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THE POST-KEYNESIAN “CROWDING-IN” POLICY MEME: GOVERNMENT-LED SEMI-AUTONOMOUS DEMAND GROWTH

Brett Fiebiger¹

ABSTRACT

A recent literature has explored the role of semi-autonomous demand growth. This paper builds on the literature by incorporating a Lernerian government semi-autonomous demand function and an endogenous supply-side. Our main purpose is threefold. First, we wish to contribute to the case for crowding-in effects, especially in the long-run. Second, we confirm the Keynesian/Kaleckian pedigree of the capital stock adjustment principle. Third, we contrast core post-Keynesian ideas on demand-led supply-side endogeneity with the alternative neo-Marxian neo-Harrodian proposition of an exogenously-given natural growth rate, and find the latter lacking.

¹ brett.fiebiger@hotmail.com

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Government-led Semi-Autonomous Demand Growth**

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Key words: Fiscal policy, Crowding-in, Semi-autonomous demand, Capital stock adjustment principle

JEL classifications: B22, B50, E11, E20, E60, O42

1. Introduction

The novel premise of the Keynesian revolution was that a sustained fiscal expansion, supported by accommodative monetary policy if need be, could push a depressed economy towards full-employment. Keynes/Henderson (1929) focused on debunking the orthodox dogma that debt-financed fiscal stimulus must necessarily ruin a nation's finances. Lerner (1943) went further than Keynes did in rejecting the traditional doctrine of "sound finance". He argued that modern economies were constrained, not by financial resources which could be created at negligible cost by domestic monetary authorities, but by real resources. Fiscal measures ought to be oriented to achieving full-employment with price stability. Whether or not policymakers adopt a functional finance approach to fiscal policy can be regarded in significant part as an ideological choice: are policymaker "animal spirits" Lernerian or Friedmanite?

The aim of this paper is to explore the crowding-in effects of pure government expenditures. The analysis proceeds in Section 2 with a literature review of the semi-autonomous demand approach. Section 3 presents a neo-Kaleckian supermultiplier model in which the equilibrium growth rate is set by the growth rate of pure government expenditures. Section 4 adds in supply-side variables along with a Lernerian government counter-cyclical fiscal reaction function. Section 5 turns to the debate over demand-led supply-side endogeneity. Concluding remarks are presented in Section 6.

2. An Overview of the Semi-Autonomous Demand Approach

A growing heterodox literature has explored the role of semi-autonomous demand growth. One strand has proceeded under the banner of the Sraffian supermultiplier (SM).¹ The SM adjustment mechanism proposes that the instability of accelerator-multiplier mechanisms can be tamed by the components of effective demand that: (1) do not create productive capacity; and, (2) are largely independent of the current cyclical position of the economy. In the long-run, the rate of capital accumulation converges to the growth rate of semi-autonomous non-capacity generating expenditures, while the utilisation rate of productive capacity is restored to a structurally-determined normal utilisation rate.²

A second strand has added semi-autonomous demand expenditures into neo-Kaleckian models.³ Besides Kaleckians and Sraffians, there are also contributions from neo-Harrodians (Fazzari *et al.* 2020) and neo-Schumpeterians (Nomaler *et al.* 2020). The demand-led nature of the SM closure underlies its recent popularity amongst heterodox scholars. Another reason is that SM models address the issue of *long-run* Harrodian instability. Harrod (1939) is credited with the idea that firms will mount a strong and potentially destabilising investment response whenever there is a significant discrepancy between the actual and normal utilisation rates (hereafter the utilisation gap).

Questions about the "endogeneity" of autonomous demand expenditures have been raised (Nikiforos 2018, Skott 2019A). Fiebiger/Lavoie (2019) use the prefix *semi* to emphasise that what portion of effective demand is more stable is country and time specific. Hein/Woodgate (2021: 391) remark in view of the 2007-2009 Global Financial Crisis that 'government expenditure growth remains the 'realistic' alternative, under certain circumstances'. Since then the 2020-2021 COVID-19 pandemic has added force to that view. The modelling proposition that some portion of effective demand grows at an exogenous rate is, of course, a simplifying assumption. Canonical growth models often seek to demonstrate the relevance of a certain mechanism while abstracting from other real world features.⁴

¹ See, for example, Cesaratto (2015), Pariboni (2016), Serrano/Freitas (2017), Brochier/Macedo e Silva (2019), Girardi/Pariboni (2019), Brochier/Freitas (2020), Freitas/Christianes (2020).

² A structurally-determined normal utilisation rate implies only that its determinants are slow-changing.

³ See Allain (2015), Lavoie (2016), Nah/Lavoie (2017, 2018, 2019A, 2019B), Dutt (2019, 2020), Cassetti (2020), Hein/Woodgate (2021).

⁴ As one example Brochier/Freitas (2020: 2) point out that the canonical neo-Kaleckian model supposes 'autonomous investment is explained by animal spirits, which in turn are not explained by the model'.

Blecker/Setterfield (2019: 366) offer another line of criticism: ‘it might be argued that Sraffian-inspired developments in supermultiplier analysis have prompted a sudden, late, and undesirable turn towards exogenous growth theory in heterodox macrodynamics’. But the question can be turned around: what role does heterodox endogenous growth theory assign to the government? Should it be a passive agent *vis-à-vis* endogenous market forces; or, should it be a pro-active agent that undertakes concerted interventions to bound and drive economic outcomes? It remains that what determines the growth rate of semi-autonomous demand is typically left unexplained in the literature. Palley (2018: 339) puts it this way: ‘the great unanswered question in super-multiplier theory is what determines the rate of growth of autonomous demand’. Our answer will be that the “animal spirits” of policymakers exert the decisive influence on long-run economic outcomes.

The SM adjustment mechanism and endogeneity in the normal utilisation rate need not be mutually exclusive; indeed, Nah/Lavoie (2018) use both mechanisms. Brochier/Macedo e Silva (2019) and Cassetti (2020) specify an interval for the normal utilisation rate. Bassi *et al.* (2020) make a case for bounded endogeneity in the normal utilisation rate. The authors note that their ‘partial hysteresis result is suggestive of an economy in which both Classical/neo-Keynesian and Kaleckian adjustment mechanisms are operative’ (*ibid*: 39). The divide between neo-Kaleckians who endorse an endogenous utilisation rate and those who utilise the SM adjustment mechanism may not be too wide.

2.1. The Capital Stock Adjustment Principle and the Keynesian/Kaleckian Message

Keynes/Henderson (1929) advanced an optimistic message on fiscal policy activism; namely, that the numbers of employed could be increased through government expenditures (and with accommodating monetary policy if required) without ruining a nation’s finances. To the extent that a fiscal expansion increases effective demand, and that in turn pushes up the rates of profit and capacity utilisation, those inducements to firm investment will ‘diminish the need for deficit spending’ (Lerner 1943: 48). The crowding-in effects of fiscal policy are amplified when firms adjust capacity to the growth path of effective demand. To illustrate the point consider this expression for the rate of capital accumulation:

$$g_k = \frac{I_k - \delta K}{K} = hu/v - \delta$$

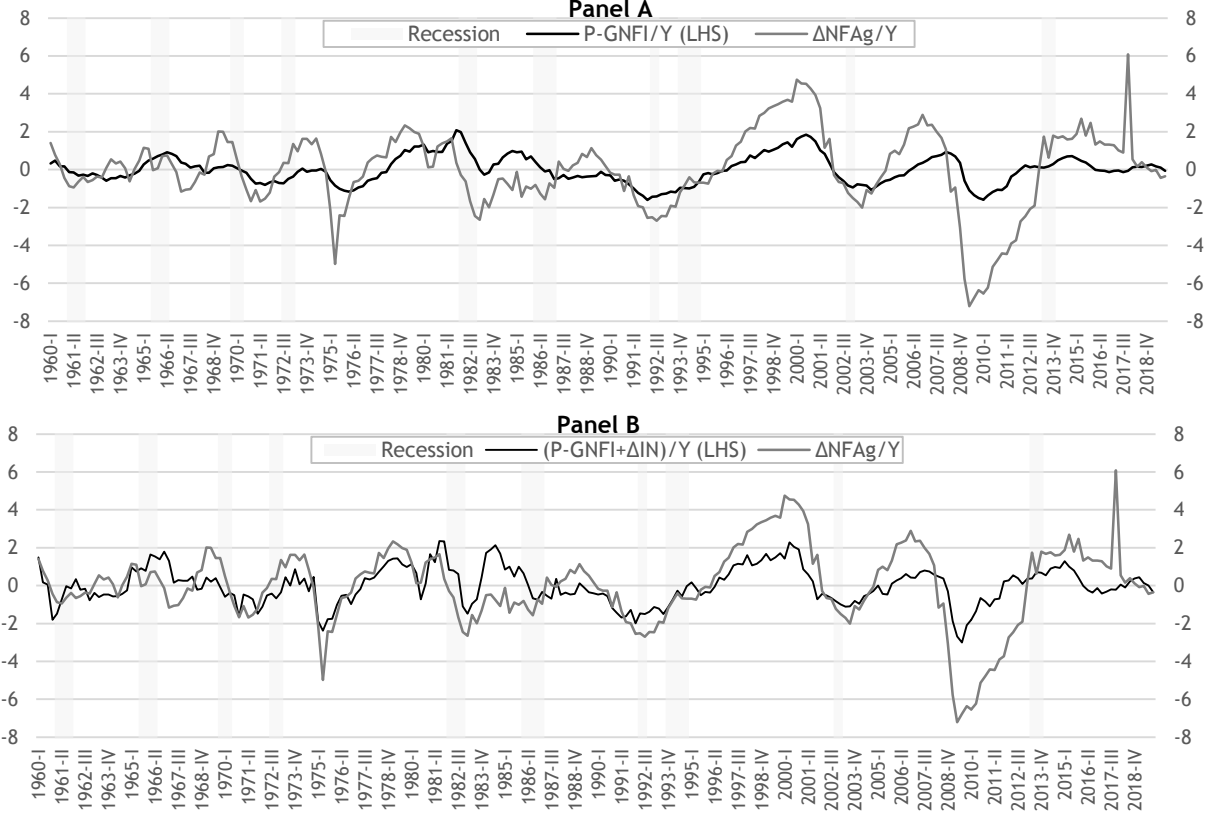
Where I_k is gross capacity investment, K is the (real net) capital stock, δ is the depreciation rate of fixed capital, h is the gross investment share, u is the capacity utilisation rate and v is the capital-to-full-capacity output ratio. Assume now that $\bar{\delta}$ and $\bar{v}$ are constants and that in the long-run the utilisation rate is restored to a structurally-determined normal rate $\bar{u}_n$; hence, $u^{**} = \bar{u}_n$.

With the above assumptions: $g_k^{**} = h^{**}\bar{u}_n/\bar{v} - \bar{\delta}$. A change in the equilibrium growth rate g_k^{**} must also change the equilibrium investment share h^{**} in the same direction. A unique feature of the SM closure is the positive (negative) long-run relation between the investment share (output share of semi-autonomous non-capacity generating expenditures) and the growth rate of the semi-autonomous non-capacity generating expenditures (Freitas/Christianes 2020). In the case of a faster growth rate for the semi-autonomous non-capacity generating expenditures, and when the government is undertaking those expenditures, the SM adjustment mechanism implies a fall in the equilibrium primary deficit-to-output ratio and thereby in the equilibrium fiscal deficit and government debt to output ratios.

