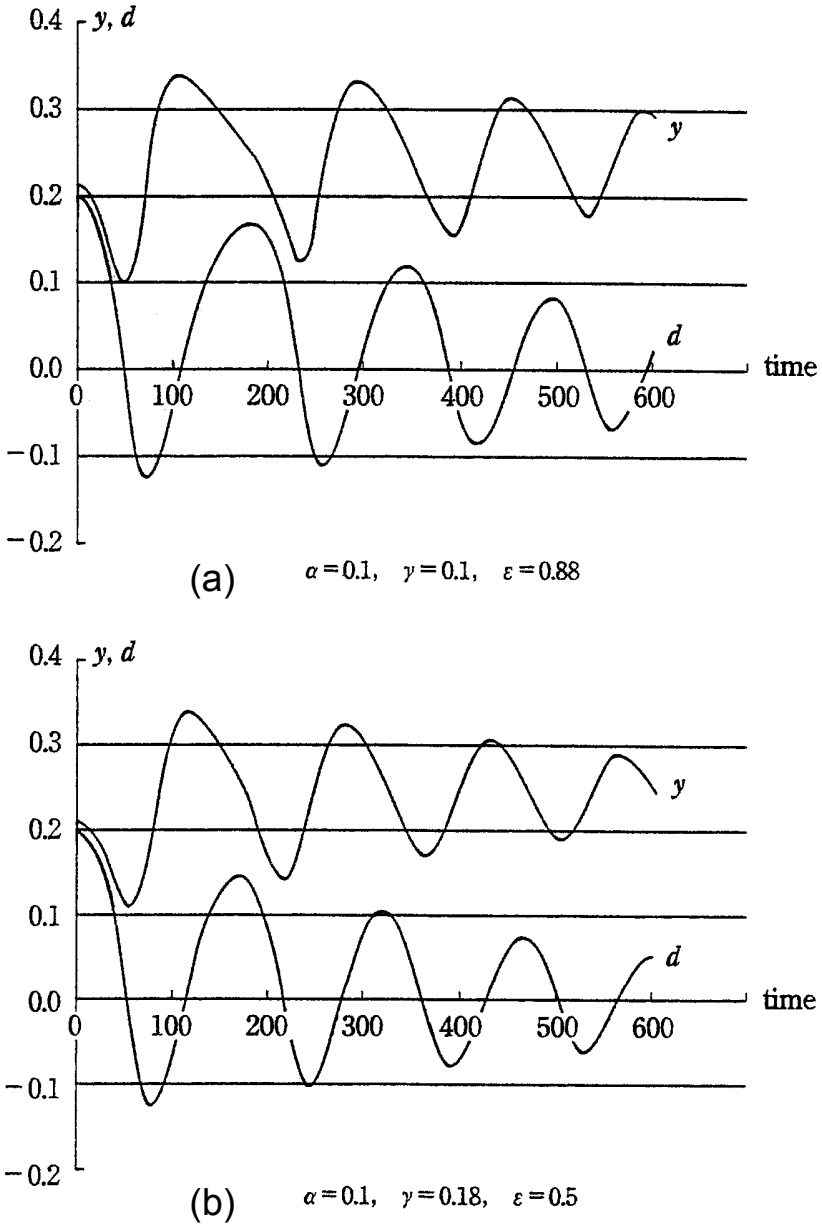


Figure4. Numerical illustration (2)
(Source : Asada 2004, p. 51)

$f_{13} > 0$ and $f_{23} > 0$) because of the following reasons. The increase of π^e implies the decrease of $\rho - \pi^e$, which induces the increase of the rate of investment and the increase of debt financing of its expenditure as well as the increase of effective demand.

We can consider the increase of the parameter value ε or γ as the increase of the 'price flexibility'. Asada(2004) proved analytically, however, that the increase of price flexibility in this sense tends to *destabilize* rather than stabilize the economic system contrary to the teaching of the mainstream economic theory, because the following destabilizing positive feedback mechanism is intensified by the increase of the parameter values ε and γ, which contributes to intensify the amplitude of macroeconomic fluctuations.

$$\pi^e \downarrow \Rightarrow (\rho - \pi^e) \uparrow \Rightarrow g \downarrow \Rightarrow y \downarrow \Rightarrow \pi \downarrow \Rightarrow \pi^e \downarrow \qquad (FM_1)$$

Figures 3 and 4 summarize the result of Asada(2004)'s numerical simulations that support the above analytical conclusion.

Incidentally, we pointed out in the previous section that the variable d plays the role of 'predator' and the variable y plays the role of 'prey' in the dynamic interaction between these two variables like the Lotka-Volterra system of mathematical biology. The role of the variable π^e is, however, not so simple. It follows from the system of equations (4) that both of the following results (A) and (B) apply.

