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Sustainable Capitalism: Full-Employment Flexicurity Growth with Real Wage Rigidities

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Abstract

In this paper we present a model of flexicurity capitalism that exhibits a second labor market with the government as an employer of first resort, where all workers not employed by firms in the private sector find meaningful employment. We show that the model exhibits a unique interior steady state which is asymptotically stable under real wage adjustment dynamics of the type considered in Blanchard and Katz (1999), and under a type of Okun's Law that links the level of utilization of firms to their hiring and firing decision. The introduction of a company pension fund can be shown to contribute to the viability of the analyzed economic system. However, when credit is incorporated in the model, in place of savings-driven supply side fluctuations in economic activity, investment-driven demand side business cycle fluctuations (of a probably much more volatile type) can take place.

Keywords: Flexicurity, employer of first resort, Solovian growth, company pension funds,

sustainability.

JEL CLASSIFICATION SYSTEM: E3, E6, H1

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1 Introduction

In the US unemployment rates had been relatively low until the beginning of the financial crisis in the late 2000's, in contrast to Europe where a great many countries have been suffering from high and persistent unemployment over the last decades. But there are also European countries that were successful in maintaining high employment rates. These are, on the one hand, the Netherlands and the Nordic Welfare states, such as Denmark, Finland and Sweden, and, on the other hand, Great Britain and Ireland. While Great Britain and Ireland have pursued an Anglo-Saxon approach with respect to its economic policy, mainly characterized by flexible hiring and firing conditions and by few social spending, the Nordic Welfare states and the Netherlands have followed a different policy. The latter allow flexible hiring and firing, too, but they have adopted high standards of social security. Thus, these countries demonstrate that flexibility and security need not be contradictory but may well be compatible and that social security does not necessarily lead to high unemployment rates or instability of the economic system. Often, the Nordic welfare system is referred to as the flexicurity model, with the term flexicurity obtained by merging the terms flexibility and security.

It is in particular in public debates, that the flexicurity model has attracted great attention, although there is no clear consensus on its definition (cf. Zhou (2007). According to Wilthagen (1998) the concept of flexicurity was launched by the sociologist and member of the Dutch Scientific Council for Government Policy Hans Adriaansens in speeches and interviews. According to Adriaansens flexicurity means a shift from 'security within a job' toward 'security of a job' (cf. Wilthagen, 1998, p. 13). In any case, an important aspect as regards flexibility on the labor market is that there is both external flexibility, i.e. hiring and firing, as well as internal flexibility, such as flexible working hours and the possibility of working overtime and part-time work (see Wilthagen et al., 2004a,b). Essential characteristics with respect to security are income security that is income protection in the event of job loss and after retiring from work, on the one hand, and the ability to combine paid work with other social responsibilities and obligations, on the other hand. Our goal in this paper is to integrate some ideas of the flexicurity model into the basic neoclassical growth model as presented by Solow (1956) and to analyze the resulting model with respect to its dynamic properties.

Solow's (1956) model of economic growth provides the basis for a variety of subsequent models analyzing the phenomenon of economic growth in Western capitalist economies. An important aspect in Solow's growth model is the assumption of a neoclassical production function with smooth factor substitution characterizing the input-output relationship that determines the laws of motion of the economy, in place of a fixed proportions technology. Solow assumed full employment and considered homogenous labor as one of the factors of production. In contrast to that, Goodwin's (1967) growth cycle model had quite a different starting point (Marx's reserve army mechanism): It assumed – as in Marx (1954, ch. 23) – a real wage Phillips curve and considered its interaction assuming an extreme variant of classical savings behavior in a technological framework with fixed proportions in production. Instead of monotonic convergence to the steady state, the Goodwin model gave rise to persistent cycles around its steady state position of a structurally unstable center dynamics type that could be easily modified towards the occurrence of stable limit cycles (as in Rose's (1967) employment cycle model).[1]

It is not difficult to combine the Solow growth model with the Goodwin growth cycle model, since the latter only introduces real wage rigidities into the Solovian framework (or smooth factor substitution into the Goodwin growth model). The resulting model features damped oscillations (close to Goodwinian cycles if the elasticity of substitution between capital and labor is low) and even monotonic convergence of the state variables (labor intensity and real wage) to the steady state in the opposite case. However, one problem of this integrated model is – if it creates periods of mass unemployment – that it implies the possibility of unemployed workers losing their skills and, thus, leading to labor market segmentation, with older workers subject to long-term or never ending unemployment and workers' families becoming degraded in their social and emotional status (a situation that is difficult to reverse). Further, there may be counteracting unemployment benefits, low wages for the degraded part of the work force and more that must be analyzed with respect to their consequences for the evolution of capitalist economies.

In this paper we will not engage into such an analysis of the consequences of mass unemployment but we will augment the above Solow-Goodwin synthesis by an employer of first (not last) resort, where all workers (and even pensioners) find reasonable employment if they are temporarily dismissed from the private sector of the economy, the sector of capitalist firms. In our model economy, that is to be seen as ideal in that respect, we only allow for two types of skill characteristics: skilled and high-skilled labor instructed in primary/secondary education and in tertiary education, respectively. Thus, by speaking of an employer of first resort we intend to underline that the skilled or high-skilled work profiles are employed in the public sector as well as in the private sector. Hence, we abstract from an employer of last resort and from the corresponding labor market where all labor is employed that is either unwilling or unable to work as skilled or high-skilled worker. By modeling the government as an employer of first resort we want to emphasize that the government needs qualified employees in order to organize the complex social security system in a flexicurity economy. This fact holds true for industrialized countries and, therefore, for Nordic countries as well so that one cannot call the government an employer of last resort. The model we build on this basis is providing a stylized theoretical basis for the Nordic Welfare approach to flexicurity, but one that is not subject to the pejorative reformulation of flexicurity as 'flexploitation' as it is sometimes referred to in evaluations of the concept of flexicurity in the political debate. Instead, we use the Solow model with the Goodwin real wage rigidity to construct full employment in this framework by means of (decentralized) government actions, with wage bargaining in the private sector and with two implied laws of motion (for employment and for the real wage) that will guarantee even monotonic convergence to the steady state in such a framework with flexible hiring and firing.

We view this model as an ideal economic system, a democratic and egalitarian society should aspire to, and towards which progress paths have to be found, confirmed by elections in a

[1]See also Solow (1990) for an interesting discussion of the Goodwin (1967) growth cycle model.

democratic society introducing ratchet effects when some parties propose to abolish such an evolution (if it has been by and large successful). It is ideal in that it combines flexible hiring and firing (and job discontinuities in the first, the private labor market) with income and employment security through a pension scheme and a second labor market that preserves the skills of the workforce and prevents their human degradation. Although being an ideal system some elements have already been integrated in real-world economies. For example, in Europe the three main pillars on which the social security system rests are the health care system, the unemployment insurance and the pension system. In our model we will take into account two of these pillars, the unemployment insurance and the pension system while neglecting health insurance. Thus, we present a model where the aspect of income security is modeled with respect to unemployment and with respect to the old age. By demonstrating that our model economy is stable, meaning that it converges to a steady state, we can show that an economy with a relatively elaborate social security system may well be a sustainable one.
We think that modern market economies are currently experiencing progress paths towards the flexicurity model, sometimes on a very low pace as the current discussion about minimum wages in Germany demonstrates. Yet, even such a discussion can be reflected from the perspective of the concept of flexicurity and may be interpreted as a step forward towards flexicurity if a general minimum level of (real) wages can be established in Germany. We have shown in Flaschel and Greiner (2009) in the context of Goodwin's growth cycle mechanism that minimum and also maximum wages (of workers) could dampen the employment fluctuations of the economy and could thus contribute to its stability after a transitory period of low employment.
Flexicurity – properly understood – may be the modern equivalent to Solow's growth model and may – in the same ideal way – provide a perspective for the future of capitalism which is compatible with the social structure of democratic societies. To demonstrate the working of flexicurity capitalism we will provide in section 2 the accounting framework for such an economy. In section 3 we will consider the behavior of the agents in such a framework in very basic terms and show on this basis the global asymptotic stability of – and even monotonic convergence to – its steady state position with respect to its central state variables, the real wage in the first labor market and the utilization rate of the workforce of firms. In section 4 we study the law of motion and the steady state positions of the (extra) company pension payments this model type allows for and, thus, consider conditions for the viability of the economy (which should allow for pension payments above the level of base pension payments). Yet, if we consider credit financing in nominal terms the investment behavior can depart from savings behavior such that the coordination of these two magnitudes leads to Keynesian effective demand problems giving rise to demand driven business cycles. This is the stage where flexicurity capitalism must prove its superiority, since there are existing business cycle fluctuations of a much larger extent than those that can originate from supply side driven full capacity growth. Such problems must however be left for future research here.