The prospect that a fiscal expansion could itself generate the conditions for an improvement in fiscal metrics is a point worth making. Turning to Nikiforos (2018: 669): ‘The [Sraffian SM] theory implies that somehow there is going to be an acceleration of growth that will stabilize the debt-to-income ratio [of the sector undertaking the semi-autonomous expenditures]’. The somehow is the capital stock adjustment principle. Figure 1 presents cyclical deviations from the long-run trend for the output shares of the U.S. private sector’s nonresidential gross fixed investment (P-GNFI) and the

general government's net lending/net borrowing (ΔNFA_g). Panel B adds in inventory investment (ΔIN). The positive relation between the output shares of private nonresidential investment and fiscal balance gives some empirical plausibility to the relations that will inform the models in this paper.

Figure 1: Filtered Trends in U.S. Private Nonresidential Investment and Government Net Lending/Borrowing*



* Long-run trend uses a Hodrick-Prescott filter with a smoothing parameter of 129,600.

Source: U.S. Bureau of Analysis, *GDP & Personal Income*, Table 1.1.5. *Integrated Macroeconomic Accounts*, Tables S.7.q-S.8.q.

2.2. Short-Run and Long-Run Harroddian Instability

Neo-Kaleckian supermultiplier (NK-SM) models can be interpreted as a response to concerns that the canonical neo-Kaleckian closure of an endogenous utilisation rate amounts to an unstable equilibrium. Skott *et al.* (2020: 23) offer on the interest in SM models: ‘the post-Keynesian literature increasingly recognizes the potential significance of Harroddian instability’. All SM models address concerns about *long-run* Harroddian instability; conversely, there are differing views on *short-run* Harroddian instability.

Harrod’s (1939) “instability principle” refers to an unstable growth path whereupon firms’ expectations of sales growth positively feedbacks on itself. Sraffian SM models preclude short-run destabilising investment dynamics by way of assigning a slow adjustment speed to the parameter for the utilisation gap in the “flexible accelerator” investment function. In contrast, as most NK-SM models assume a Harroddian expected sales adjustment mechanism in a particular form, a destabilising property is introduced. It is useful to relay the basic NK-SM investment function. It has positive parameters for firms’ expectations of the secular trend in sales growth γ and another for the utilisation gap γ_u .

$$g_k = \gamma + \gamma_u(u - u_n), \quad \bar{\gamma}_u > 0$$

Allain (2015) presents a NK-SM model with a government that undertakes semi-autonomous consumption expenditures, and also instantaneously adjusts the tax rate to uphold a balanced budget.

The latter assumption is identified as “unrealistic” and adopted merely to avoid the complications of government interest payments. In the model expected sales growth is endogenised as follows:

$$\dot{\gamma} = \lambda(g_k - \gamma) = \lambda\gamma_u(u - u_n), \quad \bar{\lambda} > 0 \quad (i)$$

Where λ is a parameter for the adjustment speed. Lavoie (2016) uses the same mechanism, referencing the growth rate of expected sales $\hat{\gamma}$, instead of its rate of change $\dot{\gamma}$. Skott (2017: 190) argues that half of the $g_k - \gamma$ gap should be closed ‘within something like 2 years’. He then estimates the output share of semi-autonomous consumption that would be needed to do so, and finds it to be implausibly high. Lavoie (2017) rejects the utility of subjecting purposefully simple pedagogical models to numerical calibration.⁵ Skott *et al.* (2020) believe otherwise as they subject Allain’s (2015) model to a similar numerical calibration. Here we will return to Skott (2017: 188, fn. 2), and his second criticism of equation (i), which is that γ should instead adjust towards the actual output growth rate:⁶

$$\dot{\gamma} = \lambda(\hat{u} + g_k - \gamma) = \lambda[\hat{u} + \gamma_u(u - u_n)] \quad (ii)$$

With equation (i) or (ii): $\partial\dot{\gamma}/\partial\gamma > 0$. The stabilising properties of semi-autonomous non-capacity generating expenditures can ensure $\dot{g}_k/\partial\gamma < 0$ even while $\partial\dot{\gamma}/\partial\gamma > 0$. However, in some NK-SM models, the Harroddian adjustment mechanism imposes sluggish dynamics (Allain 2015). The problem lies in the simple behavioural expectations: adaptive and accelerationist. Firms believe that demand shocks will have strong persistence on the secular trend in sales growth, and completely disregard the prospect of counter-cyclical policy interventions. As so equations (i) and (ii) will be labelled as “ultra-Harroddian”. Dutt (2019, 2020) instead considers the possibility that firms’ expect the long-run trend in sales growth to be the growth rate of semi-autonomous demand expenditures g_z .

$$\gamma = g_z, \quad \dot{\gamma} = 0 \quad (iii)$$

The main advantage of Dutt’s specification is a simpler dynamic system because $\dot{\gamma}$ drops out. There is nothing particularly Keynesian about assuming agent expectations conform to a mechanical adaptive formula. Suppose now that a Lernerian government were to set a secular growth rate target. One might agree that firms should promptly adjust their expectations of the secular sales growth rate in line with the target so long as they believe: (1) the target is feasible; and, (2) policymakers will act to achieve the target. Framed in this way, the problem with Lucas-type rational expectations is not the hypothesis of forward-looking agents who can quickly adjust their expectations, but everything else in the orthodox paradigm. For New Classicals, the existing growth rate is always the optimal rate, and the problem of involuntary underemployment is instead a utility-maximising choice to increase leisure.

Hein/Woodgate (2021) are unconvinced by Dutt’s (2019, 2020) proposal to discard equation (i). For them equation (iii) has “switched off” Harroddian instability’ (Hein/Woodgate 2021: 390). Here the authors presumably mean *short-run* Harroddian instability. The arbiter should be whether or not there is empirical support for destabilising investment dynamics arising from $\partial\dot{\gamma}/\partial\gamma > 0$. It is here that the Harroddian story falls down.⁷ One alternative to the ultra-Harroddian adjustment mechanism would be:

$$\dot{\gamma} = \lambda(g_z - \gamma) \quad (iv)$$

⁵ Skott’s (2017) and Skott *et al.*’s (2020) simple calculations are also askew because the authors assign to the output/capital ratio a value that is around one-half of that reported in the empirical literature (Franke 2017).

⁶ Intriguingly, as the adjustment mechanism in equation (i) is used by Ryoo/Skott (2017), one is left to wonder why it is acceptable in neo-Marxian neo-Harroddian models yet questionable in neo-Kaleckian models.

⁷ Schroder (2011) investigates various investment functions in the U.S. economy. On the hysteresis Kaleckian investment function, with endogeneity in u_n and $\dot{\gamma}$ as per equation (i), he observes ‘the data does not confirm the view that accumulation increases the faster, the higher accumulation was in the past’ (*ibid*: 16).

With equation (iv), $\dot{\gamma}$ remains a state variable, although now $\partial\dot{\gamma}/\partial\gamma < 0$. Firms' expectations are anchored by policymaking decisions instead of formed with myopia about them. The case for the new adjustment mechanism is stronger if the government, rather than private agents, is undertaking the semi-autonomous demand expenditures. In the real world governments forecast the budget for many years in advance. Fiscal policies can change unexpectedly; nonetheless, the main uncertainty that firms face over demand growth lies in the private sector rather than public sector (especially if the government is Lernerian). Another possibility is that firms take into view the growth rate of pure government expenditures and the utilisation gap when assessing the change in trend sales growth:

$$\dot{\gamma} = \lambda[g_z + \phi(u - u_n) - \gamma], \quad \bar{\phi} > 0 \quad (v)$$

Where the ϕ parameter gauges the extent to which firms reference the utilisation gap as signalling a potential long-lasting change in the secular sales growth rate. With equation (v) $\partial\dot{\gamma}/\partial\gamma \lesseqgtr 0$ and firms' sales expectations are now only partly anchored by policymaking decisions.

3. A Baseline Model with a Government Sector

The simplifying assumptions of the baseline model include no price inflation and no growth in the working population or labour productivity. The simple economy that we will consider has five sectors: non-supervisory (NS) workers, rich households, firms, private banks and general government.

Nominal output pY is comprised of consumption by NS workers pC_w and rich households pC_r , pure government expenditures pZ and gross capacity investment pI_k . The latter is equal to net capacity investment pI_k^n plus firms' depreciation allowances δpK . Fixed capital depreciates at the constant rate δ . For full-capacity output Y_{fc} we assume a Leontief function with fixed coefficients for the capital-to-full-capacity output ratio $v = K/Y_{fc}$ and labour productivity $\xi = Y/L$. An extension to the baseline model will examine the case of labour-augmenting technical change.

$$pY = pC_w + pC_r + pZ + pI_k, \quad pI_k = pI_k^n + \delta pK, \delta > 0 \quad (1)$$

$$Y_{fc} = \min(K/v, L\xi), \quad \bar{v} > 0 \quad (2)$$

The total wage bill wL is given by the employment of NS workers L_w and managers L_r times their respective nominal wage rates, w_w and w_r . The wage rate of managers is a multiple σ of that of NS workers. For brevity we will suppose that the labour demand for the two household classes depends on the current level of real output Y .⁸ A corollary is that the labour productivity of NS workers and managers is always equal to their labour productivity as calculated at the level of full-capacity output; thus, $\xi_i = \xi_i^{fc}$ where $\xi_i^{fc} = Y_{fc}/L_i^{fc}$ and L_i^{fc} is full-capacity labour demand. The symbol f will be used for the ratio of the productivity of NS workers to the productivity of managers. It is a constant equal to the ratio of managers to NS workers employed at full-capacity labour demand.

$$w = w_w(1 + \sigma f)/(1 + f), \quad \bar{\sigma} = w_r/w_w > 1, 0 < \bar{f} < 1 \quad (3)$$

$$L = L_w + L_r \quad (4)$$

$$L_w = Y/\xi_w \quad (5)$$

$$L_r = Y/\xi_r \quad (6)$$

$$f = \frac{\xi_w}{\xi_r} = \frac{\xi_w^{fc}}{\xi_r^{fc}} = \frac{Y_{fc}/L_w^{fc}}{Y_{fc}/L_r^{fc}} = \frac{L_r^{fc}}{L_w^{fc}} \quad (7)$$

⁸ More realistically, as in Lavoie (2014: Ch.5), manager labour demand could depend on Y_{fc} . When managers are always employed at the full-capacity labour input (i.e. $L_r = L_r^{fc}$) their productivity becomes a positive function of, and their wage share a negative function of, the utilisation rate. An implication is that the gross profit share is no longer a constant, but instead endogenous, and pro-cyclical with the utilisation rate.