(A) π^e is the food for d and y.
(B) y is the food for π^e.

This means that the relationship between two variables π^e and y is not simple predator-prey relationship, and these variables share their destiny. This is the source of the destabilizing positive feedback mechanism that is described schematically by the causal relationships (FM_1).

5. Modeling monetary stabilization policy

Now, we are in a position to consider how we can study the effect of monetary stabilization policy by using the analytical framework of the model that was interpreted in section 4. Let us assume that the central bank controls the nominal rate of interest by means of the following feedback monetary policy rule, which is a kind of 'Taylor rule' due to Taylor(1993).[8]

[8] Formalization of the monetary policy by means of the 'Taylor rule' became popular

$$\dot{\rho} = \begin{cases} \beta_1(\pi - \bar{\pi}) + \beta_2(y - \bar{y}) & if \quad \rho \succ 0 \\ \max[0, \beta_1(\pi - \bar{\pi}) + \beta_2(y - \bar{y}) & if \quad \rho = 0 \end{cases} \; ; \; \beta_1 > 0, \; \beta_2 > 0 \tag{9}$$

We can interpret this policy rule as a kind of 'flexible inflation targeting rule', which considers both of inflation targeting and employment targeting. In this formulation, the nonnegative constraint of nominal rate of interest is also allowed for.[9]

It will be appropriate to replace the inflation expectation formation equation (7) in section 4 with the following new equation in case in which the central bank announces the target rate of inflation $\bar{\pi}$ to the public.

$$\dot{\pi}^e = \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)] \; ; \; \gamma > 0, \; 0 \leqq \xi \leqq 1 \tag{10}$$

This equation formalizes a mixed type of inflation expectation formation hypothesis, which means that the inflation expectation formation has both of the 'forward looking' and 'backward looking' elements. The parameter ξ is the weight of the 'forward looking' element. We can interpret the value of the parameter ξ as a measure of the 'credibility' of the inflation targeting by the central bank. The more 'credible' the inflation targeting, the larger will be the value of ξ.

We obtain the following relationships substituting the equation of price Phillips curve (6) into equations (9) and (10)..

$$\dot{\rho} = f_4(y, \pi^e; \varepsilon, \beta_1, \beta_2) = \begin{cases} \beta_1\{\varepsilon(y - \bar{y}) + \pi^e - \bar{\pi}\} + \beta_2(y - \bar{y}) & if \quad \rho \succ 0 \\ \max[0, \beta_1\{\varepsilon(y - \bar{y}) + \pi^e - \bar{\pi}\} + \beta_2(y - \bar{y})] & if \quad \rho = 0 \end{cases} \tag{11}$$

$$\dot{\pi}^e = \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)\varepsilon(y - \bar{y})] = f_3(y, \pi^e; \gamma, \varepsilon, \xi) \tag{12}$$

Equations (4) (i), (4) (ii), (11) and (12) consist of the following four dimensional system of nonlinear differential equations, where ρ in equations (4) (i) and (4) (ii) is no longer constant.

(i) $\dot{d} = f_1(d, y, \pi^e, \rho; \varepsilon) \; ; \; \varepsilon > 0$

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since so called 'New Keynesian' dynamic models that are represented by Woodford(2003) and Galí(2008) took the leading position in the mainstream macroeconomics. Choice of the nominal rate of interest rather than nominal money supply as a monetary policy instrument is now common to Post Keynesian as well as New Keynesian literatures. As for the related literatures in Post Keynesian tradition, see, for example, Arestis and Sawyer(2008), Asada(2010, 2011), Asada, Chiarella, Flaschel and Franke(2010), Hein(2007), Isaac(2009) and Setterfield(2009).

[9] This type of nonnegative constraint is in particular important in Japan in the 1990s and the 2000s under long period deflationary depression and USA and European countries in the late 2000s after the serious financial crises.

(ii) $\dot{y} = f_2(d, y, \pi^e, \rho; \alpha)$; $\alpha > 0$

(iii) $\dot{\pi}^e = f_3(y, \pi^e; \gamma, \varepsilon, \xi)$; $\gamma > 0,\ 0 \leqq \xi \leqq 1$

(iv) $\dot{\rho} = f_4(y, \pi^e; \varepsilon, \beta_1, \beta_2)$; $\beta_1 > 0,\ \beta_2 > 0$ (13)

The equilibrium solution $(d^*, y^*, \pi^{e*}, \rho^*)$ of this system such that

$$\dot{d} = \dot{y} = \dot{\pi}^e = \dot{\rho} = 0 \tag{14}$$

satisfies the following relationships.

$$y^* = \bar{y},\quad \pi^{e*} = \pi^* = \bar{\pi},\quad f_1(d^*, \bar{y}, \bar{\pi}, \rho^*) = 0,\quad f_2(d^*, \bar{y}, \bar{\pi}, \rho^*) = 0$$

(15)

We can see that the equilibrium value of each variable is *independent* of the parameter values β_1, β_2, and ξ. Examining the 4×4 Jacobian matrix of the system (13) at the equilibrium point, we can prove the following proposition.[10]

Proposition 1.