2 Flexicurity societies

The flexicurity concept – primarily discussed with respect to the Nordic economies and the Netherlands – intends to combine two labor market components which – as many economists might argue – cannot be reconciled with each other, namely workplace flexibility in a very competitive environment with income and employment (but not job) security for workers in this economy. The problem here is to find the appropriate mix between the two aspects of labor market institutions, intended to overcome both the case of flexibility without security (free hiring and firing capitalism) as well as the case of security without flexibility (past Eastern socialism).

In this section we first consider some basic features of a flexicurity economy. Thereafter, the budget equations and the economic behavior within these equations are presented. On this basis we investigate the stability of balanced growth paths of such an economy and also its sustainability concerning the generation of sufficient income and pension payments.

2.1 Full-employment capitalism: Ideal, status-quo and compromises

Let us start here from a definition of the concept of flexicurity as it is discussed in the European Union.

> *The concept of 'flexicurity' attempts to find a balance between flexibility for employers (and employees) and security for employees. The Commission's 1997 Green Paper on 'Partnership for a new organization of work' stressed the importance of both flexibility and security to competitiveness and the modernization of work organization. The idea also features prominently in the 'adaptability pillar' of the EU employment guidelines, where 'the social partners are invited to negotiate at all appropriate levels agreements to modernize the organization of work, including flexible working arrangements, with the aim of making undertakings productive and competitive and achieving the required balance between flexibility and security.' This 'balance' is also consistently referred to in the Commission's Social Policy Agenda 2000-2005* COM (2000) 379 final, Brussels, 28 June 2000).[2]

The concept of 'flexicurity' was introduced in Denmark on the political level by the social democratic prime minister Poul Nyrup Rasmussen in the 1990's[3] and it was introduced into the academic literature by Ton Wilthagen, see Wilthagen (1998) and Wilthagen et al. (2004a,b) on the Dutch origins of the flexicurity model. The role of the flexicurity approach in the performance of the Danish economy is critically investigated in Anderson and Svarer (2007); for further critical assessments of the proposals for and the discussion on a flexicurity economy the reader is moreover referred to recent contributions by Funk (2008) and Viebrock and J. Clasen (2009).

We stress in this context that our following approach to flexicurity is an abstract and primarily macroeconomic one that neglects the difficulties of how to implement flexicurity coordination and incentive principles on the microlevel from the economic, the social and the juristic point of view.

[2] http://www.eurofound.europa.eu/areas/industrialrelations/dictionary/definitions/flexicurity.htm

[3] See http://www.eurofound.europa.eu/areas/industrialrelations/dictionary/definitions/FLEXICURITY.htm

Our approach to labor market institutions of the flexicurity type differs significantly from the basic income guarantee (BIG) and Employer of Last Resort (ELR) approaches of the literature as they are compared for example in Tcherneva and Wray (2005), though the intention of these and our approaches have many things in common. Our approach can be characterized as an abstract modeling of a full-employment economy comparable in intention to the Tableau Economique of Quesnay. It therefore represents an ideal economy to be compared with the status-quo of actual developed capitalist economies. Such a comparison should then allow us to formulate compromises between the ideal and the status-quo of actual economies, like the United States of America or Australia, as described in Tchernova and Wray in the first case and in Quirk et al. (2006) with respect to Job Guarantee (JG) principles in the second case. We would however argue here that these latter approaches are presenting compromises without really formulating an ideal on the basis of which these compromises can be designed.
We thus just do the opposite here which may therefore be considered as complementary to the ELR and JG approaches, though we do not assume job guarantees, but only employment security. Moreover, however, in the ideal we have dismissed the concept of un- or low-skilled labor as representing a significant portion of the working population, since we believe that an ideal schooling system can overcome this factual situation to a large degree.

2.2 Flexicurity: Basic principles and problems

Basic aspects and problems of such a combination are:

- How much flexibility in:
    - 1.1 hiring and firing and job discontinuities?
    - 1.2 wage and price setting?
    - 1.3 technical change?
    - 1.4 globalization and financial markets?
- How much security in:
    - 2.1 base income?
    - 2.2 employment?
    - 2.3 location of employment?
    - 2.4 atypical employment?

Moreover, in order to get social acceptance for such a combination of the interests of capital owners and those of workers, basic aspects of social cohesion in a modern democratic market economy must be given. Thus, the following problems must also find a positive solution:

- Is there a consent-based cooperation between capital and labor?
- Does a proper citizenship education and democratic evolution exist?
- Is the existence of equal opportunities assured?

- Is there a reflected and controlled evolution of institutions?

In this paper we will provide a model which reconciles the aspects 1.1/2 with the problems 2.1/2, where the other aspects of the enumerated points remain excluded. Further, we shall simply assume that the societal issues in the last block have been developed to such an extent that the proposed model is not only transparent to the citizens of the considered capitalist society, but have indeed led to basic agreements on how the economy is to be organized and how the society should be developed further.

2.3 Budget equations, consumption and investment

Against this background, we next design the accounting framework (formulate the budget equations) of a growth model that combines ideas of Solow (1956) and Goodwin (1967). In contrast to Goodwin, our model takes into account a second labor market which, through its institutional setup, guarantees full employment in its interaction with the first labor market, the highly flexible and competitive private sector of the economy. It goes without saying that, to start with, the model must be formulated in as simple a way as possible, however, incorporating the essential components which underlie a flexicurity economy and society. We first consider the sector of firms in such an economy:

Firms

Production and Income Account:

Uses	Resources
δK	δK
$\omega_1 L_1^d$	$C_1 + C_2 + C_r$
$\omega_2 L_{2f}^w$	G
$\Pi \qquad (= Y^f)$	$I \qquad (= Y^f)$
Y	Y
$\delta_1 R + \dot{R}$	S_1

This account is a very simple one. Firms use their capital stock K (at profit-maximizing full capacity utilization) to employ the amount of labor (in hours): L_1^d in its operation, at the real wage ω_1, the law of motion of which is to be determined in the next section from a model of the wage-price dynamics in the private sector. They in addition employ labor force $L_{2f}^w = \alpha_f L_1^d$ (in heads[4]) from the second labor market at the wage ω_2, which is a constant fraction α_ω of the real wage in the first labor market. This labor force L_{2f}^w is working the normal hours of a standard workday, while the workforce L_1^w from the first labor market may be working overtime or undertime depending on the size of the capital stock in particular. Therefore the rate $u_w = L_1^d / L_1^w$ gives the utilization rate of the workforce L_1^w in the first labor market, the private sector of the economy (all other employment comes from the working of households

[4]to be determined in the next section.

occupied in the second labor market). Note finally that the capital stock depreciates at the rate δ.

Firms produce full capacity output, augmented by the company pension payments $\delta_1 R$, out of company pension funds R, to pensioners: $Y+\delta_1 R = C_1+C_2+C_r+I+\delta K+G+S_1$, with $S_1-\delta_1 R$ giving the net inflow into the company pension fund R (see also below). Output is sold to the two types of consumers and the retired households, the investing firms and the government. The demand side of the model is formulated in a way such that this full capacity output can be sold (see the supplement below), since investment is given by profits (and since all other income is consumed or put into the company pension stock R in real terms). Deducting from this output Y of firms their real wage payments to workers in the first and the second labor market (and depreciation) we get the profits Π of firms which are assumed to be retained and completely invested into capital stock growth $\dot{K} = I = \Pi$. Thus, we have Classical (direct) investment habits but there is not yet debt or equity financing of investment in this model type. We will assume (in place of the fixed proportions technology of Flaschel, Franke and Semmler (2008) and Flaschel, Greiner, Luchtenberg and Nell (2008)) a neoclassical production function as underlying the input-output data shown in the above table and we will describe this in detail in the next section.