As firms set the price level p as a fixed mark-up θ on unit labour costs, the gross profit share π is a constant. Firms' gross profits $p\Pi$ is nominal output minus wages. Net entrepreneurial profits $p\Pi_E^n$ subtracts from $p\Pi$ depreciation allowances, a sales tax pT and the interest on bank loans to firms $i_L\mathcal{L}$. As banks earn no profits, the interest that banks receive on loans is the same as the interest they pay on deposits $i_D\mathcal{D}$.⁹ Firms pay dividends $p\Pi_D^n$ at a fixed rate i_ε on proprietors' equity $\mathcal{E}$. For simplicity the dividends rate is adjusted to the loan rate. Firms' net undistributed profits Π_U^n can be expressed as net entrepreneurial profits minus dividends or as net capacity investment minus the change in loans.¹⁰

$$p = (1 + \theta)w/\xi = (1 + \theta)(1 + \sigma f)w_w/\xi_w, \quad 0 < \bar{\theta} < 1 \quad (8)$$

$$\pi = \Pi/Y = \theta/(1 + \theta), \quad 0 < \bar{\pi} < 1 \quad (9)$$

$$p\Pi = pY - wL \quad (10)$$

$$p\Pi_E^n = p\Pi - \delta pK - pT - i_L\mathcal{L} \quad (11)$$

$$i_L\mathcal{L} = i_D\mathcal{D}, \quad i_L = i_D > 0, \mathcal{L} = \mathcal{D} \quad (12)$$

$$p\Pi_D^n = i_\varepsilon\mathcal{E}, \quad i_\varepsilon = i_L \quad (13)$$

$$p\Pi_U^n = p\Pi_E^n - p\Pi_D^n = pI_k^n - \dot{\mathcal{L}} \quad (14)$$

Dutt (2020) and Hein/Woodgate (2021) in NK-SM models, and Freitas/Christianes (2020) in a Sraffian SM model, focus on government expenditures as the semi-autonomous demand component. Differently from Hein/Woodgate (2021) who assume firms have zero undistributed profits, and from Dutt (2020) and Freitas/Christianes (2020) who abstract from firm sector financial relations, we will analyse financial dynamics in the firm sector. Firms are deficit-units that incur an increase in loans $\dot{\mathcal{L}}$ when net undistributed profits is less than net capacity investment. Proprietors' equity $\mathcal{E}$ is augmented by net undistributed profits and the current cost accounting revaluation to the capital stock $\dot{p}K$. Steindl's ([1952] 1976) gearing ratio $\mathcal{G}$ is the ratio of the nominal capital stock to proprietors' equity.

$$\dot{\mathcal{L}} = p\Pi_U^n - pI_k^n \quad (15)$$

$$\dot{\mathcal{E}} = p\Pi_U^n + \dot{p}K \quad (16)$$

$$\mathcal{G} = pK/\mathcal{E}, \quad \mathcal{E} = pK - \mathcal{L} \quad (17)$$

The government is also a deficit-unit. It incurs debt $\dot{\mathcal{B}}$ when its income from a sales tax levied on nominal output at the rate ϑ is less than its pure expenditures pZ and interest payments $i_B\mathcal{B}$.¹¹ Normalising variables to the capital stock, we use z for the government consumption-to-capital ratio and b for the government debt-to-capital ratio. In the baseline model pure government expenditures grow in real terms at the constant rate g_z . In a more complex model, the interest rate structure would be $i_\varepsilon \leq i_L > i_B > i_D$, and the interest rate on government debt i_B would be close to the central bank's base interest rate.¹² Here we adopt the simplifying assumption of a single nominal interest rate i .¹³

$$\dot{\mathcal{B}} = pZ + i_B\mathcal{B} - pT = (z + i_B b - \vartheta u/v)pK \quad (18)$$

$$pT = \vartheta pY \quad (19)$$

$$Z = Z_0 e^{g_z}, \quad \bar{g}_z > 0 \quad (20)$$

$$i = i_B = i_\varepsilon = i_L = i_D \quad (21)$$

⁹ It would be straightforward to set $i_L > i_D$ and then have banks distribute profits to rich household owners.

¹⁰ In a discrete time setting: $p\Pi_E^n = p\Pi_U^n + i_{\varepsilon-1}\mathcal{E}_{-1}$. The continuous time formulation $p\Pi_E^n = p\Pi_U^n + i_\varepsilon\mathcal{E}$ gives the misleading impression that firms could somehow pay dividends out of the profits that have yet to be generated.

¹¹ A more realistic model would include corporate taxes and proportional tax rates on household income.

¹² A Lernerian government presupposes that monetary authorities would underpin fiscal solvency by intervening in security markets when required. The ideal policy is to peg sovereign borrowing rates at desired levels, and then have the central bank buy whatever quantity of government securities are needed to maintain the targets, and in a "floor" system (where the base interest rate is set at the interest rate paid on banking system reserves).

¹³ Fiebiger (2021) analyses the case where $i_\varepsilon \leq i_L$ in a NK-SM model with no government sector.

NS workers spend their share in the total wage bill $1 - \varphi$ on consumption. Rich households consume a constant proportion α of their share φ in the wage bill plus the financial income they receive from the product of the real interest rate $r = i - \hat{p}$ times their holdings of financial wealth.¹⁴

$$C_w = (1 - \varphi)(1 - \pi)Ku/v, \quad \bar{\varphi} = 1/(1 + 1/\sigma f) \quad (22)$$

$$C_r = \alpha[\varphi(1 - \pi)u/v + r(1 + \ell)]K, \quad 0 < \bar{\alpha} < 1, r = i - \hat{p} \quad (23)$$

3.1. Short-Run

For the rate of real (net) capital accumulation we use a standard NK-SM investment function.

$$g_k = \frac{I_k^n}{K} = \gamma + \gamma_u(u - u_n), \quad \bar{\gamma}_u > 0, u_n \in [\bar{u}_n^{min}, \bar{u}_n^{max}] \quad (24)$$

Next we write the real output level with variables normalised by the capital stock:

$$Y = \langle (1 - \varphi)(1 - \pi)u/v + \alpha[\varphi(1 - \pi)u/v + r(1 + \ell)] + z + g_k + \delta \rangle K \quad (25)$$

Dividing by the real (net) capital stock and rearranging:

$$u = \frac{v[z + \alpha r(1 + \ell) + g_k + \delta]}{\pi + \varphi(1 - \alpha)(1 - \pi)} \quad (26)$$

Inserting the terms in the investment function gives this equilibrium solution for u :

$$u^* = \frac{v[\gamma + z + \alpha r(1 + \ell) + \delta - \gamma_u u_n]}{\pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v} \quad (27)$$

Note that requirement for a positive denominator is less stringent than the so-called Keynesian stability condition (Fiebiger 2021). Taking the partial derivative of u^* in respect to γ , z , α , π and φ :

$$\frac{\partial u^*}{\partial \gamma} = \frac{v}{\pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v} > 0 \quad (28)$$

$$\frac{\partial u^*}{\partial z} = \frac{v}{\pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v} > 0 \quad (29)$$

$$\frac{\partial u^*}{\partial \alpha} = \frac{v[r(1 + \ell)[\pi + \varphi(1 - \pi) - \gamma_u v] + \varphi(1 - \pi)(\gamma + z + \delta - \gamma_u u_n)]}{[\pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v]^2} > 0 \quad (30)$$

$$\frac{\partial u^*}{\partial \pi} = \frac{-v[1 - \varphi(1 - \alpha)][\gamma + z + \alpha r(1 + \ell) + \delta - \gamma_u u_n]}{[\pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v]^2} < 0 \quad (31)$$

$$\frac{\partial u^*}{\partial \varphi} = \frac{-v(1 - \alpha)(1 - \pi)[\gamma + z + \alpha r(1 + \ell) + \delta - \gamma_u u_n]}{[\pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v]^2} < 0 \quad (32)$$

A faster growth rate of $\bar{g}_z$ that increases z , an increase in firms' expectations of sales growth, a higher consumption propensity of rich households, a lower profit share and a lower wage bill share for managers, will all have a short-run expansionary effect on u^* .¹⁵

3.2. Long-Run

Over time the government consumption-to-capital ratio will be evolving as follows:

$$\dot{z} = z(\bar{g}_z - g_k) = z[\bar{g}_z - \gamma - \gamma_u(u^* - u_n)] \quad (33)$$

¹⁴ To save on notation we ignore wealth effects in rich households' consumption function.

¹⁵ A higher r , by increasing $r\ell$, could have an expansionary effect on u^* given $\alpha > 0$. We are less convinced about an expansionary effect from an increase in firms' rentier payments. A long-run contractionary effect from a higher r , ϑ or σ is possible if firms are able to recoup those payments at the expense of the wage share of NS workers.

Using equation (18) we define the growth rate of government debt g_B , and then the change in the government debt-to-capital ratio $\dot{b}$.

$$g_B = \dot{B}/B = (z - \vartheta u^*/v)/b + i \quad (34)$$

$$\dot{b} = b(g_B - g_k - \hat{p}) = b[(z - \vartheta u^*/v)/b + r - \gamma - \gamma_u(u^* - u_n)] \quad (35)$$

Steindl's ([1952] 1976) "internal accumulation" is the growth rate of proprietors' equity:

$$g_f^v = \frac{\dot{\mathcal{E}}}{\mathcal{E}} = \frac{p\Pi_U^n + \dot{p}K}{\mathcal{E}} = \frac{[(\pi - \vartheta)u^*/v - i - \delta + \hat{p}]pK}{\mathcal{E}} = G[(\pi - \vartheta)u^*/v - r - \delta] \quad (36)$$

Which we use to obtain the change in the gearing ratio:

$$\dot{G} = G(g_k + \hat{p} - g_f^v) = G(\gamma + \gamma_u(u^* - u_n) + \hat{p} - G[(\pi - \vartheta)u^*/v - r - \delta]) \quad (37)$$

Firms' sales expectations are partly anchored by policymaking decisions:

$$\dot{\gamma} = \lambda[\bar{g}_z + \phi(u^* - u_n) - \gamma], \quad \bar{\lambda} > 0, \bar{\phi} > 0 \quad (38)$$

Equations (33), (35), (37) and (38) form a 4×4 dynamic system. The Routh-Hurwitz (HR) stability conditions for a four-dimensional system are complex. Appendix A shows that local stability is possible if: (i) the denominator of u^* is positive; and, (ii) λ is sufficiently low.