(1) Suppose that all of the parameter values β_1, β_2, and ξ are sufficiently small (sufficiently close to 0). Then, the equilibrium point of the system (13) becomes locally unstable.

(2) Suppose that β_1 and β_2 are sufficiently large and ξ is sufficiently close to 1. Then, the equilibrium point of the system (13) becomes locally stable.[11]

(3) At the intermediate values of the parameters β_1, β_2 and ξ, cyclical fluctuations around the equilibrium point occur.

This proposition means that the central bank's active monetary policy that is

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[10] We omit the proof because we need some tedious long calculations to prove this proposition that is related to such a high dimensional dynamic system. We can use the method of the proof that is used in Asada, Chiarella, Flaschel and Franke(2010) extensively.

[11] The concept of stability and instability in our model is the traditional one that is applicable to the case in which all initial values of the endogenous variables are pre-determined, unlike the mainstream 'New Keynesian' models in which some variables are treated as not-pre-determined 'jump' variables. Namely, we consider that the equilibrium point is locally stable *if and only if* all the roots of the characteristic equation at the equilibrium point have negative real parts. As for the critique of the 'New Keynesian' methodology that allows for the 'jump' variables, see Asada(2010), Asada, Chiarella, Flaschel and Franke(2010), and Asada, Flaschel, Mouakil and Proaño(2011).

combined with the 'credible' inflation targeting can 'stabilize an unstable economy'.[12]

6. Analysis of the policy mix of monetary and fiscal stabilization policies

Finally, we shall try to construct a theoretical framework that can analyze the dynamic effect of the policy mix of monetary and fiscal policies. For this purpose, the following two equations play important roles.

$$M/(pK) = m(\rho)H/(pK) = \phi(\rho)y \;\; ; \;\; m_\rho = dm/d\rho > 0, \;\; \phi_\rho = d\phi/d\rho < 0 \qquad (16)$$

$$pT + \dot{B} + \dot{H} = pG + \rho B \qquad (17)$$

where $M = mH =$ nominal money stock, $H =$ nominal high-powered money, $m =$ money multiplier > 1, $p =$ price level, $T =$ real tax, $G =$ real government expenditure, and $B =$ stock of nominal public debt.

Eq. (16) is nothing but standard type of LM equation that describes the equilibrium condition of the money market. We can see, however, that the high-powered money per capital stock $H/(pK)$ in Eq. (16) becomes an endogenous variable if the central bank controls nominal rate of interest ρ rather than money stock according to the 'Taylor rule' that is expressed by Eq. (9). Eq. (17) is the budget constraint of the 'consolidated government' that includes the central bank. This equation shows that the government expenditure including the interest payment of the public debt must be financed by (1) tax, (2) new issue of the public debt, and (3) money financing of the central bank.[13]

It follows from Eq. (17) that we can obtain the following dynamic of the public debt-capital ratio $b = B/(pK)$.

$$\frac{\dot{b}}{b} = \frac{\dot{B}}{B} - \frac{\dot{p}}{p} - \frac{\dot{K}}{K} = \frac{pG + \rho B - pT - \dot{H}}{B} - \pi - g(y, \rho - \pi^e, d) \qquad (18)$$

Rewriting this equation, we have the following relationship.

$$\dot{b} = \frac{G}{K} - \frac{T}{K} - \frac{\dot{H}}{pK} - \{\rho - \pi - g(y, \rho - \pi^e, d)\}b \qquad (19)$$

If we neglect the indirect effects of the change of b on $\dot{b}$ through the changes of T/K and other variables, we have

$$\partial\dot{b}/\partial b = \rho - \pi - g. \qquad (20)$$

We can see from this result that the inequality

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[12] Needless to say, this sentence is the quotation of the title of Minsky's book (Minsky 1986). See also Asada, Chiarella, Flaschel, Mouakil, Proaño and Semmler(2010).

[13] We assume that the capitalists purchase both of government and private bonds.

real rate of interest $= \rho - \pi > g =$ real rate of capital accumulation (21)

or equivalently, the inequality

nominal rate of interest $= \rho > \pi + g =$ nominal rate of capital accumulation (22)

acts as a *destabilizing* factor of the system, and the inequality

real rate of interest $= \rho - \pi < g =$ real rate of capital accumulation (23)

or equivalently, the inequality

nominal rate of interest $= \rho < \pi + g =$ nominal rate of capital accumulation (24)

acts as a *stabilizing* factor of the system.