We next consider the household sector of our flexicurity model which is composed of worker households working in the first labor market and the remaining ones (and pensioners) that are all working in the second labor market.

Households I and II (primary and secondary labor market)

Income Account (Households I):

Uses	Resources
$C_1 = c_{h1}(1-\tau_h)\omega_1 L_1^d$	
$\omega_2 L_{2h}^w = c_{h2}(1-\tau_h)\omega_1 L_1^d$	
$T = \tau_h \omega_1 L_1^d$	
$\omega_2(L-(L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$	
$\omega_2 L^r, L^r = \alpha_r L$	
S_1	$\omega_1 L_1^d$
$Y_1^w = \omega_1 L_1^d$	$Y_1^w = \omega_1 L_1^d$

Income Account (Households II):

Uses	Resources
C_2	$\omega_2 L_2^w, L_2^w = L - L_1^w$
Y_2^w	Y_2^w

Households of type I consume goods of amount C_1 and simple labor services from the second labor market L_{2h}^w. They pay an (in fact all) income taxes T and they pay in addition – via

further tax transfers – all workers' income that is not coming from firms, from them and the government (which is equivalent to an unemployment insurance). Moreover, they pay the pensions of the retired households ($\omega_2 L^r$) and accumulate their remaining income S_1 in the form of a company pension into a fund R that is administrated by firms (with inflow S_1 and outflow $\delta_1 R$). Thus, we have a pay-as-you-go pension system and a company pension fund that assure the income of retired persons.
The transfer $\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w)$ can be considered as solidarity payments, since workers from the first labor market that lose their job will automatically be employed in the second labor market where full employment is guaranteed by the government. We consider this employment as skill preserving, since it can be viewed as ordinary office or handicraft work (subject only to learning by doing when such workers return to the first labor market, i.e, to employment in the production process of firms).
The second sector of households is modelled in the simplest way that is available: Households employed in the second labor market, i.e, $L_2^w = L_{2f}^w + L_{2h}^w + L_{2g}^w$ pay no taxes and totally consume their income. We have thus Classical saving habits in this household sector, while households of type I may have positive or negative savings S_1 as residual from their income and their expenditures. We assume that they can accumulate these savings (or dissave in case of a negative S_1) from the stock of commodities they have accumulated as pension fund inventories in the past. In order to have a coherent distribution of the funds R that are accumulated by households of type I on the basis of their savings S_1, we have to formulate the law of motion of such funds R as follows:

$$\dot{R} = S_1 - \delta_1 R$$

where δ_1 is the rate by which these funds are depreciated through company pension payments to the 'officially retired' workers L^r, assumed to be a constant fraction of the 'active' workforce $L^r = \alpha_r L$. These worker households are added here as not really inactive, but offer work according to their still existing capabilities that can be considered as a (voluntary) addition to the supply of work organized by the government $L - (L_1^w + L_{2f}^w + L_{2h}^w)$, i.e. the working potential of the officially retired persons remains an active and valuable contribution of the working hours that are supplied by the members of the society. We consider this principle of 'active aging' an important feature of a flexicurity economy. It is obvious that the proper allocation of the work hours under the control of the government needs thorough reflection from the microeconomic and the social point of view, which however cannot be a topic in a paper on the macroeconomics of such an economy.
As the income account of the retired households shows (see below) they receive pension payments as if they would work in the second labor market and they get in addition individual transfer income (company pensions) from the accumulated funds R in proportion to the time they have been active in the first labor market and as an aggregate household group of the total amount $\delta_1 R$ by which the pension funds R are reduced in each period.

Income Account (Retired Households):

Uses	Resources
C_r	$\omega_2 L^r + \delta_1 R, L^r = \alpha_r L$
Y^r	Y^r

There is finally the government sector which is also formulated in a very basic way:

The Government

Income Account: Fiscal Authority / Employer of First Resort

Uses	Resources
$G = \alpha_g T$	$T = \tau_h \omega_1 L_1^d$
$\omega_2 L_{g2}^w = (1-\alpha_g)T$	
$\omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w))$	$\omega_2 L_r^w$
$\omega_2 L^r$	$\omega_2 \alpha_r L$
Y^g	Y^g

The government receives income taxes, the solidarity payments (employment benefits) for the second labor market paid from workers in the first labor market and old-age pension payments. It uses the taxes to finance government goods demand G and the surplus of taxes over these government expenditures to actively employ the core workers in the government sector. In addition it employs the workers receiving employment benefits from the households in first labor market and it in fact also employs the 'retired' persons to the extent they are willing and can still contribute to the various employment activities. We thus have that the total labor force in the second labor market is employed by firms, by households of type I and the remainder through the government as is obvious from the solidarity payments of households working in the first labor market. The income payments to workers in the second labor market ($\omega_2 L_2^w$) that are not originating from their services to firms, to households of type I or through an excess of income taxes over government commodity expenditures (base government employment) are paid out of transfers from the household sector I that works in the private sector to the government under competitive conditions. On the basis of these payments the remaining work in the second labor market is organized by government (in the way it does this in the administration of the state in all modern market economies).

In sum we get that workers are employed either in the first labor market and if not there then by doing auxiliary work within firms, services for households of type I or services in the government sector concerning public administration, infrastructure services, educational services or other public services. In addition there is a potential labor supply $\alpha_r L$ from the retired households, which due to their long life-expectancy in modern societies can remain effective suppliers of specific work over a considerable span of their life time after their official retirement. In this way the whole workforce is always fully employed in this model of social growth (and the retired persons according to their willingness and capabilities) and, thus, does not suffer from human degradation through prolonged unemployment in particular. Of course, there are a variety of issues concerning state organized work that point to (incentive) problems in the organization of such work, but all such problems exist also in actual industrialized market economies in one way or another.

We have formulated in this section the skeleton of a flexicurity growth model of the economy where full employment is not assumed, but actively constructed. The next sections augment the accounts of the considered economy by basic behavioral relationships concerning production, employment and the law of motion of real wages.

3 Smooth factor substitution, Okun's law and real wage rigidities

Our synthesis of the growth models of Solow (1956) and Goodwin (1967) into a model of the flexicurity variety consists of three basic building blocks, the three factor production function of the private sector, Okun's law that relates the utilization of the workforce L_1^w to the hiring and firing decision of firms and the dynamic of the real wages of the workers in the first labor market, describing the degree of labor market rigidity existing in the private sector of the economy.

The module that describes the growth dynamics of the model therefore consists of the following three structural equations:

$$
\begin{aligned}
Y &= F(K, L_1^d, L_{2f}^w), \quad \omega_1 = F_2(K, L_1^d, L_{2f}^w), \quad \omega_2 = F_3(K, L_1^d, L_{2f}^w) && (1)\\
\dot{L}_1^w &= \beta_e(L_1^d - L_1^w) + nL_1^w && (2)\\
\hat{w}_1 &= \beta_w(u_w - 1) + \hat{p}, \quad u^w = L_1^d/L_1^w && (3)
\end{aligned}
$$

The first (set of) equation(s) provide a three factor neoclassical production function, built on standard assumptions, coupled with the conditions for profit maximization with respect to its two variable inputs, the labor hours worked by workers L_1^w in the first labor market segment and the normal working hours supplied by the workforce L_{2f}^w that is employed by firms from the second labor market. We stress that workers L_1^w of type I are providing over- or under-time work according to the needs of profit-maximizing firms (who will recruit additional workers or dismiss employed ones in view of the discrepancy $L_1^d - L_1^w$ described by Okun's law later on).[5]

We should also like to point out that there is one good in this economy that is produced with two types of labor input supplied by households of type I and of type II, respectively.

The second equation describes, as already indicated, Okun's law in the given environment where only L_1^w is over- or under-employed (since capital is always fully employed and since we have an employer of first resort with respect to the second labor market). The time rate of change $\dot{L}_1^w$ is following the excess measure $L_1^d - L_1^w$ with an adjustment speed described by β_e, taking into account that the natural growth rate n, a constant, of the total workforce L must be used as trend term in this law of motion to provide a steady state solution later on (if further adjustment equations are to be avoided in this baseline model for reasons of simplicity).