Next we investigate the nature of the long-run equilibrium. It is a "fully-adjusted position" where the actual and normal utilisation rates are equal. And so too are the real growth rates of pure government expenditures, capital accumulation, government debt and firms' internal accumulation.¹⁶ Firms' assessment of trend sales growth also converges to the equilibrium growth rate set by $\bar{g}_z$.

$$u^{**} = u_n \quad (39)$$

$$\bar{g}_z = g_k^{**} = \gamma^{**} = g_B^{**} - \hat{p}^{**} = g_f^{v**} - \hat{p}^{**} \quad (40)$$

The long-run equilibrium value of z^{**} can be obtained by rearranging the definitional equation of u in equation (26) and then inserting $\bar{g}_z$ for g_k :

$$z^{**} = [\pi + \varphi(1 - \alpha)(1 - \pi)]u_n/v - \alpha r(1 + b^{**}) - \bar{g}_z \quad (41)$$

A faster $\bar{g}_z$ will lower z^{**} , although the extent depends on the long-run equilibrium value of the government debt-to-output ratio, which we now obtain by setting $\dot{b} = 0$ in equation (35):

$$b^{**} = \frac{z^{**} - \vartheta u_n/v}{\bar{g}_z - r} = \frac{[\pi + \varphi(1 - \alpha)(1 - \pi) - \vartheta]u_n/v - \alpha r - \bar{g}_z}{\bar{g}_z - (1 - \alpha)r} \quad (42)$$

Freitas/Christianes (2020: 323) refer to when the trend output growth rate is greater than the average (real) interest rate on government debt as the 'Domar stability or sustainability condition'. The condition underscores the importance of monetary policy to long-run fiscal metrics. We will focus on the case where the Domar stability/sustainability condition holds (i.e. $\bar{g}_z > r = i - \hat{p}^{**}$) and the government runs a primary deficit. In this case b^{**} and $\bar{g}_z$ are negatively related. Next we obtain the equilibrium values of the primary deficit-to-capital ratio p_d and fiscal deficit-to-capital ratio f_d :

$$p_d^{**} = z^{**} - \vartheta u_n/v \quad (43)$$

$$f_d^{**} = z^{**} + (r - \hat{p}^{**})b^{**} - \vartheta u_n/v \quad (44)$$

With a flat tax rate on output, a faster $\bar{g}_z$ must improve p_d^{**} . So too will f_d^{**} improve due to: (1) a lower p_d^{**} ; and, (2) an increase in $\bar{g}_z$ above $r = i - \hat{p}^{**}$. In the long-run $u^{**} = u_n$, and if the normal

¹⁶ In the baseline model $\hat{p}^{**} = 0$. In extensions $\hat{p}^{**}$ will equal the policymaker target inflation rate.

utilisation rate is fully-exogenous, a lower ℓ^{**} entails a commensurate fall in $\bar{\phi}_d^{**}$. One interpretation would be that the government has—through the principle of effective demand and the capital stock adjustment principle—crowded-in economic activity to such an extent there is a paradoxical long-run improvement in fiscal metrics. If instead $u_n \in [\bar{u}_n^{min}, \bar{u}_n^{max}]$, and u^{**} shifts upwards from $\bar{u}_n^{min}$ to $\bar{u}_n^{max}$, such endogeneity will lessen the improvement in the government's long-run fiscal metrics.

Another formal property of SM models is the positive long-run relation between $\bar{g}_z$ and the investment share h . Drawing again on the definitional equation of the utilisation rate in equation (26), and recalling $g_k = hu/v - \delta$, we obtain the long-run equilibrium investment share:

$$h^{**} = \pi + \varphi(1 - \alpha)(1 - \pi) - [z^{**} + \alpha r(1 + \ell^{**})]u_n/v \quad (45)$$

In the expansionary case of a faster $\bar{g}_z$, as z^{**} and ℓ^{**} will be lower at the end of the traverse, then h^{**} will be higher. And so too the gearing ratio which we derive by setting $\dot{g} = 0$ in equation (37):

$$g^{**} = \frac{g_z^{**}}{r_U^{**} + \hat{p}^{**}}, \quad g_z^{**} = \bar{g}_z + \hat{p}^{**}, r_U^{**} = \Pi_U^n/pK = (\pi - \vartheta)u^{**}/v - i - \delta \quad (46)$$

In a model with explicit financial relations for the firm sector, the SM closure can be cast as long-run endogeneity in the entrepreneurial gearing ratio (Fiebiger 2021).

4. An Endogenous Supply-Side

The baseline model will now be extended to include supply-side variables. The natural growth rate g_N is the growth rate of the active working population g_A plus the growth rate of labour productivity g_ξ . The active working population A is comprised of employed labour plus the unemployed. The inactive working population includes discouraged, sick and disabled workers.

$$g_N = g_A + g_\xi \quad (47)$$

$$e = L/A \quad (48)$$

Following Nah/Lavoie (2019B) and Fazzari *et al.* (2020) we posit a positive relation between the active working population growth rate and the rate of employment e . High rates of employment may encourage a higher rate of labour force participation amongst domestic workers (and perhaps also from net immigration). An additional influence on g_A from the rate of capital accumulation is included. One justification for the Y_k parameter is that g_k , as a proxy for real output growth, will capture the positive effects on a society's capacity to work from a faster growth rate of healthcare spending.¹⁷

$$g_A = Y_0 + Y_e e + Y_k g_k, \quad \bar{Y}_0 > 0, \bar{Y}_e > 0, \bar{Y}_k > 0 \quad (49)$$

$$g_\xi = \Lambda_0 + \Lambda_e e + \Lambda_k g_k, \quad \bar{\Lambda}_0 > 0, \bar{\Lambda}_e > 0, \bar{\Lambda}_k > 0 \quad (50)$$

With labour-augmenting technical change the capital-to-full-capacity labour ratio $K/L_{fc} = v\xi$ will be increasing over time with g_ξ . The determinants of labour productivity growth have symmetry with those of g_A . The Λ_e parameter can be motivated by labour market hysteresis effects: worker skills develop (atrophy) with a high (low) e . Another justification is that firms have 'greater incentive to innovate when labor markets are tight and unemployment is low' (Palley 2018: 337). Finally, the Λ_k parameter captures Kaldor-Verdoon effects, which work through the rate of capital accumulation.

In analytical neo-Kaleckian models the growth rate of output is proxied by the short-run equilibrium rate of capital accumulation. As so the rate of change in the employment rate is given by.

$$\dot{e} = e(g_k - g_N) \quad (51)$$

¹⁷ Other government expenditures that have a positive effect on labour participation could be included such as subsidies for childcare, training and education, as well as various initiatives to assist disadvantaged social groups.

Orthodoxy subscribes to a monetarist accelerationist view of price inflation. Our preference for price dynamics would be a non-linear Phillips curve with a flat section, although simpler arguments will be presented here.¹⁸ Both worker types have the same wage aspirations. Nominal wage growth $\hat{w}$ is anchored by the policymaker target rate of inflation $\hat{p}^T$. It also depends positively on the growth rate of labour productivity, the utilisation gap and the gap between e and the natural employment rate e_n . When setting prices firms reference the target inflation rate along with the utilisation gap and the employment gap. The sensitivity of nominal wage growth to the utilisation gap and employment gap is given respectively by the parameters Ω_u and Ω_e and, of prices, by the parameters Ψ_u and Ψ_e . The case $\Omega_u + \Omega_e > \Psi_u + \Psi_e$ ($\Omega_u + \Omega_e < \Psi_u + \Psi_e$) is known as radical (Cambridge) price adjustments. In this paper the complications of a conflicting claims approach to distribution will be put to the side.¹⁹ Thus, in the limiting case $\Omega_u + \Omega_e = \Psi_u + \Psi_e$, the profit share will remain constant.

$$\hat{w} = \hat{p}^T + g_\xi + \Omega_u(u^* - u_n) + \Omega_e(e - e_n), \quad \bar{\Omega}_u > 0, \bar{\Omega}_e > 0, \hat{p}^T \in [\hat{p}^{TL}, \hat{p}^{TV}], e_n \in [\bar{e}_n^{min}, \bar{e}_n^{max}] \quad (52)$$

$$\hat{p} = \hat{p}^T + \Psi_u(u^* - u_n) + \Psi_e(e - e_n), \quad \bar{\Psi}_u > 0, \bar{\Psi}_e > 0 \quad (53)$$

Next we define the Lernerian government's semi-autonomous demand function:

$$g_z = \psi_0 - \psi_{\hat{p}}(\hat{p} - \hat{p}^T) = \psi_0 - \psi_{\hat{p}}[\Psi_u(u^* - u_n) + \Psi_e(e - e_n)], \quad \bar{\psi}_{\hat{p}} > 0 \quad (54)$$

$$\psi_0 = \psi_s + \psi_m, \quad \bar{\psi}_s > 0, \bar{\psi}_m > 0 \quad (55)$$

Where ψ_0 is a constant parameter that is determined by structural factors ψ_s (e.g. slow-moving population demographics) and by the macro preference of policymakers ψ_m . In short we are suggesting that policymakers have their own “animal spirits”. The $\psi_{\hat{p}}$ parameter measures the degree to which the Lernerian government adjusts its pure expenditures to close the gap between the actual and target inflation rates. The latter is not a single point but an interval: $\hat{p}^T \in [\hat{p}^{TL}, \hat{p}^{TV}]$. So too is the natural rate of the employment subject to an interval: $e_n \in [\bar{e}_n^{min}, \bar{e}_n^{max}]$.

Cassetti (2020: 22) models a pre-globalisation framework in which central banks maintain ‘constant and low nominal interest rates’ [emphasis original]. A Lernerian government has a so-called fiscal reaction function, through which it moderates z to stabilise output growth around its trend, while $\hat{p}$ is an accommodating variable as policymakers have no target. As Lerner (1943) urged the use of fiscal measures to target full-employment with price stability, it is doubtful a Lernerian government would consent to price inflation taking any value. Godley/Lavoie’s (2007) fiscal reaction function is instead oriented to achieving a target inflation rate. The central bank sets a neutral monetary policy by way of adjusting the nominal base interest rate in tandem with the inflation rate to maintain a constant real base interest rate. We will likewise consider neutral monetary policy: $\bar{r} = i - \hat{p}$. The case of active monetary policy would require modifications to the baseline model.²⁰ For expediency we will limit ourselves to the case of active fiscal stabilisation and neutral monetary policy.

It would be possible albeit complicated to add $\dot{e}$ into the four-dimensional baseline model. Higher order systems are more difficult to interpret. To simplify the model we will discard the

¹⁸ One reason for a non-linear Phillips curve is that firms incur quasi-fixed overhead costs—such as manager salaries (see fn. 8), administration staff, advertising, marketing, rent—the proportion of which in total unit costs decrease, as the utilisation rate increases. The fall in unit overhead costs, as u rises, moderates upward pressures on prices.

¹⁹ Many contributions focus narrowly on firms and workers. Nowadays much of the distributive conflict occurs between managers and NS workers. Further, as the State is a large employer, the quantity of public sector jobs and wage rates should be included amongst the determinants of labour’s bargaining power in the private sector.