Needless to say, these inequalities are only partial conditions for instability/stability. Nevertheless, these conditions provide us some insight concerning instability/stability of the system. The (partial) stability condition (23) or (24) is called 'Domar condition' after Domar(1957). It must be noted that the nominal rate of interest ρ cannot become negative. Therefore, it is likely that the destabilizing 'anti Domar condition' (21) or (22) is satisfied if the rate of inflation π is negative and the real rate of capital accumulation g is negative or almost zero. In fact, this is the case of the Japanese economy in the 1990s and the 2000s under long term deflationary depression, during which the public debt-capital ratio b increased rapidly.[14]

Now, let us formulate the fiscal policy rule as follows.

$$\dot{v} = \beta_3[\theta(\bar{y} - y) + (1-\theta)(\bar{b} - b)] \ ; \ \beta_3 > 0,\ 0 < \theta < 1, \tag{25}$$

where $v = G/K$ and $\bar{b}$ is the target public debt-capital ratio that is set by the government. This is the feedback rule of the government expenditure that considers both of the stabilization of real national income (employment) and stock of public debt. The parameter θ is the weight of the relative importance of national income (employment) consideration compared with public debt consideration that is attached by the government.

The equations (16), (19) and (25) together with Eq. (13) consists of six-dimensional system of nonlinear differential equations with six endogenous variables $(d, y, \pi^e, \rho, b, v)$.[15] Equations (13) (iv) and (25) formalize the policy mix of monetary and fiscal policies in this model.

Proposition 1 in the previous section also applies to this extended system.

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[14] In Japan in this period, the nominal annual rate of interest of long term national debt was in average between 1 percent and 2 percent, and the nominal annual rate of economic growth was in average only 0.1 percent. This means that the destabilizing 'anti Domar condition' (22) was in fact satisfied in Japan in this period (cf. Asada 2011).

[15] We assume that T/K is a function of y and b. Furthermore, we suppose that both of $\dot{d}$ and $\dot{y}$ are not independent of the variables b and v.

Furthermore, we can show analytically that the *large* value of the parameter θ (value of θ that is close to 1) is a *stabilizing* factor and the *small* value of θ (value of θ that is close to 0) is a *destabilizing* factor. The intuitive explanation of this conclusion is as follows.

Suppose that the government expenditure responds actively to the changes of real national income (employment). Then, the following stabilizing negative feedback mechanism will work.

$$y\downarrow \Rightarrow v\uparrow \Rightarrow \text{(effective demand per capital stock)}\uparrow \Rightarrow y\uparrow \qquad (FM_2)$$

On the other hand, the following two destabilizing positive feedback mechanisms will work if the government expenditure responds to the changes of the stock of public debt excessively.

$$y\downarrow \Rightarrow (T/K)\downarrow \Rightarrow b\uparrow \Rightarrow v\downarrow \Rightarrow \text{(effective demand per capital stock)}\downarrow \Rightarrow y\downarrow \qquad (FM_3)$$

$$b\uparrow \Rightarrow v\downarrow \Rightarrow y\downarrow, (T/K)\downarrow, (H/p)\downarrow \Rightarrow b\uparrow \qquad (FM_4)$$

These destabilizing feedback mechanisms produce the paradoxical situation in which the *decrease* of the government expenditure per capital stock induces the *decrease* of real national income per capital stock (employment) and the *increase* of debt-capital ratio, contrary to the government's subjective intention.

This theoretical reasoning is consistent with the experience of the Japanese economy in the 1990s and the 2000s, which is called the 'lost twenty years'. We already noted that the destabilizing 'anti Domar condition' was satisfied in Japan in this period. In fact, in Japan in this period of serious deflationary depression with high unemployment, the sharp *increase* of the public debt-capital ratio b coexisted with the *decrease* rather than increase of both of government expenditure-capital ratio v and income-capital ratio y together with the *decline* of the growth rate of money stock(cf. Asada 2011). These apparently paradoxical behaviors of some key variables in Japan in recent twenty years are quite consistent with the theoretical reasoning of the model in this section, which means that the Japanese government (the Ministry of Finance) and the Bank of Japan have been carrying out quite inappropriate policy mix of fiscal and monetary policies during such a long period of twenty years, probably because they have been sticking to some irrational ideologies.

7. Concluding remarks

In this paper, we presented some dynamic models that can contribute to formalize Minsky's idea of financial instability hypothesis, and furthermore presented some

models that can contribute to the dynamic analysis of the macroeconomic stabilization policies by means of monetary and fiscal policies. We hope that these models could successfully give an answer to the question "what policy mix is appropriate to stabilize an inherently unstable economy in the sense of Minsky?"

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