[5] We assume that the normal supply of labor by individual workers is measure by '1' for notational simplicity.

The third equation is a standard money-wage Phillips curve, solely based on the actual inside employment of workers working on the first labor market, and on myopic perfect foresight concerning price inflation.[6] This latter assumption avoids the explicit consideration of a price Phillips curve, since we can then reduce the wage-price dynamics of this model type to a real wage dynamics $\hat{\omega}_1, \omega_1 = w_1/p$ and need not consider nominal effects in the chosen framework. For the real wage of workers in the second labor market we simply assume that it is a constant fraction of the real wage ω_1, which means that income distribution is driven by the insiders in the first labor market solely. Outsiders (the second labor market) play no role in the wage bargaining process.

Since aggregate demand is always equal to aggregate supply in the chosen framework, since all savings is product-oriented and since all profits are invested (i.e., Say's law holds), we have that actual output can be and is supply driven, depending on the level of real wages ω_1, ω_2 and on the capital stock K.

Supplement: On the validity of Say's law in Solovian flexicurity growth.

In our social growth model, we have assumed that workers of type II consume their whole income (they pay no taxes). With respect to the other type of workers we have assumed the following consumption function

$$\begin{aligned} C_1 &= c_{h1}(1-\tau_h)\omega_1 L_1^d, && c_h \text{ propensity to consume}, \tau_h \text{ tax rate} \\ \omega_2 L_{2h}^w &= c_{h2}(1-\tau_h)\omega_1 L_1^d && \text{consumption of household services} \end{aligned}$$

Savings of households of type I is, on the basis of our accounting relationships, given by:

$$S_1 = \omega_1 L_1^d - C_1 - \omega_2 L_{2h}^w - \omega_2(L - (L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w)) - \omega_2 L^r$$

due to the assumed solidarity contribution they provide to the second labor market.

Since the government, workers from the second labor market and pensioners do not save and since all tax transfers are turned into consumption and the savings of households of type I into commodity inventories of firms from which company pensions are to be deducted and since finally all profits are invested it can easily be shown that we must have at all times:

$$Y + \delta_1 R = C_1 + C_2 + C_r + I + \delta K + G + S_1, \quad C_1 = c_{h1}(1-\tau_h)\omega_1 L_1^d,$$

if firms produce at full capacity Y (which they can and will do in this case). Thus, there are no demand problems on the market for goods and, therefore, no need to discuss a dynamic multiplier process as in Keynesian type models. Note, moreover, that we have by construction for our social growth model at all points in time:

$$L = L_1^w + L_{2f}^w + L_{2h}^w + L_{2g}^w + L_r^w = L_1^w + L_2^w \qquad L_r = \alpha_r L$$

[6]See Blanchard and Katz (1999) for its micro-foundation and note that we do not use Blanchard and Katz (1999) error correction terms here which however – when added – would not modify our stability results obtained in this paper.

We have assumed that households of type I must pay a solidarity contribution (employment benefits) to those workers of type II, whose wages are not paid by firms, through services of households of type II to households of type I and through the core employment in the government sector. The government employs in addition as administrative workers and infrastructure workers (public work and education) the remaining workforce in the second labor market (plus the L^r services from pensioners). This completes the discussion of the behavioral equations of the social growth model.

4 Global convergence towards balanced reproduction

Normalizing level magnitudes by dividing through the capital stock K, as usual, and using lower case letters for the ratios thereby obtained, we get the reduced form equations as:

$$y = F(1, l_1^d, l_{2f}^w), \quad \omega_1 = F_2(1, l_1^d, l_{2f}^w), \quad \omega_2 = F_3(1, l_1^d, l_{2f}^w) \tag{4}$$

$$\hat{l}_1^w = \beta_e(l_1^d/l_1^w - 1) + n - \hat{K}, \quad \hat{K} = \rho = \Pi/K \tag{5}$$

$$\hat{\omega}_1 = \beta_w(l_1^d/l_1^w - 1) \tag{6}$$

Since ω_2 is a constant fraction of ω_1 we get from the profit maximization condition of firms the following proposition (due to $\rho = y - \delta - \omega_1 l_1^d - \omega_2 l_{2f}^w$):

Proposition 1:

Assume that the production function F satisfies:

$$F_2 > 0, \quad F_3 > 0, \quad F_{22} < 0, \quad F_{33} < 0, \quad F_{23} \geqq 0, \quad \Delta = F_{22}F_{33} - (F_{23})^2 > 0.$$

Then, the profit maximizing behavior of firms implies the relationships:[7]

$$l_1^d = l_1^d(\omega_1), \quad (l_1^d)'(\omega_1) < 0 \tag{7}$$

$$l_{2f}^w = l_{2f}^w(\omega_1), \quad (l_{2f}^w)'(\omega_1) < 0 \tag{8}$$

$$y = y(\omega_1), \quad y'(\omega_1) < 0 \tag{9}$$

$$\rho = \rho(\omega_1), \quad \rho'(\omega_1) < 0 \tag{10}$$

Proof: See the mathematical appendix.

Given proposition 1, the dynamics implied by the model can be reduced to two nonlinear laws of motion for the state variables $\omega_1, l_1^w > 0$ of the type:

$$\hat{\omega}_1 = \beta_w(l_1^d(\omega_1)/l_1^w - 1) \tag{11}$$

$$\hat{l}_1^w = \beta_e(l_1^d(\omega_1)/l_1^w - 1) + n - \rho(\omega_1) \tag{12}$$

[7] In case of a Cobb-Douglas production function $K^\alpha (L_1^d)_1^\beta (L_2^w)_2^\beta$ we have:

$$l_{2f}^w = \frac{\beta_2}{\beta_1 \alpha_\omega} l_1^d, \quad l_1^d = [\frac{\beta_2 \alpha_\omega}{\beta_1} \frac{\beta_2}{\beta_1}{}_2^\beta \omega_1]^{\frac{1}{\beta_1 + \beta_2 - 1}}.$$

The two differential equations (11) and (12) determine the dynamic behavior of our flexicurity model. With these two equations we can prove the following two propositions.

Proposition 2:

Assume that there holds $\lim_{\omega_1 \to 0} \rho(\omega_1) = +\infty$ *and* $\lim_{\omega_1 \to +\infty} \rho(\omega_1) = 0$. *Then: The above dynamical system has a unique interior steady state that is given by:*

$$\omega_1^o = \rho^{-1}(n), \quad l_1^{wo} = l_1^d(\omega_1^o) \tag{13}$$

Proof: See the mathematical appendix.

Proposition 3:

1. *The above dynamical system can be reformulated as a planar system defined on the whole plane by using the logs* $\varpi_1, \ell_1^w > 0$ *of the state variables* $\omega_1, l_1^w > 0$ *which implies the equivalent system of differential equations*

$$\dot{\varpi}_1 = \beta_w(\exp(\ell_1^d)(\exp(\varpi_1))/\exp(\ell_1^w) - 1) \tag{14}$$
$$\dot{\ell}_1^w = \beta_e(\exp(\ell_1^d)(\exp(\varpi_1))/\exp(\ell_1^w) - 1) + n - \rho(\exp(\varpi_1)) \tag{15}$$

2. *The unique interior steady state of these laws of motion is globally asymptotically stable*

Proof: An application of Olech's theorem (see the mathematical appendix)

Our model of social economic growth or flexicurity growth model always converges to its unique balanced growth path. Just as the original Solow (1956) model it is based on supply side conditions solely, while the demand side is only of importance for the savings decision of households of type I and, therefore, for the evolution of company pension funds, to be considered in detail in the next section.

Supplement: Solovian labor intensity $l = L/K$ dynamics (solely an appended law of motion under flexicurity growth):

By definition we have the following further law of motion for labor intensity l in our model of flexicurity growth.

$$\hat{l} = n - \rho(\omega_1)$$

For the Jacobian of the resulting 3D dynamics evaluated at the steady state we get from the laws of motion for ω_1, l_1^w, l:[8]

[8] Note again that the $\pm$ term does not give rise to an ambiguous sign for the determinant of J^o (which is always positive).

$$J^o = \begin{pmatrix} - & - & 0 \\ \pm & - & 0 \\ + & 0 & 0 \end{pmatrix}$$

Proposition 4:

The above 3D dynamics are globally asymptotically stable, but exhibits zero root hysteresis with respect to the state variable l.