²⁰ In the baseline model: $\partial u^* / \partial r > 0$. A negative short-run relation between r and u^* could be obtained by adding a negative investment function parameter for the discrepancy between r and its long-run trend; or, by making household expenditures depend negatively on r . The latter is more realistic given the greater empirical sensitivity of dwelling investment and debt-financed consumption to lending rates. We also note that macro stabilisation via monetary policy will be ineffective if the required r is unattainable due to the zero nominal lower bound on i .

expected sales adjustment mechanism. Dropping $\dot{\gamma}$ as a state variable is preferable to other options.²¹ One possibility is that firms are able to discern the secular sales growth rate and take into account that counter-cyclical fiscal actions will only have a temporary effect on the long-run trend:

$$\gamma = \psi_0$$

It is not obvious why firms should have the type of accelerationist behavioural expectations that transform any change in sales growth into a cumulative divergence. Still, notwithstanding the above points, we will explore the alternative possibility that firms' sales expectations are informed by the growth rate of pure government expenditures and the utilisation gap:

$$\gamma = g_z + \mu(u^* - u_n), \quad \bar{\mu} > 0 \quad (56)$$

As in Dutt (2019, 2020) $\dot{\gamma}$ is no longer a state variable; however, it is not assumed that firms possess perfect foresight on the growth path of effective demand. Through equation (56) the items in the Lernerian semi-autonomous demand function now enter into the investment function and u^* .

$$g_k = \psi_0 + (\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)(u - u_n) - \psi_{\hat{p}}\Psi_e(e - e_n), \quad (24a)$$

$$u^* = \frac{v[\psi_0 + z + \alpha\tau(1 + \ell) - \psi_{\hat{p}}\Omega_e(e - e_n) - (\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)u_n]}{\pi + \varphi(1 - \alpha)(1 - \pi) - v(\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)} \quad (27a)$$

We now have a 4×4 dynamic system given by $(\dot{z}, \dot{\ell}, \dot{g}, \dot{e})$.

$$\dot{z} = -z(\gamma_u + \mu)(u^* - u_n) \quad (33a)$$

$$\dot{\ell} = \ell[(z - \vartheta u^*/v)/\ell + r - \psi_0 - (\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)(u^* - u_n) + \psi_{\hat{p}}\Psi_e(e - e_n)] \quad (35a)$$

$$\begin{aligned} \dot{g} = & g(\psi_0 + \hat{p}^T + [\gamma_u + \mu + \Psi_u(1 - \psi_{\hat{p}})](u^* - u_n) + \Psi_e(1 - \psi_{\hat{p}})(e - e_n) \\ & - g[(\pi - \vartheta)u^*/v - r - \delta]) \end{aligned} \quad (37a)$$

$$\dot{e} = e[(1 - Y_k - \Lambda_k)(\psi_0 + (\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)(u^* - u_n) - \psi_{\hat{p}}\Omega_e(e - e_n)) - Y_0 - \Lambda_0 - e(Y_e + \Lambda_e)] \quad (51a)$$

Appendix B shows that local stability is possible if: (i) $[\pi + \varphi(1 - \alpha)(1 - \pi) + \psi_{\hat{p}}\Psi_u]/v > \gamma_u + \mu$; and, (ii) the $(\gamma_u + \mu)$ parameters are sufficiently low. The new features of the long-run equilibrium are:

$$g_z^{**} = \psi_0 = g_k^{**} = \gamma^{**} = g_B^{**} - \hat{p}^{**} = g_f^{v**} - \hat{p}^{**} \quad (40a)$$

$$\hat{p}^{**} = \hat{p}^T \quad (57)$$

$$e^{**} = e_n^{**} = \frac{g_z^{**}(1 - Y_k - \Lambda_k) - Y_0 - \Lambda_0}{\Lambda_e + Y_e} \quad (58)$$

There may be a trade-off between the policymaker choice of the target for the inflation rate and real output growth; nonetheless, it may be useful to contemplate a long-run equilibrium where the rate of price inflation falls within a range acceptable to policymakers. It also seems amiss to speak of a “fully-adjusted position” unless the goals of entrepreneurs and policymakers are aligned. The case for a functional finance approach to fiscal policy may also be boosted by including the channels through which the supply-side responds to the demand-side, and thereby moderates inflationary pressures.

4.1. A Bounded Semi-Endogenous Semi-Autonomous Demand Function

The extended model describes a macro economy that is bounded by the exogenously-given constraints $\hat{p}^T \in [\hat{p}^{TL}, \hat{p}^{TU}]$, $u_n \in [\bar{u}_n^{min}, \bar{u}_n^{max}]$ and $e_n \in [\bar{e}_n^{min}, \bar{e}_n^{max}]$. The endogeneity of the natural employment rate within the limits $\bar{e}_n^{min} \leftrightarrow \bar{e}_n^{max}$ has been left implicit. An explicit mechanism would be:

$$\dot{e}_n = \eta(e - e_n), \quad \bar{\eta} > 0 \quad (59)$$

²¹ Ryoo/Skott (2017) obtain a simpler dynamic system via the unrealistic assumptions of an exogenously-given natural growth rate, constant firm debt-to-capital ratio and constant government consumption-to-capital ratio.

Obviously: $\partial \dot{e}_n / \partial e_n < 0$. Next we allow the Lernerian government's long-run growth objectives to become more (less) ambitious when the natural rate of employment is higher (lower).

$$g_z = \psi_0 - \psi_{\hat{p}}(\hat{p} - \hat{p}^T) + \psi_{e_n} e_n, \quad \bar{\psi}_{e_n} > 0 \quad (54a)$$

The ψ_{e_n} parameter implies that any variable which raises e (e.g. lower π , lower φ , higher α), and therefore also e_n via equation (59), will generate permanent positive growth effects within the limits of endogeneity in e_n . Such a possibility is what Bassi *et al.* (2020) have in mind, although working through bounded endogeneity in u_n , rather than through e_n as here.

4.2. Alternative Accommodating Variables

In the extended model a faster growth rate of pure government expenditures is allied with a higher equilibrium employment rate. It can be queried to what extent e^{**} can be an accommodating variable. The rate of unemployment in this model differs from the official unemployment statistics that count underemployed workers as employed (even if they work only one hour per week) and leave out the long-term unemployed. There is meaningful scope for the full-time employment rate to be increased even in countries with a low official unemployment rate. As things stand it must be acknowledged that e^{**} is quite sensitive to changes in the equilibrium growth rate. One step towards greater realism is to explicitly model the labour participation rate, $l = A/P$, where P is the total population. To do so the supply-side equations must be amended to be responsive to the *effective* rate of employment e_f :

$$e_f = \frac{L}{P} = \frac{L}{A} \frac{A}{P} = el \quad (60)$$

$$g_A = Y_0 + Y_{e_f} el + Y_k g_k, \quad \bar{Y}_{e_f} > 0 \quad (49a)$$

$$g_\xi = \Lambda_0 + \Lambda_{e_f} el + \Lambda_k g_k, \quad \bar{\Lambda}_{e_f} > 0 \quad (50a)$$

$$e^{**} = e_n^{**} = \frac{g_z^{**}(1 - Y_k - \Lambda_k) - Y_0 - \Lambda_0}{l^{**}(\Lambda_{e_f} + Y_{e_f})} \quad (58a)$$

An endogenous labour participation rate would substantially reduce the sensitivity of the equilibrium rate of employment to changes in the equilibrium growth rate. We cannot think of a reason why the growth rates of the active working population g_A and total population g_P would ever converge. As $\dot{l} = l(g_A - g_P)$ should always be changing there could not be a proper steady-state equilibrium. It is for this reason that the extended model excludes endogeneity in l . There is evidence that endogeneity in the labour participation rate has made advanced economies much less mature (Margin 2017).

A more sophisticated treatment of technical innovation could also reduce the sensitivity of e^{**} to changes in g_z^{**} . The Λ_k Kaldor-Verdoon parameter could itself be a positive function of the rate of capital accumulation. If $\Lambda_k = \Lambda_k(g_k)$ and $\Lambda'_k(g_k) > 0$, a faster g_k^{**} will increase Λ_k , and thereby reduce the extent to which a faster g_z^{**} would otherwise need to be accommodated by a higher e^{**} . In an open-economy context it is worth noting that Kaldor-Verdoon effects lessen external constraints by increasing trade competitiveness. An increase in the technical sophistication of production processes may enable domestic firms to expand market share abroad, or regain domestic market share, *vis-à-vis* foreign firms. A more efficient capital stock could also reduce the need for inputs produced abroad.

Our discussion of inflation dynamics has overlooked raw materials. The prices of raw materials are more sensitive to supply and demand conditions in global markets than manufactured products. Raw materials can be added into the Leontief function and firms' mark-up pricing equation as follows:

$$Y_{fc} = \min(K/v, L\xi, M\zeta) \quad (2a)$$

$$p = (1 + \theta)(1 + j)(1 + \sigma f)w_w/\xi_w \quad (8a)$$

Where M is raw materials, ζ is the output-to-raw material coefficient and j is an index of the ratio of unit raw material costs to unit labour costs. Suppose now that j depends positively on the utilisation gap and negatively on ζ , while the output-to-raw material ratio is itself a positive function of the equilibrium growth rate. We would then have a Kaldor-Verdoon type mechanism—operating via the cost-minimisation incentive for firms to use technical innovation to curb the usage of raw materials in production processes—that functions to moderate upward pressures on domestic prices and improve the external balance. A strong mechanism would also be welcomed given the urgency of raising the resource-efficiency of production processes to mitigate multifaceted environmental challenges.

4.3. Marx-Goodwin Luddite Firms or Marx-Hicks Innovative Firms

Neo-Marxian neo-Harrodians (NMNH) claim that Kaleckians neglect supply-side constraints in general, and the distinction between “mature” labour-constrained and “dual” labour-unconstrained economies, in particular.²² The canonical neo-Kaleckian model advances the reasonable hypothesis that u can be a proxy for e ; still, the exclusion of labour markets may appear to give credence to the NMNH critique. We can explore these issues by adding the employment gap into the investment function:

$$g_k = \gamma + \gamma_u(u - u_n) + \gamma_e(e - e_n), \quad \bar{\gamma}_e \leq 0 \quad (24b)$$

Skott’s (2010) NMNH mature-economy posits a negative relation between e and g_k (through a so-called output expansion function); hence, $\gamma_e < 0$. One rationalisation for $\gamma_e > 0$ is that firms may take a positive employment gap as indicating a need to scale up their investment in capital equipment with the latest labour-saving technology in order to lower unit costs and improve price competitiveness. Even if by some miraculous coincidence all firms were operating their plants at the cost-minimising normal utilisation rate, there may be still some firms optimistic of gaining of a higher market share, and others fearful of losing market share.²³ For those firms a decision to invest in capital equipment with the latest labour-saving technology to improve competitiveness may be the optimal strategy.

The inclusion of the employment gap into the investment function can be regarded as a step towards a more general mechanism through which wage costs and technical innovation influence the rate of capital accumulation. Assume now radical price adjustments so that the wage share ϖ is a positive function of the utilisation and employment gaps. Next replace γ_e with a parameter for the discrepancy between ϖ and a normal wage share ϖ_n determined by slow-changing social conventions.

$$g_k = \gamma + \gamma_u(u - u_n) + \gamma_\varpi(\varpi - \varpi_n), \quad \gamma_\varpi \leq 0 \quad (24c)$$

The above investment function could be interpreted in a similar way to the post-Kaleckian conceptual dichotomy of wage-led/profit-led demand regimes. Firms with $\gamma_\varpi < 0$ will be labelled as “Marx-Goodwin” and those with $\gamma_\varpi > 0$ as “Marx-Hicks”.²⁴ Marx-Goodwin type firms fit in well with a profit-led demand regime. Neo-Marxian neo-Goodwinians and NMNH claim that profit-squeeze theory fits U.S. data; however, it is overlooked that the typical impetus to cyclical upswings and downswings comes from residential investment and debt-financed consumer spending (Fiebiger/Lavoie 2019). It can also be observed that Marx-Goodwin firms are a pessimistic Luddite lot. If buoyant demand conditions lead to positive utilisation and employment gaps, and thus to $\varpi - \varpi_n > 0$, there is another option available to profit-maximising firms than to curb investment in spite of buoyant demand conditions. The alternative is to expand investment in capital equipment with the latest labour-saving technology.

²² See Skott (2010, 2016, 2018, 2019A, 2019B, 2020), Ryoo/Skott (2017), Skott *et al.* (2020).

²³ One would still expect capacity concerns to dominate so that $\gamma_u \gg \gamma_e$.

²⁴ Storm/Naastepad (2012) emphasise Marx-Hicks effects and formalise it by way of supposing that the growth of real wages has a positive effect on the growth rate of labour productivity.

Marx-Hicks type firms with $\gamma_{\omega} > 0$ may fit a wage-led demand regime. Yet, regardless of the positive effect on capital accumulation from the wage-led causal chain (that runs from a higher $\omega \rightarrow$ faster consumption growth $\rightarrow$ higher u), these firms are motivated to invest in new technology in order to restore the normal relation between costs and prices. The story requires firms to not only innovate in view of a high e but also invest. One option is to include g_{ξ} in the investment function (Palley 2018, Nah/Lavoie 2019A). The alternative option we are discussing works instead through the gap between the actual and normal wage shares. Over time the wage share will be changing as follows:

$$\dot{\omega} = \hat{\omega} - \hat{p} - g_{\xi} = (\Omega_u - \Psi_u)(u^* - u_n) + (\Omega_e - \Psi_e)(e - e_n)$$

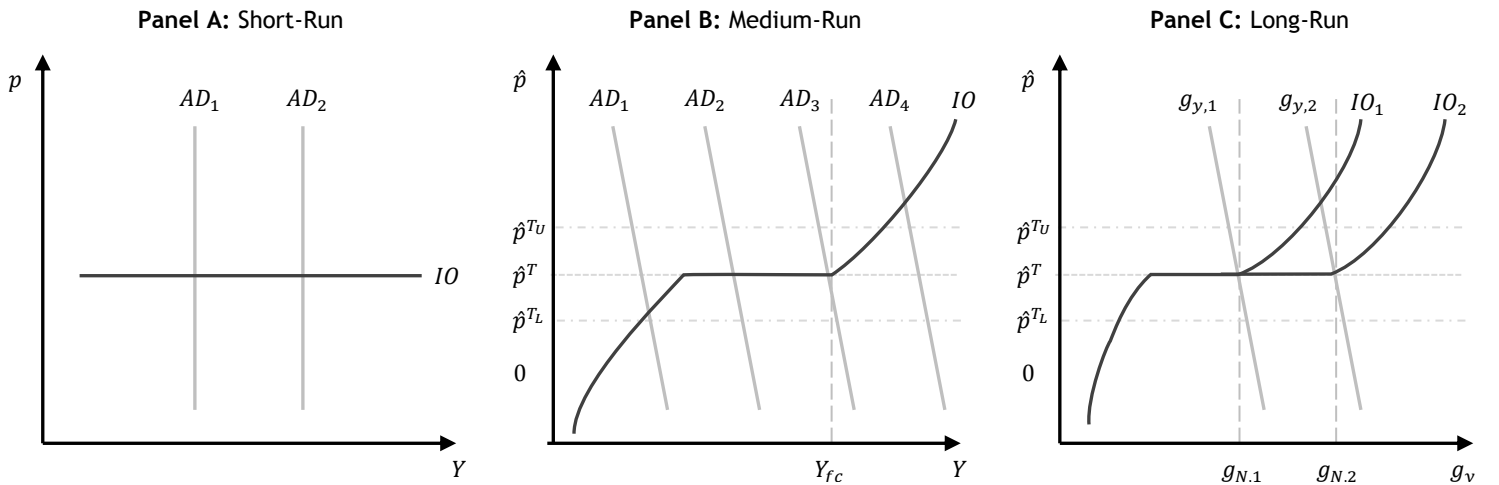
With radical price adjustments: $\partial \dot{\omega} / \partial e > 0$. A positive relation between e and g_k would occur if Hicks-Marx firms are more numerous than Marx-Goodwin firms. The prospect seems likely given that the Luddite Marx-Goodwin firms will be losing market share to the innovative Marx-Hicks firms.

5. The Debate over Demand-Led Supply-Side Endogeneity

A core post-Keynesian claim is that the growth rate of effective demand is a major determinant of economic outcomes in both the short-run *and* long-run. Post-Keynesians have long emphasised the endogeneity of the natural growth rate: ‘Mr. Harrod’s G_n is not a natural datum, but an object for policy and organisation’ (Robinson 1949: 85). The rationale for extending the Keynesian/Kaleckian principle of effective demand into the long-run is that the supply-side variables which constitute the natural growth rate are responsive to demand-side variables within reasonable confines (Lavoie 2014).

Figure 2 presents an aggregate demand / inflation-output AD/IO diagram over stylised runs. With a fixed price level in the short-run, the AD curve is vertical, while the IO curve is horizontal (Panel A). In the medium-run the IO curve is now non-linear (Panel B). The effects of a fiscal stimulus on the inflation rate depend on where the AD curve is relative to full-capacity output. An expansion in the demand curve within the horizontal section of the non-linear IO curve; from AD_2 to AD_3 , will have negligible effects on $\hat{p}$ as firms can utilise spare productive capacity to increase real output.

Figure 2: Post-Keynesian Aggregate Demand / Inflation-Output Diagrams



In the long-run a fiscal expansion that increases the growth path of real output g_y beyond the initial growth path of the natural growth, and which would otherwise lead to a positive inflation gap, gets resolved through an endogenous demand-led increase in the growth path of potential output (Panel C). Within an empirically-relevant range of growth rates, aggregate supply can accommodate a faster growth rate of aggregate demand while the inflation rate remains within a tolerable range of the policymaker target rate, which is the sentiment of Fazzari *et al.* (2020).

The story that post-Keynesians tell on demand-led supply-side endogeneity is not considered very interesting to NMNH. The standard proposition in mature NMNH models is an exogenously-given natural growth rate (Skott 2010, 2016, 2019A, 2019B, Ryoo/Skott 2017); thus, $g_k^{**} = \bar{g}_N$ à la Solow-Swan neoclassical growth model. NMNH concede on occasions that g_N may be endogenous. Skott (2016: 173, fn. 2, 2018: 7, 2019A: 242, fn. 12) refers the reader to Flaschel/Skott (2006), whose consideration of endogeneity in the natural growth rate which they denote by n , is limited to noting ‘the possibility could be captured by assuming that $n = n(e), n'(e) \geq 0$ ’ (*ibid*: 328). Two sentences later the reader is back again in the Harrodian/Solovian world: ‘in the interest of expositional simplicity we focus... on the case where $n'(e) = 0$ ’ (*ibid*: 329). On other occasions NMNH downplay endogeneity in g_N :

Realistically, the sensitivity of the natural growth rate to variations in employment is likely to be small. Firms may have an enhanced incentive to invest in R&D and to search for labor-saving changes in production if the labor market is tight. But for economies that are already at or close to the technological frontier, the effect surely is limited. The endogeneity of the labor supply may show greater promise. High demand will pull new groups of workers into the labor force, and the potential growth rate of the labor supply through immigration may seem almost unlimited. Until... one considers the political constraints (Skott 2018: 7).

What if the “technological frontier” is not static but can be sped up by policy interventions as Robinson (1949) would have it? And what if the domestic labour supply can be meaningfully enlarged by policy interventions that lessen/remove health and social barriers to participation? A demand-led increase in the actual and natural growth rates from say 2.0% per annum up to 3.0% per annum would, if sustained over time, have a demonstrable effect on the living standards of a society that achieves it. So why downplay and trivialise demand-led endogeneity in the natural growth rate?

This brings us to one of the more bizarre NMNH claims. Apparently, SM proponents not only reject Lerner’s (1943) functional finance, but advocate a perpetually balanced government budget. Skott *et al.* (2020) present a version of Allain’s (2015) model, and then compare two policy regimes. The “Sraffian supermultiplier (SSM) regime” has $\bar{g}_z$ while the “functional finance regime” allows g_z to be moderated to stop u from rising above a threshold. In both regimes the budget is always balanced.

SSM proponents could object that the cards have been stacked against the SSM policy: the simulations of functional finance presume an unrealistic ability of policy makers to control and fine tune the economy ... [But policymakers] can do better, surely, than keep constant the growth rate of government consumption and maintain a balanced budget. If an economy is in deep recession, then presumably Keynesian economists would recommend aggressive stimulus, rather than balanced budgets and the continuation of the previous trend in government spending (Skott *et al.* 2020: 25).

It is preposterous to infer that Allain’s (2015) self-admitted unrealistic assumption of a continuously balanced budget amounts to a policymaking recommendation that is universally endorsed by SM proponents. Skott *et al.* (2020: 26, fn. 27) are aware of SM models where the government runs budget deficits, as they reference Freitas/Christianes (2020) and Hein/Woodgate (2021) in a footnote. So why invent a fictitious SM proponent who advocates balanced budgets even in deep recessions?

Further, as shown by Casetti (2020), it is possible to combine a Lernerian government with the SM adjustment mechanism. In the Section 4 model, the $\psi_{\bar{p}}$ parameter in the Lernerian government’s semi-autonomous demand function is geared to short-run macro-stabilisation, while the ψ_0 parameter determines the long-run growth rate. Without the $\psi_0 = \psi_s + \psi_m$ parameters the model would need to be closed by some other means. We already know the NMNH answer for mature-economies: $g_k^{**} = \bar{g}_N$. The NMNH answer for dual-economies is more complicated and lacking Keynesian sensibilities.

Harrod (1960) believed that dual-economies were constrained by “under-saving”. The solution he advised was to increase the aggregate saving rate ‘by a budget surplus or compulsory levy’ (*ibid*: 289). The orthodox fable of thrift-driven growth is touted by Harrodians: ‘[in Harrodian-type] theory it is the savings rate (thrift) that principally drives growth’ (Shaikh 2009: 476). It is difficult to interpret an

“aggregate saving rate” (Fiebiger 2021); nonetheless, it can be imputed that acts of “thrift” by the household and public sectors supply “loanable funds”. Turning now to Skott/Ryoo (2008), and their dual-economy specification, the authors argue that any increase in the variables that make-up an aggregate saving function ‘must increase the amount of financial resources available to firms—raising the rate of capital accumulation’ (*ibid*: 847). So, in dual-economies where thrift is a virtue through the orthodox loanable funds channel, budget deficit are a vice that hinder economic development.

In a capital constrained [dual] economy, a sustained increase in the rate of accumulation requires reductions in the shares of private or public consumption (or an increase in net imports). Simply boosting aggregate demand is not a viable development strategy (Skott 2019B: 12).

High saving rates do not cause structural aggregate demand problems in dual economies ... [S]uccessful development requires high saving, and a sensible aggregate demand will typically avoid persistent deficits and high public debt (Skott 2020: 2).

The fiscal austerity that is envisaged for a dual-economy will only depress the utilisation rate. A miracle is then required to turn a short-run contractionary effect into a long-run expansionary effect. The textbook Harroddian thrift-driven dual-economy lacks a plausible traverse.