Proof: See the mathematical appendix.

5 Monotonic adjustment processes through flexible hiring and firing

We now derive a local condition for the occurrence of monotonic convergence to the steady state of our model of flexicurity growth. According to the last proposition, we basically only have to investigate the first two laws of motion. It suffices therefore to consider the following matrix with respect to its eigenvalues:

$$J^o = \begin{pmatrix} J^o_{11} & J^o_{12} \\ J^o_{21} & J^o_{22} \end{pmatrix} = \begin{pmatrix} - & - \\ \pm & - \end{pmatrix}$$

where the $\pm$ sign reduces to $+$ in the calculation of the determinant of this matrix. It is obvious that we always have locally asymptotically stable dynamics (i.e. trace $J^o < 0, \det J^o > 0$). Furthermore, the condition trace $J^o = 4 \det J^o$, i.e.

$$(J^o_{11} + J^o_{22})^2 = 4(J^o_{11} J^o_{22} + J^o_{21} J^o_{12})$$

separates monotonic convergence (for parameters β_e sufficiently large) from cyclical convergence (parameters β_e sufficiently small). Reformulated, this condition reads:

$$|J^o_{22}| = |J^o_{11}| + 2\sqrt{|J^o_{21} J^o_{12}|}, \quad i.e. \quad sign\beta_e^H = sign[|J^o_{11}| + 2\sqrt{|J^o_{21} J^o_{12}|}]$$

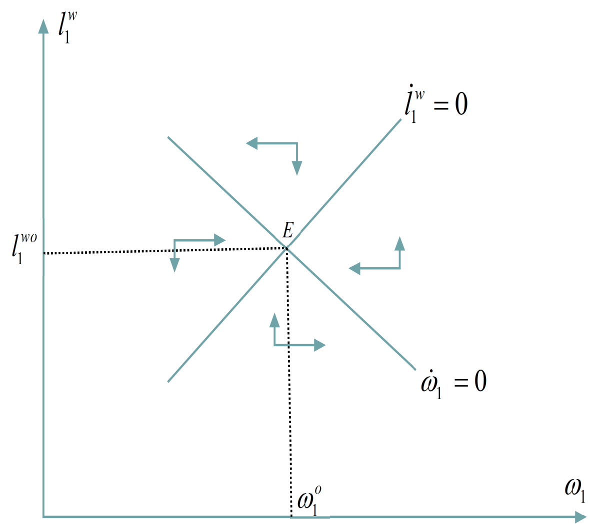

Figure 1: The case of a small parameter β_e

This gives:

Proposition 5:

Assume that the parameter β_w fulfills the inequality:

$$\beta_w \omega_1^o \{(l_1^d)'(\omega_1^o)\}^2 < -4 l_1^d(\omega_1^o) \underset{(-)}{\rho'(\omega_1^o)} l_1^{wo}$$

Then, there exists a unique bifurcation value $\beta_e^H > 0$ that separates monotonic from cyclical convergence. Cyclical convergence to the balanced growth path occurs for all $\beta_e \in (0, \beta_e^H)$, and monotonic convergence to the balanced growth path occurs for all $\beta_e \in (\beta_e^H, +\infty)$.

Proof: See the mathematical appendix.

The case of monotonic convergence is shown in figure 2, while the case of cyclical convergence to the steady state is given in figure 1. Thus, we get that economic fluctuations can be avoided in this type of economy if wages in the first labor market respond relatively sluggishly to demand pressure in this market (as measured by the utilization rate of the insiders) and if hiring and firing is a sufficiently flexible process as envisaged by the concept of flexicurity capitalism. The critical value for the hiring and firing speed parameter in our model of social growth is the larger, the larger the reaction of money wage inflation with respect to workforce utilization, i.e. the larger the parameter β_w becomes.

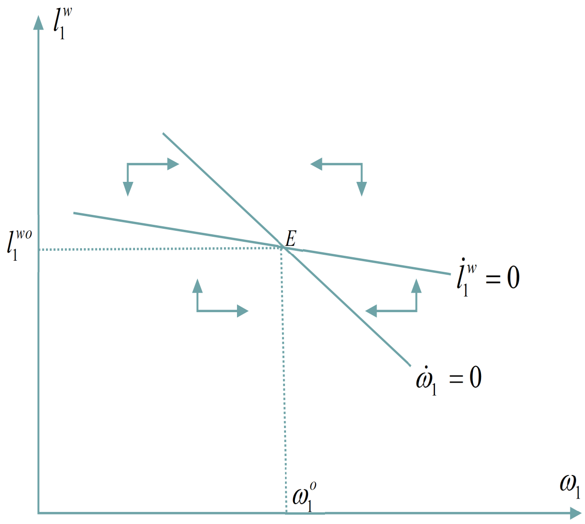

Figure 2: The case of a large parameter β_e

6 A further law of motion: Company pension funds

There is a further law of motion in the background of the model that needs to be considered in order to provide an additional statement on the viability of the considered model of flexicurity capitalism. This law of motion describes the evolution of the pension fund per unit of the capital stock $\eta = \frac{R}{K}$ and is obtained from the defining equation $\dot{R} = S_1 - \delta_1 R$ as follows:

$$\begin{aligned}\hat{\eta} &= \hat{R} - \hat{K} = \frac{\dot{R}}{K}\frac{K}{R} - \rho = \frac{S_1 - \delta_1 R}{K}/\eta - \rho, \quad i.e.: \\ \dot{\eta} &= \frac{S_1}{K} - (\delta_1 + \rho)\eta = s_1 - (\delta_1 + \rho)\eta\end{aligned}$$

For reasons of simplicity we now assume a Cobb-Douglas production function. Then, we know that there is a constant $\alpha_f > 0$ such that $l^w_{2f} = \alpha_f l^d_1$ holds. Savings of households of type I and profits of firms per unit of capital are given by:

$$\begin{aligned}s_1 &= (1 - (c_{h1} + c_{h2})(1 - \tau_h) - \tau_h)\omega_1 l^d_1(\omega_1) - \alpha_\omega \omega_1 (l^w_x + l^r) \\ &\quad l^w_x = l - (l^w_1 + l^w_{2f} + l^w_{2h} + l^w_{2g}) \\ &\quad l^r = \alpha_r l, \quad i.e. \text{ due to the financing of the employment terms } \quad l^w_{2h} + l^w_{2g}: \\ s_1 &= (1 - c_{h1}(1 - \tau_h) - \alpha_g \tau_h)\omega_1 l^d_1(\omega_1) - ((1 + \alpha_r)l - (l^w_1 + l^w_{2f}))\alpha_\omega \omega_1, \quad l^w_{2f} = \alpha_f l^d_1(\omega_1) \\ \rho &= y - (1 + \alpha_\omega \alpha_f)\omega_1 l^d_1(\omega_1)] - \delta\end{aligned}$$

For reasons of analytical simplicity we also assume that the economy has already reached its steady state with respect to the variables ω_1, l^w_1 and that we also have a given ratio $l = L/K = const$. This is a simplifying assumption that must be accompanied later on by the assumption that the actual value of $l = \bar{l}$ must be chosen in a neighborhood of a base value l^o (see below). The above of course also provides us with a steady state value for the rate of profit $\bar{\rho}(\bar{\omega}_1) = \rho^o(\omega^o)$. Moreover we also assume for simplicity $\delta_1 = \delta$ for the depreciation rates of the capital stock and for the stock of pension funds.
This gives for the law of motion of the pension fund per unit of capital the following differential equation:

$$\dot{\eta} = (1 - c_{h1}(1 - \tau_h) - \alpha_g \tau_h)\bar{\omega}_1 l^d_1(\bar{\omega}_1) - ((1 + \alpha_r)\bar{l} - (\bar{l}^w_1 + \alpha_f l^d_1(\bar{\omega}_1)))\alpha_\omega \bar{\omega}_1 - (\delta + \bar{\rho})\eta$$

which gives a single linear differential equation for the ratio η. This dynamical equation is globally asymptotically stable around its steady state position given by:

$$\eta_o = \frac{(1 - c_{h1}(1 - \tau_h) - \alpha_g \tau_h)\bar{\omega}_1 l^d_1(\bar{\omega}_1) - ((1 + \alpha_r)\bar{l} - (1 + \alpha_f) l^d_1(\bar{\omega}_1))\alpha_\omega \bar{\omega}_1}{\delta + \bar{\rho}}$$