Even in an economy with abundant labour: $g_N = g_A + g_\xi$. An elastic g_A does not make g_ξ somehow redundant. Developing economies do not lack capital *per se* but capital in efficiency units. The factors that impel firms in developing economies to invest and innovate are no different to those in advanced economies: the capital stock adjustment principle and cash flow. In all economies alike the investment-innovation-productivity nexus is driven by the realised and expected growth path of effective demand. An abundant labour supply says nothing about the skills of workers, and it is difficult to see how fiscal austerity in the form of curbing government expenditures on healthcare, education, scientific research, infrastructure, utilities and so forth, could be a successful development strategy.

External constraints do tend to be more binding in the periphery than in advanced economies. The principles of functional finance require a government to issue debt in the domestic currency unit. A policymaker priority is to promote the development of the domestic financial system to increase: (1) the contribution of domestic demand to output growth; and, (2) the supply of local currency borrowing for domestic sectors. A Lernerian government can also use progressive tax initiatives ‘in its program of financing government spending to maintain full employment’ (Lerner 1943: 49).

6. Conclusion

Lerner (1943) assigned a central role to fiscal policy in driving and stabilising an unstable economy. Budget deficits put a floor under aggregate demand and employment in recessions. The improvement in the fiscal balance during upswings moderates upward instability in growth processes including inflationary pressures. The growth rate of pure government expenditures is a key driver of long-run output growth and, as aggregate supply is partially endogenous to aggregate demand, then so too the natural growth rate. A durable recovery from the COVID-19 pandemic, and a rapid transition to a low carbon economy, will require a Lernerian government to lead the way.

This paper has explored the crowding-in effects of semi-autonomous government expenditures. A case has been made for the Keynesian/Kaleckian pedigree of the capital stock adjustment principle. Our analysis also suggests caution on the short-run Harroddian instability argument given the absence of empirical evidence and behavioural foundations to support it. There is also no need to follow NMNH in disregarding demand-led supply-side endogeneity. Endogeneity in the utilisation rate, the natural rate of employment, the labour participation rate and technical progress, are all limited closures insofar as no one should imagine that aggregate supply can accommodate any growth rate of aggregate demand. Collectively those closures can provide the space—i.e. be operative over an empirically-relevant range of growth rates—wherein the principle of effective demand determines long-run economic outcomes.

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Appendix A: Baseline Model

Appendix A analyses the stability conditions of the baseline model. Equations (33), (35), (37) and (38), with u^* given by equation (27), define a 4×4 dynamic system. The Jacobian matrix takes the form:

$$J^{**}(\dot{z}, \dot{\ell}, \dot{g}, \dot{\gamma}) = \begin{bmatrix} \frac{-z^{**}\gamma_u v}{A} & \frac{-z^{**}\alpha\gamma_u v}{A} & 0 & -z^{**}\left(1 + \frac{\gamma_u v}{A}\right) \\ \frac{-(B-A)}{A} & -\left(\frac{p_d^{**}}{\ell^{**}} + \frac{\alpha B}{A}\right) & 0 & -\left(\ell^{**} + \frac{B}{A}\right) \\ \frac{-g^{**}C}{A} & \frac{-g^{**}\alpha C}{A} & -g^{**}(r_u^{**} + \hat{p}^{**}) & g^{**}\left(1 - \frac{C}{A}\right) \\ \frac{\lambda\phi v}{A} & \frac{\alpha\lambda\phi v}{A} & 0 & \frac{-\lambda(A - \phi v)}{A} \end{bmatrix} \quad (A1)$$

$$A = \pi + \varphi(1 - \alpha)(1 - \pi) - \gamma_u v$$

$$B = \ell^{**}\gamma_u v + \vartheta$$

$$C = g^{**}(\pi - \vartheta) - \gamma_u v$$

From equation (46) we know $g^{**}(r_u^{**} + \hat{p}^{**})$ equals the equilibrium *nominal* growth rate of pure government expenditures g_Z^{**} . Next we list the HR stability conditions for a four-dimensional system:

$$\begin{aligned} \alpha_1 &= -TrJ^{**} > 0 \\ \alpha_2 &= DetJ_1^{**} + DetJ_2^{**} + DetJ_3^{**} + DetJ_4^{**} + DetJ_5^{**} + DetJ_6^{**} > 0 \\ \alpha_3 &= -(DetJ_A^{**} + DetJ_B^{**} + DetJ_C^{**} + DetJ_D^{**}) > 0 \\ \alpha_4 &= DetJ^{**} > 0 \\ \alpha_5 &= \alpha_1\alpha_2\alpha_3 - \alpha_1^2\alpha_4 - \alpha_3^2 = \alpha_3(\alpha_1\alpha_2 - \alpha_3) - \alpha_1^2\alpha_4 > 0 \end{aligned}$$

The 1st HR condition requires that the trace of the Jacobian matrix be negative. We assume $z^{**} > 0$, $p_d^{**} > 0$, $\ell^{**} > 0$, $g_Z^{**} > 0$ and $A > 0$. As so the requirement for $\alpha_1 > 0$ will always be satisfied if $A - \phi v > 0$ and, therefore, if $\gamma_u + \phi < [\pi + \varphi(1 - \alpha)(1 - \pi)]/v$.

$$\alpha_1 = \frac{z^{**}\gamma_u v + \alpha B + \lambda(A - \phi v)}{A} + p_d^{**}/\ell^{**} + g_Z^{**} > 0 \quad (A2)$$

Alternatively, if $A - \phi v < 0$ while $A > 0$, the trace could still be negative so long as:

$$\phi < \frac{A}{v} + \frac{z^{**}\gamma_u v + p_d^{**}A/\ell^{**} + \alpha B + g_Z^{**}A}{\lambda v}$$

The 2nd HR condition specifies that the sum of the six 2nd-order principal minors be positive. The requirement for $\alpha_2 > 0$ will always be met if $A - \phi v > 0$. It could also be satisfied if $A - \phi v < 0$.

$$DetJ_1^{**}(\dot{z}, \dot{\ell}) = \frac{z^{**}\gamma_u v(p_d^{**}/\ell^{**} + \alpha\gamma)}{A} \quad (A3)$$

$$DetJ_2^{**}(\dot{z}, \dot{g}) = \frac{g_Z^{**}z^{**}\gamma_u v}{A} \quad (A4)$$

$$DetJ_3^{**}(\dot{z}, \dot{\gamma}) = \frac{z^{**}\lambda v(\gamma_u + \phi)}{A} \quad (A5)$$

$$DetJ_4^{**}(\dot{\phi}, \dot{g}) = \frac{g_z^{**}(\mathcal{P}_d^{**}A/\mathcal{B}^{**} + \alpha r B)}{A} \quad (A6)$$

$$DetJ_5^{**}(\dot{\phi}, \dot{\gamma}) = \frac{\lambda[(A - \phi v)\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r(B + \mathcal{B}^{**}\phi v)]}{A} \quad (A7)$$

$$DetJ_6^{**}(\dot{g}, \dot{\gamma}) = \frac{g_z^{**}\lambda(A - \phi v)}{A} \quad (A8)$$

$$\alpha_2 = \frac{z^{**}\gamma_u v(\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r + g_z^{**}) + g_z^{**}(\mathcal{P}_d^{**}A/\mathcal{B}^{**} + \alpha r B) + \lambda\theta_1}{A} > 0 \quad (A9)$$

$$\theta_1 = z^{**}v(\gamma_u + \phi) + (\mathcal{P}_d^{**}/\mathcal{B}^{**} + g_z^{**})(A - \phi v) + \alpha r(B + \mathcal{B}^{**}\phi v)$$

The 3rd HR condition requires that the sum of the four 3rd-order principal minors be negative. The requirement for $\alpha_3 > 0$ will always be met if $A - \phi v > 0$. It could also be satisfied if $A - \phi v < 0$.

$$DetJ_A^{**}(\dot{z}, \dot{\phi}, \dot{g}) = \frac{-g_z^{**}z^{**}\gamma_u v(\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r)}{A} \quad (A10)$$

$$DetJ_B^{**}(\dot{z}, \dot{\phi}, \dot{\gamma}) = \frac{-z^{**}\lambda v(\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r)(\gamma_u + \phi)}{A} \quad (A11)$$

$$DetJ_C^{**}(\dot{z}, \dot{g}, \dot{\gamma}) = \frac{-g_z^{**}z^{**}\lambda v(\gamma_u + \phi)}{A} \quad (A12)$$

$$DetJ_D^{**}(\dot{\phi}, \dot{g}, \dot{\gamma}) = \frac{-g_z^{**}\lambda[(A - \phi v)\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r(B + \mathcal{B}^{**}\phi v)]}{A} \quad (A13)$$

$$\alpha_3 = \frac{g_z^{**}z^{**}\gamma_u v(\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r) + \lambda\theta_2}{A} > 0 \quad (A14)$$

$$\theta_2 = z^{**}v(\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r + g_z^{**})(\gamma_u + \phi) + g_z^{**}[(A - \phi v)\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r(B + \mathcal{B}^{**}\phi v)]$$

The 4th HR condition specifies that the determinant of the Jacobian matrix must be positive:

$$\alpha_4 = \frac{g_z^{**}z^{**}\lambda\gamma_u v[(A - \phi v)\mathcal{P}_d^{**}/\mathcal{B}^{**} + \alpha r(B + \mathcal{B}^{**}\phi v)]}{A^2} > 0 \quad (A15)$$

The determinant will always be positive if $A - \phi v > 0$ or if $A - \phi v < 0$ and $\mathcal{P}_d^{**}/\mathcal{B}^{**} < \alpha r\mathcal{B}^{**}$. Otherwise, if $A - \phi v < 0$ and $\mathcal{P}_d^{**}/\mathcal{B}^{**} > \alpha r\mathcal{B}^{**}$, then if $\phi < (\mathcal{P}_d^{**}A/\mathcal{B}^{**} + \alpha r B)/v(\mathcal{P}_d^{**}/\mathcal{B}^{**} - \alpha r\mathcal{B}^{**})$.

The 5th HR condition will be approached by obtaining an expression for $(\alpha_1\alpha_2 - \alpha_3)$.

$$\begin{aligned} \alpha_1\alpha_2 - \alpha_3 &= (\alpha_1 - g_z^{**})(\alpha_2 + g_z^{**2}) - DetJ_B^{**} \\ \Rightarrow g_z^{**2}(\alpha_1 - g_z^{**}) + \alpha_2 &\left[\frac{z^{**}\gamma_u v + \lambda(A - \phi v)}{A} \right] + \frac{z^{**}\alpha r\lambda v(\gamma_u + \phi)(B - A)}{A^2} \\ &+ \left(\frac{\mathcal{P}_d^{**}A/\mathcal{B}^{**} + \alpha r B}{A} \right) (DetJ_1^{**} + DetJ_2^{**} + DetJ_4^{**} + DetJ_5^{**} + DetJ_6^{**}) \end{aligned} \quad (A16)$$

The term $(\alpha_1\alpha_2 - \alpha_3)$ will always be positive if $A - \phi v > 0$. It would require lengthy and difficult to interpret algebra to show the upper limit that the ϕ parameter could take for $(\alpha_1\alpha_2 - \alpha_3) > 0$ in the case where $A - \phi v < 0$. Here we note that it is possible for $(\alpha_1\alpha_2 - \alpha_3) > 0$ even when $A - \phi v < 0$.