In this case we have monotonic adjustment of the pension-fund capital ratio to its steady state position, while in general we have a non-autonomous adjustment of this ratio that is driven by the real wage and employment dynamics of the first labor market. The steady state level of η is positive iff there holds for the full employment labor intensity ratio:

$$\bar{l} < \frac{(1 - c_{h1}(1 - \tau_h) - \alpha_g \tau_h)\bar{\omega}_1 l^d_1(\bar{\omega}_1) + ((1 + \alpha_f) l^d_1(\bar{\omega}_1))\alpha_\omega \bar{\omega}_1}{(\delta + \bar{\rho})(1 + \alpha_r)\alpha_\omega \bar{\omega}_1}$$

We assume in addition that the additional company pension payments to pensioners should add the percentage $100\alpha_c$ to their base pension $\omega_2 \alpha_r \bar{l}$ per unit of capital. This gives as a further restriction for the steady state position of the economy:

$$\delta \eta_o = \alpha_c \omega_2 \alpha_r \bar{l}, \quad \omega_2 = \alpha_\omega \bar{\omega}_1$$

Inserting the value for η_o then gives

$$\alpha_c = \delta \frac{(1-c_{h1}(1-\tau_h)-\alpha_g\tau_h)\bar{\omega}_1 l_1^d(\bar{\omega}_1) - ((1+\alpha_r)\bar{l} - (1+\alpha_f)l_1^d(\bar{\omega}_1))\alpha_\omega\bar{\omega}_1}{(\delta+\bar{\rho})\omega_2\alpha_r\bar{l}}$$

In this way, we get that a target value for α_c demands a certain labor intensity ratio $\bar{l}$ and vice versa. For a given total labor intensity ratio there is a given percentage by which company pensions compare to base pension payments. This percentage is the larger the smaller the ratio $\bar{l}_1^w/\bar{l}$ due to the following reformulation of the α_c formula:

$$\alpha_c = \delta \frac{[(1-c_{h1}(1-\tau_h)-\alpha_g\tau_h)\bar{\omega}_1 + (1+\alpha_f)\alpha_\omega\bar{\omega}_1]\bar{l}_1^w/\bar{l} - (1+\alpha_r)\alpha_\omega\bar{\omega}_1}{(\delta+\bar{\rho})\alpha_r\alpha_\omega\bar{\omega}_1} \tag{16}$$

If this value of the total employment labor intensity ratio prevails in the considered economy (where it is of course as usually assumed that $c_{h1}(1-\tau_h)+\alpha_g\tau_h < 1$ holds) we have that core pension payments to pensioners are augmented by company pension payments by a percentage that is given by the parameter α_c and that these extra pension payments are distributed to pensioners in proportion to the time that they have worked in the private sector of the economy. This implies a trade-off between the ratios $\bar{l}, \alpha_c$, as expressed by the relationship (16). It also shows that the total working population must make a certain ratio of the capital stock in order to allow for a given percentage of extra company pension payments.
Due to $\delta\eta_o = \alpha_c\omega_2\alpha_r\bar{l}$ and $s_1^o = (\delta+\bar{\rho})\eta_o$ we also have the equivalence between positive savings per unit of capital of households of type I and positive values for α_c, η_o. Moreover, these values are indeed positive if the following holds:[9]

$$\bar{l} \quad < \quad \frac{(1-c_{h1}(1-\tau_h)-\alpha_g\tau_h)\bar{\omega}_1 l_1^d(\bar{\omega}_1) + (1+\alpha_f)l_1^d(\bar{\omega}_1))\alpha_\omega\bar{\omega}_1}{(\delta+\bar{\rho})(1+\alpha_r)\alpha_\omega\bar{\omega}_1}$$

This inequality set limits to the total labor-supply capital-stock ratio $\bar{l}$ which allows for positive savings of households of type I in the steady state and thus for extra pension payments to them later on. Households of type I are by and large financing the second labor market through taxes and employment benefits (besides their contribution to the base income of the retired people). Since firms have a positive rate of profit in the steady state, since the government budget is always balanced and since only households of type I save in this economy, we have derived the condition under which such an economy accumulates not only capital, but also pensions funds to a sufficient degree – under appropriate restrictions on labor supply.

7 Conclusion and outlook

We have shown in this paper that a model of flexicurity capitalism can be formulated, exhibiting a second labor market (and an employer of first resort) where all workers not employed by private firms find meaningful employment. This economy is characterized by viable and attracting balanced reproduction schemes. Hence, we have been able to demonstrate that a

[9] Note that the numerator is easily shown to be positive and even larger than 1 under standard Keynesian assumptions on expenditure and taxation rates.

capitalistic system which is characterized by both flexibility and social security is a stable and, thus, sustainable system. Social security in our model means that retired workers get pensions from a pay-as-you-go system and from firms that administer a pension fund that is built up through savings of active workers. In addition, workers not employed by firms receive solidarity payments and meaningful occupation from the government. This shows that our social security system is rather complex but, nevertheless, allows convergence to a steady state. In case of a sufficiently flexible labor market the convergence is even monotonic implying that transitory business cycle fluctuations can be avoided.

In technical terms, the model exhibits a unique interior steady state which is globally asymptotically stable. We could show this by concentrating on the private sector of the economy, the dynamics of which are characterized by insider real wage adjustment dynamics of the type considered in Blanchard and Katz (1999),[10] and on a type of Okun's Law that linked the level of utilization of the insiders of firms to their hiring and firing decision. Since both of these laws of motion only refer to the first labor market and, thus, only to a part of the whole workforce, the fundamental equation of the Solow (1956) growth model has appeared only as an appendix to this core dynamics, describing the evolution of the total labor supply per unit of capital in addition.

A further fundamental law concerning the viability of the economy was the law of motion of company pension funds per unit of capital which was shown to lead to a viable steady state level of it when the labor-supply capital ratio is bounded by above in an appropriate way. The existence of such pension funds allows in principle to add credit (out of these funds) to the considered flexicurity model which, when such credits are delivered in physical form, would not question the supply side orientation of the model, see Flaschel, Greiner, Luchtenberg and Nell (2008) for details.

This, however, changes when paper credit is added to the model implying that investment demand can now depart from the supply of savings in which case we get an IS-equilibrium on the market for goods that generally differs from the supply of goods through profit-maximizing firms. In place of savings-driven supply side fluctuations in economic activity we then have investment driven demand side business cycle fluctuations of a probably much more volatile type. This situation is modeled and analyzed in Flaschel et al. (2008). It represents one litmus test for the proper working of flexicurity capitalism, since supply side growth may be too stable a situation in order to really test the strength of economies that are designed on the basis of the flexicurity approach. In such a situation it has to be tested in detail, also numerically (since the resulting 5D dynamics are of a fully interdependent type), how the hiring and firing parameter β_e influences the performance of the economy. In addition, prudent fiscal and monetary policy may then be needed in order to preserve the stability features we have shown to exist for our supply side version of flexicurity growth in this paper. The investigation of such topics must be left for future research here however.

[10] if myopic perfect foresight is added to their discussion of a wage Phillips curve and its theoretical underpinnings.