Now, after having established that under some conditions $(\alpha_1\alpha_2 - \alpha_3) > 0$, we observe that the term $\alpha_1^2\alpha_4$ in $\alpha_5 = \alpha_3(\alpha_1\alpha_2 - \alpha_3) - \alpha_1^2\alpha_4$ depends positively and linearly on the value of the λ parameter through the determinant α_4 . If we set $\lambda = 0$, then $\alpha_1^2\alpha_4 = 0$, while the term $\alpha_3(\alpha_1\alpha_2 - \alpha_3) > 0$ and so too therefore $\alpha_5 > 0$. The term $\alpha_3(\alpha_1\alpha_2 - \alpha_3)$ would be positive because α_3 and $(\alpha_1\alpha_2 - \alpha_3)$ both contain positive terms that are independent of λ . A sufficiently low value for λ could ensure $\alpha_5 > 0$.

Appendix B: Extended Model

Appendix B presents stability analysis for the extended model. Equations (33a), (35a), (37a) and (51a), with u^* given by equation (27a), form a 4×4 dynamic system that has the following Jacobian matrix:

$$J^{**}(\dot{z}, \dot{\ell}, \dot{g}, \dot{e}) = \begin{bmatrix} \frac{-z^{**}v(\gamma_u + \mu)}{A} & \frac{-z^{**}\alpha r v(\gamma_u + \mu)}{A} & 0 & \frac{z^{**}\psi_{\hat{p}}\Omega_e v(\gamma_u + \mu)}{A} \\ \frac{-(B - A)}{A} & -\left(\frac{p_d^{**}}{\ell^{**}} + \frac{\alpha r B}{A}\right) & 0 & \frac{\psi_{\hat{p}}\Omega_e(A\ell^{**} + B)}{A} \\ \frac{-g^{**}C}{A} & \frac{-g^{**}\alpha r C}{A} & -g^{**}(r_U^{**} + \hat{p}^{**}) & g^{**}\left[\Psi_e(1 - \psi_{\hat{p}}) - \frac{\psi_{\hat{p}}\Omega_e C}{A}\right] \\ \frac{e^{**}vD(\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)}{A} & \frac{e^{**}\alpha r vD(\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)}{A} & 0 & -e^{**}E \end{bmatrix} \quad (B1)$$

$$A = \pi + \varphi(1 - \alpha)(1 - \pi) - v(\gamma_u + \mu - \psi_{\hat{p}}\Psi_u)$$

$$B = \ell^{**}v(\gamma_u + \mu - \psi_{\hat{p}}\Psi_u) + \vartheta$$

$$C = g^{**}(\pi - \vartheta) - v[\gamma_u + \mu + \Psi_u(1 - \psi_{\hat{p}})]$$

$$D = 1 - \Upsilon_k - \Lambda_k$$

$$E = \frac{\psi_{\hat{p}}\Omega_e D[\pi + \varphi(1 - \alpha)(1 - \pi)]}{A} + \Upsilon_e + \Lambda_e$$

As noted in Appendix A, the term $g^{**}(r_U^{**} + \hat{p}^{**})$ is equal to the nominal growth rate of pure government expenditures g_z^{**} . The HR stability conditions for a four-dimensional system are also listed in Appendix A. The 1st HR condition requires that the trace of the Jacobian matrix be negative. As we assume $z^{**} > 0$, $p_d^{**} > 0$, $\ell^{**} > 0$, $g_z^{**} > 0$, $e^{**} > 0$ and $A > 0$, this condition will always be satisfied.

$$\alpha_1 = \frac{z^{**}v(\gamma_u + \mu) + \alpha r B}{A} + p_d^{**}/\ell^{**} + g_z^{**} + e^{**}E > 0 \quad (B2)$$

The 2nd HR condition requires that the sum of the six 2nd-order principal minors be positive.

$$DetJ_1^{**}(\dot{z}, \dot{\ell}) = \frac{z^{**}v(p_d^{**}/\ell^{**} + \alpha r)(\gamma_u + \mu)}{A} \quad (B3)$$

$$DetJ_2^{**}(\dot{z}, \dot{g}) = \frac{g_z^{**}z^{**}v(\gamma_u + \mu)}{A} \quad (B4)$$

$$DetJ_3^{**}(\dot{z}, \dot{e}) = \frac{z^{**}e^{**}v(\Upsilon_e + \Lambda_e + \psi_{\hat{p}}\Omega_e D)(\gamma_u + \mu)}{A} \quad (B5)$$

$$DetJ_4^{**}(\dot{\ell}, \dot{g}) = \frac{g_z^{**}(p_d^{**}A/\ell^{**} + \alpha r B)}{A} \quad (B6)$$

$$DetJ_5^{**}(\dot{\ell}, \dot{e}) = \frac{e^{**}(p_d^{**}AE/\ell^{**} + \alpha r[B(\Upsilon_e + \Lambda_e) + \vartheta\psi_{\hat{p}}\Omega_e D])}{A} \quad (B7)$$

$$DetJ_6^{**}(\dot{g}, \dot{e}) = g_z^{**}e^{**}E \quad (B8)$$

$$\alpha_2 = \frac{z^{**}v(g_z^{**} + p_d^{**}/\ell^{**} + \alpha r)(\gamma_u + \mu) + g_z^{**}(p_d^{**}A/\ell^{**} + \alpha r B) + e^{**}\Gamma_1}{A} > 0 \quad (B9)$$

$$\Gamma_1 = [z^{**}v(\gamma_u + \mu) + \alpha r B](\Upsilon_e + \Lambda_e) + \psi_{\hat{p}}\Omega_e D[z^{**}v(\gamma_u + \mu) + \alpha r \vartheta] + AE(g_z^{**} + p_d^{**}/\ell^{**})$$

The 3rd HR condition requires that the sum of the four 3rd-order principal minors be negative.

$$DetJ_A^{**}(\dot{z}, \dot{\ell}, \dot{g}) = \frac{-g_z^{**}z^{**}v(p_d^{**}/\ell^{**} + \alpha r)(\gamma_u + \mu)}{A} \quad (B10)$$

$$DetJ_B^{**}(\dot{z}, \dot{\ell}, \dot{e}) = \frac{-z^{**}e^{**}v(p_d^{**}/\ell^{**} + \alpha r)(\Upsilon_e + \Lambda_e + \psi_{\hat{p}}\Omega_e D)(\gamma_u + \mu)}{A} \quad (B11)$$

$$DetJ_C^{**}(\dot{z}, \dot{g}, \dot{e}) = \frac{-g_Z^{**} z^{**} e^{**} v(Y_e + \Lambda_e + \psi_{\hat{p}} \Omega_e D)(\gamma_u + \mu)}{A} \quad (B12)$$

$$DetJ_D^{**}(\dot{b}, \dot{g}, \dot{e}) = \frac{-g_Z^{**} e^{**} \langle p_d^{**} A E / \ell^{**} + \alpha r [B(Y_e + \Lambda_e) + \vartheta \psi_{\hat{p}} \Omega_e D] \rangle}{A} \quad (B13)$$

$$\alpha_3 = \frac{g_Z^{**} z^{**} v(p_d^{**} / \ell^{**} + \alpha r)(\gamma_u + \mu) + e^{**} \Gamma_2}{A} > 0 \quad (B14)$$

$$\Gamma_2 = z^{**} v(g_Z^{**} + p_d^{**} / \ell^{**} + \alpha r)(Y_e + \Lambda_e + \psi_{\hat{p}} \Omega_e D)(\gamma_u + \mu) + g_Z^{**} DetJ_5^{**} A$$

The 4th HR condition requires that the determinant of the Jacobian matrix be positive.

$$\alpha_4 = \frac{g_Z^{**} z^{**} e^{**} v(\gamma_u + \mu) \langle p_d^{**} A E / \ell^{**} + \alpha r [B(Y_e + \Lambda_e) + \vartheta \psi_{\hat{p}} \Omega_e D] \rangle}{A^2} > 0 \quad (B15)$$

It is clear that $\alpha_2 > 0$, $\alpha_3 > 0$ and $\alpha_4 > 0$ will always be satisfied if z^{**} , p_d^{**} , ℓ^{**} , g_Z^{**} , e^{**} and A are positive. For the 5th HR condition, $\alpha_5 = \alpha_3(\alpha_1 \alpha_2 - \alpha_3) - \alpha_1^2 \alpha_4 > 0$, we begin with:

$$\begin{aligned} \alpha_1 \alpha_2 - \alpha_3 &= (\alpha_1 - g_Z^{**})(\alpha_2 + g_Z^{**2}) - DetJ_B^{**} > 0 \quad (B16) \\ \Rightarrow g_Z^{**2}(\alpha_1 - g_Z^{**}) + \alpha_2 \left[\frac{z^{**} v(\gamma_u + \mu)}{A} + e^{**} E \right] &+ \frac{z^{**} e^{**} \alpha r v(Y_e + \Lambda_e + \psi_{\hat{p}} \Omega_e D)(\gamma_u + \mu)(B - A)}{A} \\ &+ \left(\frac{p_d^{**} A / \ell^{**} + \alpha r B}{A} \right) (DetJ_1^{**} + DetJ_2^{**} + DetJ_4^{**} + DetJ_5^{**} + DetJ_6^{**}) \end{aligned}$$

With our assumptions the term $(\alpha_1 \alpha_2 - \alpha_3)$ is necessarily positive. Next we observe that α_4 depends positively and linearly on $(\gamma_u + \mu)$. If $(\gamma_u + \mu) = 0$, then $\alpha_1^2 \alpha_4 = 0$, while $\alpha_3(\alpha_1 \alpha_2 - \alpha_3) > 0$ because α_3 and $(\alpha_1 \alpha_2 - \alpha_3)$ both contain positive terms that are independent of the value assigned to the $(\gamma_u + \mu)$ parameters.²⁵ A sufficiently low value for $(\gamma_u + \mu)$ could ensure $\alpha_5 > 0$.

²⁵ In respect to α_3 note that $DetJ_D^{**} = g_Z^{**} DetJ_5^{**}$ contains terms that are completely independent of the value assigned to the $(\gamma_u + \mu)$ parameters as well as terms that are negatively affected by those parameters:

$$DetJ_D^{**} = g_Z^{**} e^{**} \left\{ \frac{p_d^{**} (Y_e + \Lambda_e)}{\ell^{**}} + \frac{\alpha r B(Y_e + \Lambda_e) + \psi_{\hat{p}} \Omega_e D \langle [\pi + \varphi(1 - \alpha)(1 - \pi)] p_d^{**} / \ell^{**} + \alpha r \vartheta \rangle}{\pi + \varphi(1 - \alpha)(1 - \pi) - v(\gamma_u + \mu - \psi_{\hat{p}} \Psi_u)} \right\}$$

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Macroeconomic Policy Institute (IMK) of Hans-Böckler-Foundation, Georg-Glock-Str. 18,
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