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Mathematical Appendix

A. Proof of Proposition 1

Since ω_2 is a constant fraction of ω_1, we set $\omega_2 = \alpha\omega_1$, where α is a positive constant. We can express then the rate of profit ρ as follows:

$$\rho = y - \delta - \omega_1 l_1^d - \omega_2 l_{2f}^w = F(1, l_1^d, l_{2f}^w) - \omega_1 l_1^d - \alpha\omega_1 l_{2f}^w \tag{A1}$$

The first order conditions for the maximization of ρ with respect to l_1^d and l_{2f}^w are given by the following set of equations:

$$\partial\rho/\partial l_1^d = F_2(1, l_1^d, l_{2f}^w) - \omega_1 = 0 \tag{A2}$$
$$\partial\rho/\partial l_{2f}^w = F_3(1, l_1^d, l_{2f}^w) - \alpha\omega_1 = 0 \tag{A3}$$

where $F_2 = \partial F/\partial l_1^d$ and $F_3 = \partial F/\partial l_{2f}^w$. The second order conditions for the maximization of ρ can be written as follows:

$$F_{22} < 0, \quad F_{33} < 0, \quad \Delta = \begin{vmatrix} F_{22} & F_{23} \\ F_{23} & F_{33} \end{vmatrix} = F_{22}F_{33} - (F_{23})^2 > 0 \tag{A4}$$

where $F_{22} = \partial^2 F/\partial l_1^{d2}$, $\quad F_{33} = \partial^2 F/\partial l_{2f}^{w2}$, and $F_{23} = \partial^2 F/\partial l_{2f}^w \partial l_1^d = \partial^2 F/\partial l_1^d \partial l_{2f}^w$.

Assumption A1

$$F_2 > 0, \quad F_3 > 0, \quad F_{22} < 0, \quad F_{33} < 0, \quad F_{23} \geqq 0, \quad \Delta = F_{22}F_{33} - (F_{23})^2 > 0.$$

Remark A1.
Suppose that the production function is of the Cobb-Douglas type such that

$$Y = AK^{1-\beta_1-\beta_2}(L_1^d)^{\beta_1}(L_{2f}^w)^{\beta_2} \quad (0 < \beta_1 < 1, 0 < \beta_2 < 1).$$

We then have

$$y = A(l_1^d)^{\beta_1}(l_{2f}^w)^{\beta_2}, \quad y = Y/K.$$

In this case, all of the inequalities in Assumption 1 are satisfied (and $F_{23} > 0$).

Proposition 1
All of the relationships (7) – (10) in the text are satisfied under Assumption A1.

Proof: Solving equations (A2) and (A3), we have $l_1^d = l_1^d(\omega_1)$ and $l_{2f}^w = l_{2f}^w(\omega_1)$. Totally differentiating equations (A2) and (A3), we get

$$\begin{pmatrix} F_{22} & F_{23} \\ F_{23} & F_{33} \end{pmatrix} \begin{pmatrix} dl_1^d/d\omega_1 \\ dl_{2f}^w/d\omega_1 \end{pmatrix} = \begin{pmatrix} 1 \\ \alpha \end{pmatrix} \tag{A5}$$

Solving this equation, we obtain the following inequalities.

$$(l_1^d)'(\omega_1) = dl_1^d/d\omega_1 = \begin{vmatrix} 1 & F_{23} \\ \alpha & F_{33} \end{vmatrix} /\Delta = (\underset{(-)}{F_{33}} - \alpha \underset{(-or0)}{F_{23}})/\underset{(+)}{\Delta} < 0 \tag{A6}$$

$$(l_{2f}^w)'(\omega_1) = dl_{2f}^w/d\omega_1 = \begin{vmatrix} F_{22} & 1 \\ F_{23} & \alpha \end{vmatrix} /\Delta = (\alpha \underset{(-)}{F_{22}} - F_{23}^2)/\underset{(+)}{\Delta} < 0 \tag{A7}$$

Therefore, we have $y = F(1, l_1^d(\omega_1), l_{2f}^w(\omega_1)) = y(\omega_1)$ and

$$y'(\omega_1) = dy/d\omega_1 = \underset{(+)}{F_2}\, \underset{(-)}{(dl_1^d/d\omega_1)} + \underset{(+)}{F_3}\, \underset{(-)}{(dl_{2f}^w/d\omega_1)} < 0. \tag{A8}$$

It follows from eq. (A1) that ρ is a linear decreasing function of ω_1 for any given values of $l_1^d > 0$ and $l_{2f}^w > 0$. Therefore, the graph of the function $\rho = \rho(\omega_1)$ becomes the outer envelope of downward sloping straight lines. This means that we have $\rho'(\omega_1) = d\rho/d\omega_1 < 0$.

Next, let us consider the phase diagram for the system (11), (12) in the text (see there figure 1). The locus of $\dot{\omega}_1 = 0$ is given by the following equation:

$$l_1^d(\omega_1) = l_1^w \tag{A9}$$

Totally differentiating this equation and rearranging terms, we get

$$\left.\frac{dl_1^w}{d\omega_1}\right|_{\dot{\omega}_1=0} = (l_1^d)'(\omega_1) < 0 \tag{A10}$$

The locus of $\dot{l}_1^W = 0$ is given by

$$\{l_1^d(\omega_1)/l_1^w - 1\} + \{n - \rho(\omega_1)\}/\beta_e = 0 \tag{A11}$$

Totally differentiating this equation and rearranging terms, we obtain the following relationship.

$$\left.\frac{dl_1^w}{d\omega_1}\right|_{\dot{l}_1^w=0} = \{(l_1^d)'(\omega_1) - \frac{\rho'(\omega_1)l_1^w}{\beta_e}\}(\frac{l_1^w}{l_1^d(\omega_1)}) \tag{A12}$$

Since $(l_1^d)'(\omega_1) < 0$ and $\rho'(\omega_1) < 0$, we have the following results from equations (A11) and (A12):

(1) $\left.\frac{dl_1^w}{d\omega_1}\right|_{\dot{l}_1^w=0}$ is a continuous decreasing function of the parameter value $\beta_e > 0$.

(2) $\lim_{\beta_e \to 0} \left.\frac{dl_1^w}{d\omega_1}\right|_{\dot{l}_1^w=0} = +\infty$

(3) $\lim_{\beta_e \to +\infty} \left.\frac{dl_1^w}{d\omega_1}\right|_{\dot{l}_1^w=0} = (l_1^d)'(\omega_1) < 0$

In other words, the slope of the locus of $\dot{l}_1^w = 0$ is positive for all sufficiently small values of β_e, and it is negative for all sufficiently large values of β_e, and the locus of $\dot{l}_1^w = 0$ coincides with that of $\dot{\omega}_1 = 0$ if β_e is infinitely large. On the other hand, we can easily see that $\partial\hat{\omega}_1/\partial\omega_1 < 0$ and $\partial\hat{l}_1^w/\partial l_1^w < 0$. Therefore, we obtain two types of the phase diagrams (see the figures 1 and

2 in the text) depending on the magnitude of the parameter value β_e.

B. Proof of Proposition 2

Assumption B1.

$\lim_{\omega_1 \to 0} \rho(\omega_1) = +\infty$ and $\lim_{\omega_1 \to +\infty} \rho(\omega_1) = 0$.

Remark B1.
The Assumption B1 is in fact satisfied if the production function is of Cobb-Douglas type.

Proposition 2

The two-dimensional dynamical system that is described by equations (11) and (12) has a unique interior steady state that is given by Eq. (13) in the text.

Proof: The equilibrium solution of this dynamical system is characterized by the following set of equations with the two unknowns ω_1 and l_1^w :

$$l_1^d(\omega_1) = l_1^w \tag{B1}$$

$$\rho(\omega_1) = n \tag{B2}$$

Eq. (B2) has the unique solution $\omega_1^o = \rho^{-1}(n) > 0$ because of Assumption B2 and the fact that the function $\rho(\omega_1)$ is a decreasing function. Substituting $\omega_1 = \omega_1^o$ into Eq. (B1), we moreover get $l_1^{wo} = l_1^d(\omega_1^o) > 0$.

C. Proof of Proposition 3

Let us define

$$\tilde{\omega}_1 = \ln \omega_1, \quad \tilde{l}_1^d = \ln l_1^d, \quad \tilde{l}_1^w = \ln l_1^w. \tag{C1}$$

We can then transform the dynamical system that is given by equations (11) and (12) in the text as follows:

$$\dot{\tilde{\omega}}_1 = \beta_w\{\exp(\tilde{l}_1^d)(\exp(\tilde{\omega}_1))/\exp(\tilde{l}_1^w) - 1\} = G_1(\tilde{\omega}_1, \tilde{l}_1^w) \tag{C2}$$

$$\dot{\tilde{l}}_1^w = \beta_e\{\exp(\tilde{l}_1^d)(\exp(\tilde{\omega}_1))/\exp(\tilde{l}_1^w) - 1\} + n - \rho(\exp(\tilde{\omega}_1)) = G_2(\tilde{\omega}_1, \tilde{l}_1^w) \tag{C3}$$

This system is well-defined for all $(\tilde{\omega}, \tilde{l}_1^w) \in R^2$. The Jacobian matrix of this system is given by

$$J_1 = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \tag{C4}$$

where $G_{11} = \frac{\partial G_1}{\partial \tilde{\omega}_1} = \beta_w\{\frac{\partial(\exp(l_1^d))}{\partial(\exp(\tilde{\omega}_1))}\frac{\exp(\tilde{\omega}_1)}{\exp(\tilde{l}_1^w)}\} < 0$, $G_{12} = \frac{\partial G_1}{\partial \tilde{l}_1^w} = -\beta_w\{\frac{\exp(\tilde{l}_1^d)}{\exp(\tilde{l}_1^w)}\} < 0$, $G_{21} = \frac{\partial G_2}{\partial \tilde{\omega}_1} = [\beta_e\{\frac{\partial(\exp(\tilde{l}_1^d))}{\partial(\exp(\tilde{\omega}_1))}\frac{1}{\exp(\tilde{l}_1^w)}\} - \frac{d\rho}{d(\exp(\tilde{\omega}_1)}]\exp(\tilde{\omega}_1)$, and $G_{22} = -\beta_e\{\frac{\exp(\tilde{l}_1^d)}{\exp(\tilde{l}_1^w)}\} < 0$ for all $(\tilde{\omega}_1, \tilde{l}_1^w) \in R^2$.

Therefore, we have the following set of inequalities for all $(\tilde{\omega}_1, \tilde{l}_1^w) \in R^2$:

$$trace J_1 = \underset{(-)}{G_{11}} + \underset{(-)}{G_{22}} < 0 \tag{C5}$$

$$\det J_1 = G_{11}G_{22} - G_{12}G_{21} = \underset{(-)}{G_{12}}(\exp(\tilde{\omega}_1))\,(d\rho/\underset{(-)}{d(}\exp(\tilde{\omega}_1)) > 0 \tag{C6}$$

$$G_{11}G_{22} \neq 0 \tag{C7}$$

This set of inequalities implies that all of Olech's sufficient conditions for global asymptotic stability of the two-dimensional system of differential equations are satisfied (cf. Gandolfo, 1996, 354 – 355). This proves all assertions of Proposition 3.[11]

D. Proof of Proposition 4

Let us consider the global stability of the system of dynamical equations (11) and (12) in the text when appended by the following law of motion for l.

$$\hat{l} = n - \rho(\omega_1) \tag{D1}$$

This system is a decomposable system, and we already know that the unique equilibrium point (ω_1^o, l_1^{wo}) of the independent subsystem that consists of equations (11) and (12) is globally stable (cf. Proposition 3). In other words, we have $\lim_{t\to+\infty} \omega_1(t) = \omega_1^o$ so that we have $\lim_{t\to+\infty} \rho(\omega_1(t)) = \rho(\omega_1^o) = n$ (cf. Proposition 2). Therefore, we obtain

$$\lim_{t\to+\infty} \hat{l}(t) = n - \lim_{t\to+\infty} \rho(\omega_1(t)) = n - n = 0. \tag{D2}$$

This means that the whole system is globally stable in the sense that l also converges to some value although $\lim_{t\to+\infty} l(t)$ depends on the initial condition $l(0)$. We can prove the dependency of $\lim_{t\to+\infty} l(t)$ on $l(0)$ as follows. Let us define $\tilde{l} = \ln l$. Then, we can rewrite eq. (D1) as

$$\dot{\tilde{l}} = n - \rho(\omega_1). \tag{D3}$$

Integrating this equation with respect to time, we obtain

$$\tilde{l}(t) = \tilde{l}(0) + \int_0^t \{n - \rho(\omega_1(\tau)\}d\tau \tag{D4}$$

so that we have

$$\lim_{t\to+\infty} \tilde{l}(t) = \tilde{l}(0) + \int_0^\infty \{n - \rho(\omega_1(\tau)\}d\tau. \tag{D5}$$

This proves the assertion.

E. Proof of Proposition 5

We can rewrite the system of dynamical equations (11) and (12) in the text as follows:

$$\dot{\omega}_1 = \omega_1\beta_w\{l_1^d(\omega_1)/l_1^w - 1\} = \tilde{G}_1(\omega_1, l_1^w) \tag{E1}$$

$$\dot{l}_1^w = l_1^w[\beta_e\{l_1^d(\omega_1)/l_1^w - 1\} + n - \rho(\omega_1)] = \tilde{G}_2(\omega_1, l_1^w) \tag{E2}$$

[11] See also Flaschel (1984) for the application of Olech's theorem in the context of models of economic growth.

The Jacobian matrix of this system at the equilibrium point (ω_1^o, l_1^{wo}) can be written as follows:

$$J^o = \begin{pmatrix} J_{11}^o & J_{12}^o \\ J_{21}^o & J_{22}^o \end{pmatrix} \tag{E3}$$

where $J_{11}^o = \omega_1^o \beta_w (l_1^d)'(\omega_1^o)/l_1^{wo} < 0$, $J_{12}^o = -\omega_1^o \beta_w l_1^d(\omega_1^o)/(l_1^{wo})^2 < 0$, $J_{21}^o = l_1^{wo}[\beta_e\{(l_1^d)'(\omega_1^o)\}/l_1^{wo} - \rho'(\omega_1^o)]$, and $J_{22}^o = -\beta_e l_1^d(\omega_1^o)/l_1^{wo} < 0$.
Then, the characteristic equation of this system becomes

$$\Gamma^o(\lambda) = |\lambda I - J^o| = \lambda^2 + a_1\lambda + a_2 = 0 \tag{E4}$$

where

$$a_1 = -trace J^o = -J_{11}^o - J_{22}^o = [-\omega_1^o \beta_w (l_1^d)'(\omega_1^o)\} + \beta_e l_1^d(\omega_1^o)]/l_1^{wo} > 0, \tag{E5}$$

$$a_2 = \det J^o = J_{11}^o J_{22}^o - J_{12}^o J_{21}^o = -\omega_1^o \beta_w l_1^d(\omega_1^o)\rho'(\omega_1^o)/l_1^{wo} > 0. \tag{E6}$$

The discriminant(D) of this system can be written as

$$D = a_1^2 - 4a_2 = D(\beta_e). \tag{E7}$$

It is now obvious that cyclical fluctuations around the equilibrium point occur if and only if $D < 0$ holds true.

Assumption E1

The parameter β_w is so small that we have the following inequality (E8):

$$\beta_w \omega_1^o \{(l_1^d)'(\omega_1^o)\}^2 < -4 l_1^d(\omega_1^o) \underset{(-)}{\rho'(\omega_1^o)} l_1^{wo} \tag{E8}$$

Proposition 5

Under Assumption E1, we get for the uniquely determined bifurcation value that separates monotonic from cyclical convergence the inequality $\beta_e^H > 0$. Cyclical convergence to the balanced growth path occurs for all $\beta_e \in (0, \beta_e^H)$, and monotonic convergence to the balanced growth path occurs for all $\beta_e \in (\beta_e^H, +\infty)$.

Proof: We can easily see that $D(\beta_e)$ is a monotonically increasing continuous function of β_e with the following properties.

$$\lim_{\beta_e \to +\infty} D(\beta_e) = +\infty \tag{E9}$$

$$D(0) = (\omega_1^o \beta_w / l_1^{wo})[\beta_w \omega_1^o (l_1^d)'(\omega_1^o)^2 / l_1^{wo} + 4 l_1^d(\omega_1^o)\rho'(\omega_1^o)] \tag{E10}$$

Assumption E1 implies that $D(0) < 0$. In this case, there exists unique positive value β_e^H such that we have $D(\beta_e^H) = 0$, $D(\beta_e) < 0$ for all $\beta_e \in (0, \beta_e^H)$, and $D(\beta_e) > 0$ for all $\beta_e \in (\beta_e^H, +\infty)$. This proves the assertion because we already proved the global convergence of the solution to the balanced growth path(cf. Proposition 3)

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